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# Modelling the dynamics of stock market in the gulf cooperation council countries: evidence on persistence to shocks

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## Abstract

This study examines the statistical properties required to model the dynamics of both the returns and volatility series of the daily stock market returns in six Gulf Cooperation Council countries, namely Bahrain, Oman, Kuwait, Qatar, Saudi Arabia, and the United Arab Emirates, under different financial and economic circumstances. The empirical investigation is conducted using daily data from June 1, 2005 to July 1, 2019. The analysis is conducted using a set of double long-memory specifications with some significant features such as long-range dependencies, asymmetries in conditional variances, non-linearity, and multiple seasonality or time-varying correlations. Our study indicates that the joint dual long-memory process can adequately estimate long-memory dynamics in returns and volatility. The in-sample diagnostic tests as well as out-of-sample forecasting results demonstrate the prevalence of the Autoregressive Fractionally Integrated Moving Average and Hyperbolic Asymmetric Power Autoregressive Conditional Heteroskedasticity modeling process over other competing models in fitting the first and the second conditional moments of the market returns. Moreover, the empirical results show that the proposed model offers an interesting framework to describe the long-range dependence in returns and seasonal persistence to shocks in conditional volatility and strongly support the estimation of dynamic returns that allow for time-varying correlations. A noteworthy finding is that the long-memory dependencies in the conditional variance processes of stock market returns appear important, asymmetric, and differ in their volatility responses to unexpected shocks. Our evidence suggests that these markets are not completely efficient in processing regional news, thus providing a sound alternative for regional portfolio diversification.

**Keywords:** Long-memory, Volatility process, Asymmetric power, Seasonality, Forecast performance, Stock market

**JEL Classification:** G12, F31, C32

## Introduction and literature review

Financial theory stipulates that increased stock market movements lower the benefits of internationally diversified portfolios and, thus, cause domestic capital markets to be more vulnerable to external shocks. The literature measuring stock market movements has rapidly gained ground; the necessity of searching for reliable measures of

movements has prompted researchers to use alternative approaches to measure movements in international stock markets and to evaluate the persistence structure among the stock markets using a technique to consider the problem of stylized facts. In this regard, predicting the stock market is not an impertinent task. On the contrary, it is one of the most challenging applications in economics and finance. An ideal pattern in economics and finance, the efficient market hypothesis (EMH) (Fama 1965) or the random walk behavior, does not support stock market predictability.

Nevertheless, as per the EMH implications, security prices or returns move randomly and indicate market participants' rationality. Moreover, most of the empirical studies conducted in this stream of literature have examined the presence of serial dependence and persistence volatility in financial time series, which have raised questions on the validity of the EMH. In reality, volatility and dynamics of persistence effect could be explained by the tail dependence of the underlying assets, which exhibits extreme events simultaneously. Fluctuation, volatility, the dynamics of persistence, and tail dependence are interrelated in the analysis of the dependence structure of international equity markets. Further, the correlations between consecutive returns decay slowly, that is, long-range dependence in returns is exhibited. Therefore, it is necessary to test for long-range persistence before attempting to analyze the financial markets.

The long-range dependence phenomenon has raised a challenging problem in financial time series analysis, which has been the subject of extensive theoretical and empirical investigation over the past few decades. The presence of long-memory components in the generating mechanism of stock market returns is a key issue with important implications for risk management, the econometric modeling and forecasting of asset prices, portfolio allocation strategy, and testing for market efficiency hypothesis (see Al-Shboul and Alsharari 2019). Additionally, the long-memory property describes the high-order correlation structure of a given time series (Hosking 1984). There is persistent temporal dependence even among distant observations if a series exhibits long memory. Such series are characterized by a slowly decaying autocovariance function and an unbounded spectral density function at the null frequency. Thus, long-memory models are very effective tools that can be used as diagnostic tests to explore, analyze, and understand the nature of the underlying dynamics in stock returns.

Several literature reviews have been concentrated on analyzing the characteristics of assets and the long-range dependence in financial data. Usually, this modeling of asset returns is adopted in univariate and multivariate contexts, and financial modelers are confronted with the task of measuring the dependencies within or between asset returns. Many studies have confirmed that the dependence structure within and among a set of series will vary substantially, ranging from independence to complex forms of non-linear dependence that would cover all these features (Kang and Yoon 2007; Adnan and Erdost 2007). To account for this typical behavior, Granger and Joyeux (1980) and Hosking (1981) introduced the Autoregressive Fractionally Integrated Moving Average (ARFIMA) process. Beran (1994) and Guégan (1994) conducted a review of the statistical properties of this flexible class of processes. The ARFIMA model succinctly captures the long-term dependence pattern by allowing the integration order of the conventional Autoregressive integrated moving average (ARIMA) models to take non-integer (i.e., fractional) values.

Apart from the conditional mean of a time series, long-range dependence effects in the volatility process have also been widely investigated. Taylor (1986), Lobato and Savin (1988), Ding et al. (1993), Crato and Ray (2000), Ling et al. (2021), and Liang et al. (2021), among others, found strong empirical evidence for long (hyperbolic)-memory in the squared and absolute values of asset returns that are used as proxies for unobserved volatility. Baillie et al. (1996a, 1996b) proposed the Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) model, which combines high temporal dependencies in second conditional moments with the virtues of a parsimonious parameterization. Since then, the intuitive concept of a fractional unit root being present in the variance equation has been extended to other Generalized Autoregressive Conditional Heteroskedasticity (GARCH) type specifications resulting in a collection of long-memory adaptations such as the Fractionally Integrated Exponential GARCH (FIEGARCH) of Bollerslev and Mikkelsen (1996), Fractionally Integrated Asymmetric Power Autoregressive Conditional Heteroskedasticity (FIAPARCH) of Tse (1998), and Hyperbolic GARCH (HYGARCH) of Davidson (2004).

Recently, empirical research has focused on long-memory dynamics in the context of the conditional mean or conditional variance of financial time series. In a pioneering work, Teyssière (1997) introduced the double long-memory ARFIMA-FIGARCH model, which generates long-range dependencies in the first and second conditional moments. The hybrid specification is more than a simple juxtaposition of two fractional processes, that is, the joint estimation of the ARFIMA and FIGARCH components in the mean and variance equations; it proves to be crucial for estimation and forecasting issues. Teyssière (1997) employed two dual long-memory specifications, namely ARFIMA-FIGARCH and ARFIMA-FIEGARCH, to model the absolute returns on Treasury bill futures. Applications to high-frequency exchange rate returns have also been proposed by Teyssière (1998) and Beine et al. (2002). Baillie et al. (2002) showed that the ARFIMA-FIGARCH model is useful for describing monthly CPI inflation rates across several industrialized countries. Karanasos et al. (2006) investigated the integration properties of monthly US real interest rate and its uncertainty using an ARFIMA-FIAPARCH process.

Nevertheless, these models are not fully efficient in modeling the volatility of financial time series. The main feature of such data is the strong evidence of cyclical patterns in volatility. Empirical evidence highlights the importance of modeling the periodic dynamics of volatility. To fulfill this aim, Bordignon et al. (2007, 2009) suggested a new category of GARCH models characterized by periodic long-memory behavior. This category of models introduces Gegenbauer polynomials into the equation of the standard GARCH model, which are considered as the generalized periodic long-memory filters to estimate time-varying volatility. These processes are termed as periodic long-memory GARCH (PLM-GARCH) and generalized long-memory GARCH (G-GARCH). In the literature, G-GARCH models are applied to estimate the financial time series, for example, the estimation of the exchange rate using Monte Carlo simulations (Bordignon et al. 2007; Caporin and Lisi 2007).

Several empirical studies have been developed to examine the linkages among stock market returns using various econometric approaches. Previous studies relied on linear time series models to study short-term dynamics while other studies adopted multivariate techniques to test for a stable long-term relationship among stock

market indices. An important precondition for validating the linear models is the stability of the models and the invariability of the parameters over time. In particular, our approach considers that the processes are fractionally integrated and sufficiently powerful to distinguish between short-range dependence and long-range dependence, thus providing more robust results than conventional modeling methodology. Hence, it is essential to specify the model to obtain meaningful forecasts when studying the financial market.

Many investigations have been conducted on the use of forecast models and their ability to generate a better forecast. Particularly, forecasting financial assets and their volatility has considerably drawn researchers' attention. Predictive abilities and the forecasting performances of models are the fundamentals of risk management and investment analysis (Poon and Granger 2005). Specifically, this study extends Yalama and Celik (2013), Duppati et al. (2017), Borup and Jakobsen (2019), Ma et al. (2019), Duftinena and Pynnönen (2020), Kaya Soylu et al. (2020), Abuzayed and Al-Fayoumi (2021), and Dum et al. (2021) by considering the class of long-memory models that has the advantage of capturing, in addition to time-varying volatility and asymmetric, leverage effect, and long-term seasonal component. Consequently, we contribute to the study by selecting a model that integrated stylized facts to reproduce the inherent characteristics of stock markets series. Additionally, this study contributes to the existing studies by comparing the volatility forecasting models' performances. Moreover, it examines the underlying features of some GARCH-type models to assess their predictive performance at different horizons.

This study aims to assess the statistical properties and volatility series of the daily stock market returns. We evaluate the performance of various dual long-memory processes in detecting several significant features such as long-range dependencies, asymmetries, non-linearity, and multiple seasonality or time-varying correlations. We include numerous stylized facts in the modeling approach to check the performance of these new models in reproducing and identifying characteristic features of stock market indexes. Further, most empirical studies were devoted to the developed markets; this study contributes to the limited literature on the emerging stock markets in general and the Gulf Cooperation Council (GCC) markets in particular. Moreover, the findings of this study would contribute to future studies on GCC stock markets by adding further insights into the inherent dynamics of these markets.

This study investigates the presence of fractional dynamics in the returns and volatilities of six GCC stock market indexes, which include Bahrain, Oman, Kuwait, Qatar, Saudi Arabia, and the United Arab Emirates (UAE). Accordingly, a set of dual long-memory models reproducing an assortment of stylized features is used to fit the dynamic structure of the analyzed series. This study aims to evaluate the forecast performance of various GARCH-type models at different horizons forecasting over the horizons ranging from 5 days, 10 days, and 15 days. Further, it studies the intrinsic characteristics that drive the forecasting performance of these GARCH models and their predictive abilities. The originality of this study lies in the following. First, we use the most recent sample period that allows us to account for different financial and economic circumstances. Second, we establish the new development of long-memory models in the analysis to examine the efficiency of GCC markets.

The remainder of the paper is organized as follows. “[Competing dual long-memory models](#)” section summarizes the basic features of the dual long-memory specifications used in this study. “[Statistical properties of the stock market return](#)” section describes the datasets and emphasizes on their statistical properties. “[Results](#)” section presents the empirical results, models’ estimates, and a comparative study of the out-of-sample forecasting performances of the selected models. “[Conclusions](#)” section provides the summary and concluding remarks.

### Competing dual long-memory models

This paragraph presents a collection of dual long-memory specifications that have been widely discussed in the literature and emphasizes some of their salient features. We successively consider the ARFIMA-FIGARCH, ARFIMA-HYGARCH, ARFIMA-FIAPARCH, ARFIMA-HYAPARCH, and ARFIMA-G-GARCH models.

#### The ARFIMA-FIGARCH model

The double long-memory ARFIMA-FIGARCH process models the analyzed series by inserting fractional filters in both the mean and variance equations. For the conditional mean, we fit an ARFIMA( $p, d, q$ ) specification considered by Granger and Joyeux (1980) and Hosking (1981) defined as follows:

$$\theta(L)(1-L)^{d_m}(r_t - \mu) = \phi(L)\varepsilon_t, \quad \varepsilon_t | \Psi_{t-1} \sim \mathcal{D}(0, h_t), \quad (1)$$

where  $r_t$  is the stock market return series,  $\mu$  is the mean of the series, and  $d_m$  is a fractional number;  $\theta(L) = 1 - \theta_1 L - \dots - \theta_p L^p$  and  $\phi(L) = 1 + \phi_1 L + \dots + \phi_q L^q$  are the AR and MA polynomials in the lag operator of the respective orders  $p$  and  $q$  (with all roots lying outside the unit circle), which constitute the short-memory parameters and affect only the short-run dynamics of the process, while the fractional integration parameter  $d_m$  detects the long-memory behavior of the process. Various cases are possible: if  $-0.5 < d_m < 0$ , the process is anti-persistent memory; if  $0 < d_m < 0.5$ , the process is stationary long-memory and possesses shocks that disappear hyperbolically; and if  $0.5 \leq d_m < 1$ , the process is non-stationary but mean-reverting with finite impulse response weights. When  $d_m = 0$ , the process reduces to the standard ARMA; when  $d_m = 1$ , the process becomes ARIMA and implies infinite persistence of the mean to a shock in the returns.  $\Psi_{t-1}$  stands for the information set available at time  $t - 1$ , whereas the residuals are assumed to follow the conditional distribution ( $\mathcal{D}$ ).

In the variance equation, we retain a FIGARCH( $P, \delta, Q$ )-type adaptation,

$$h_t = \omega + \left\{ 1 - (1 - \beta(L))^{-1} \varpi(L)(1-L)^{d_v} \right\} \varepsilon_t^2, \quad (2)$$

where  $h_t$  is the conditional variance of  $r_t$ ,  $\omega$  is the mean of the process,  $d_v$  is the fractional degree of integration of  $h_t$ , and  $\beta(L)$  and  $\varpi(L)$  are lag polynomials of respective orders  $P$  and  $Q$ . An interesting feature of the FIGARCH model is that it nests both the GARCH model (Bollerslev 1986) for  $d_v = 0$  and the IGARCH specification (Engle and Bollerslev 1986) for  $d_v = 1$ . In the first case, shocks to the conditional variance decay at an exponential rate with the lag length, whereas in the second case, shocks remain important for all forecast horizons, thus revealing infinite persistence behavior. If  $0 < d_v < 1$ , there is a

long-term dependence in the conditional variance indicated by a hyperbolic decay of the autocorrelation and autocovariance functions.

It is noteworthy that FIGARCH-type processes, although strictly stationary and ergodic for  $d_v \in [0, 1]$ , are not covariance stationary. Furthermore, the interpretation of the long-memory parameter  $\delta$  is challenging in the FIGARCH setup (see Davidson 2004 for additional details).

#### The ARFIMA-HYGARCH model

To cope with the deficiencies inherent to the FIGARCH framework, Davidson (2004) introduced a more general class of long-memory GARCH processes called hyperbolic GARCH (HYGARCH). These processes allow for a faster non-geometric (hyperbolic) decay rate for which covariance stationarity would still be achievable.

The ARFIMA-HYGARCH model adopts Eq. (1) in the first conditional moment, whereas the conditional volatility is modeled using a HYGARCH parameterization given formally by

$$h_t = \omega + \left\{ 1 - (1 - \beta(L))^{-1} \varpi(L) \left[ 1 + \alpha \left( (1 - L)^{d_v} - 1 \right) \right] \right\} \varepsilon_t^2. \quad (3)$$

Parameters  $\alpha$  and  $d_v$  are assumed to be non-negative. Under the condition  $\alpha < 1$ , if the GARCH component obeys the usual covariance stationarity restrictions (Bollerslev 1986), then the resulting stochastic process is weakly stationary (see Davidson 2004 for further details).

The HYGARCH model nests the FIGARCH process for  $\alpha = 1$  and the stable GARCH process for  $\alpha = 0$ . It is notable that in the latter case, the fractional parameter  $d_v$  is unidentified, thus raising a problem in constructing hypotheses tests. Davidson (2004) stated that when  $d_v = 1$ , the parameter  $\alpha$  reduces to an autoregressive root reproducing geometric memory case, that is, GARCH models for  $\alpha < 1$  and integrated GARCH specifications for  $\alpha = 1$ . Hence, testing the restriction  $d_v = 1$  allows discriminating between geometric memory and hyperbolic memory dynamics. In this case, the GARCH or IGARCH type specifications will correspond to  $\alpha < 1$  and  $\alpha = 1$ , respectively.

#### The ARFIMA-FIAPARCH model

It should be noticed that the models FIGARCH and HYGARCH disregard an important stylized fact inherent to financial markets: the “leverage effect” (Black 1976), which corresponds to negative correlations between past returns and future volatility. To consider asymmetric volatility responses to positive and negative shocks and volatility persistence behavior, Tse (1998) extended the Asymmetric Power GARCH model of Ding et al. (1993) by incorporating a fractional filter in the conditional variance equation. The obtained model is known as FIAPARCH.

In the following, we present the ARFIMA-FIAPARCH model, which generates long-memory properties in both the first and (power transformed) second conditional moments. In this dual long-memory framework, the conditional mean is fitted by an ARFIMA-type adaptation (Eq. (1)), whereas the conditional variance equation is expressed as a powerful transformation of the standard deviation as follows:



$$h_t^{\delta/2} = \omega + \left\{ 1 - (1 - \beta(L))^{-1} \varpi(L)(1 - L)^{d_v} \right\} (|\varepsilon_t| - \gamma \varepsilon_t)^\delta, \quad (4)$$

where  $-1 < \gamma < 1$  and  $\delta > 0$ . Here, the power term  $\delta$  plays the role of a Box-Cox transformation of the conditional standard deviation  $h_t^{1/2}$ , while  $\gamma$  denotes the asymmetry coefficient accounting for the leverage effect. When  $\gamma > 0$ , negative shocks give rise to higher volatility than positive shocks. The reverse applies if  $\gamma < 0$ ; the magnitude of the shocks is captured by the term  $(|\varepsilon_t| - \gamma \varepsilon_t)$ .

It is noteworthy that using the power term  $\delta$  allows us to go beyond the Gaussian assumption. If the datasets are assumed to follow a conditional normal density, then the first two moments (i.e., the mean and variance) completely typify the distribution of returns. This justifies the common use of a squared term  $\delta = 2$  and, hence, a measure of the variance to characterize the volatility structure. However, since asymmetry and heavy tails are both stylized facts of financial asset return, the normality hypothesis seems unrealistic, and higher-order moments such as skewness and kurtosis are required to specify the true underlying distribution. In such a context, considering the variance as a measure of the volatility process (i.e., setting  $\delta = 2$ ) can adversely affect our models' estimation results and forecasting performances. To overcome this issue, Ding et al. (1993) suggested estimating the volatility measure in the form of a power transformation by allowing an optimal power term  $\delta$  to be endogenized and freely determined from the data. It should be noted that the FIAPARCH process reduces to the FIGARCH process when  $\gamma = 0$  and  $\delta = 2$ .

#### The ARFIMA-HYAPARCH model

The HYGARCH model introduced by Davidson (2004) allows for long memory in the process of conditional volatilities. However, no asymmetry can be described through this model. Furthermore, due to no stationarity, the FIAPARCH process exhibits infinite conditional second moments, and no statements about the autocovariance function can be derived. Therefore, in this study, the extension of FIAPARCH processes to the hyperbolic APARCH (HYAPARCH) process is considered, which can be a representation of the asymmetric conditional volatilities.

$$h_t^{\delta/2} = \omega + \left\{ 1 - (1 - \beta(L))^{-1} \varpi(L) \left[ 1 + \alpha \left( (1 - L)^{d_v} - 1 \right) \right] (1 - L)^{d_v} \right\} (|\varepsilon_t| - \gamma \varepsilon_t)^\delta. \quad (5)$$

The HYAPARCH model corresponds to the HYGARCH model for  $\tau = 0$  and  $\delta = 2$  and to the FIAPARCH model for  $\alpha = 1$ . Compared to the FIGARCH or HYGARCH models, the HYAPARCH model has the advantage of capturing important stylized features such as fat tails and leverage effects that correspond to negative correlations between past returns and future volatility. In sum, the HYAPARCH process couples the flexibility of a varying exponent with the asymmetry coefficient, thus capturing asymmetric volatility structure and letting the data determine the power of the heteroskedastic equation. Moreover, it is covariance stationarity, and it enhances the long-memory aspect of the conditional volatility via the fractional differencing parameter  $d_v$ .

### The ARFIMA-G-GARCH model

The fundamental idea of this model is to include the generalized long-memory process into the equation describing the evolution of conditional variance in a GARCH framework. This new class of models is called Gegenbauer-GARCH (G-GARCH). Thus, we consider the following ARFIMA process with G-GARCH-type innovations to include the presence of a time-varying conditional variance.

$$y_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t z_t, \quad (6)$$

where  $\mu_t$  is the conditional mean of  $y_t$  modeling using the ARFIMA process and  $\varepsilon_t/I_{t-1} \sim \mathcal{D}(0, \sigma_t^2)$ .  $\sigma_t^2$  is the conditional variance,  $I_{t-1}$  is the information observed up to time  $(t-1)$ ,  $z_t$  is an *i.i.d* random variable with zero mean and unit variance, and  $\mathcal{D}(\cdot)$  is a probability density function.

To specify the dynamics of the conditional variance, the starting point is the dynamics of  $\varepsilon_t^2$ . We assume that  $\varepsilon_t^2$  follows a  $k$ -factor GARMA model, which describes a cyclical pattern of length  $S$ .

$$\left[ (I - L)^{d_{v,0}} (I + L)^{d_{v,k}I(E)} \prod_{i=1}^{k-1} (I - 2\nu_{v,i}L + L^2)^{d_{v,i}} \right] \varpi(L) \varepsilon_t^2 = \omega + [I - \beta(L)] \vartheta_t, \quad (7)$$

$$P_v(L) \psi(L) \varepsilon_t^2 = \omega + [I - \beta(L)] \vartheta_t. \quad (8)$$

Here,  $\varpi(L) = 1 - \sum_{i=1}^Q \varpi_i L^i$  and  $\beta(L) = 1 - \sum_{i=1}^P \beta_i L^i$  are suitable polynomials in the lag operator  $L$ , and  $\vartheta_t = \varepsilon_t^2 - \sigma_t^2$  is a martingale difference;  $d_{v,0} = d_v/2$ ,  $I(E) = 1$  if  $S$  is even, and zero otherwise. Considering this assumption, the corresponding GARCH-type dynamics for conditional variance is given by

$$\sigma_t^2 = \gamma + \beta(L) \sigma_t^2 + \left\{ I - \beta(L) - \left[ (I - L)^{d_{v,0}} (I + L)^{d_{v,k}I(E)} \prod_{i=1}^{k-1} (I - 2\nu_{v,i}L + L^2)^{d_{v,i}} \right] \varpi(L) \right\} \varepsilon_t^2. \quad (9)$$

This implies that in the G-GARCH framework, each frequency has been modeled using a specific long-memory parameter  $d_{v,i}$  (differencing parameter of the conditional variance). When  $d_{v,0} = d_{v,1} = \dots = d_{v,k}$ , all the involved frequencies have the same degree of memory.

Model (9) may present most of the existing GARCH models. For example, the standard GARCH models (included seasonal GARCH (Bollerslev and Hodrick 1992) can be obtained by putting  $d_{v,i} = 0$ ,  $i = 0, 1, \dots, k$ . Similarly, the FIGARCH model is equivalent to  $S = 1$  and  $0 < d_{v,0} < 1$ . Interestingly, generalized long-memory filters, in principle, may be applied to any category of the GARCH structure. Nonetheless, due to the constraints needed for conditional variance positivity, G-GARCH models are not always feasible; for this reason, Bordignon et al. (2007) proposed to model the logarithm of the conditional variances. Therefore, a practical computing solution is to apply the filter to a generalized log-GARCH model. Therefore, consider the following equation:

$$P_v(L) \varpi(L) \left[ \ln(\varepsilon_t^2) - \tau \right] = \gamma + [I - \beta(L)] \vartheta_t. \quad (10)$$



Here,  $P_v(L)$  is the generalized long-memory filter introduced into a GARCH structure,  $\vartheta_t = \ln(\varepsilon_t^2) - \tau - \ln(\sigma_t^2)$  is a martingale difference, and  $\tau = E[(\ln(z_t^2))]$ . The expected  $\tau$  value depends on the distribution of the idiosyncratic shock and ensures that  $\vartheta_t$  is a martingale difference, given that  $\ln(\varepsilon_t^2) = \ln(\sigma_t^2) + \ln(z_t^2)$ , under the Gaussian assumption  $\tau = -1.27$ . The expression for the conditional variance implied by (9) is

$$\ln(\sigma_t^2) = \gamma + \beta(L) \ln(\sigma_t^2) + [I - \beta(L) - P_v(L)\varpi(L)] [\ln(\varepsilon_t^2) - \tau]. \quad (11)$$

Since we are modeling  $\ln(\sigma_t^2)$  instead of  $\sigma_t^2$ , no constraints for variance positivity are necessary. A further approach to bypass the parameter constraints is to adopt EGARCH versions of our model.

## Statistical properties of the stock market return

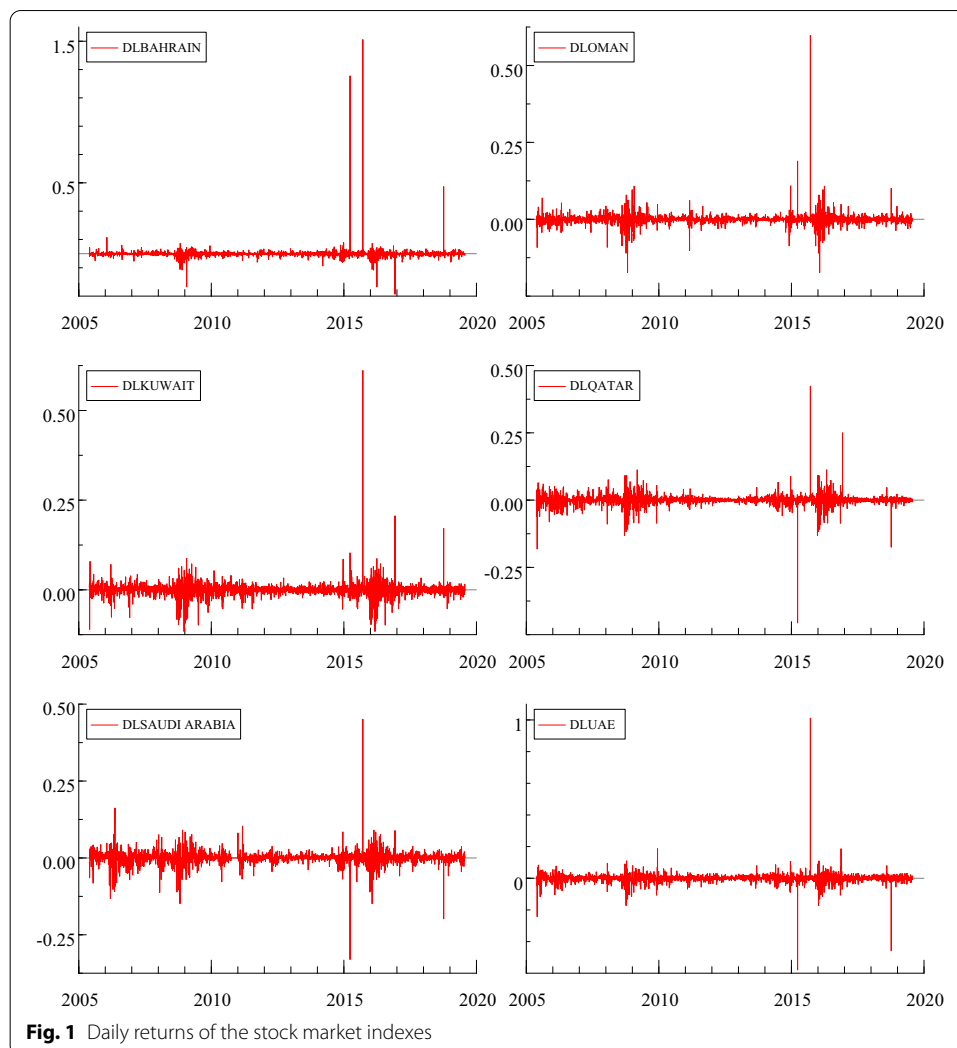
### Data

Our dataset consists of the daily stock market indices of six GCC countries, namely, Bahrain, Kuwait, Oman, Qatar, Saudi Arabia, and the UAE from June 1, 2005 to July 1, 2019, which corresponds to  $T = 3545$  observations. The chosen period allows us to consider the effect of different financial and economic circumstances. These countries are divided into two groups: (1) the Organization of Petroleum Exporting Countries (OPEC), including Kuwait, Qatar, Saudi Arabia, and the UAE and (2) non-OPEC, including Bahrain and Oman. As a proxy for stock markets, we use the major stock market index for each country extracted from Morgan Stanley Capital International. These data are transformed into their logarithmic form and considered in the first difference; so the series obtained correspond to stock market returns. Specifically, we define the stock market returns as  $r_t = \ln x_t - \ln x_{t-1}$ , for  $t = 1, 2, \dots, T$ . Here,  $x_t$  represents the closing stock market index.

Furthermore, the application of standard unit root tests and unit root tests (Dickey and Fuller 1979, 1981; Perron 1988; Phillips and Perron 1988) show evidence of stationarity,<sup>1</sup> which is a standard finding in the literature on such series. At first sight, the stock market indexes appear to have a non-stationary behavior in the sense that they do not converge toward their long-term means and exhibit great instability. The return series illustrated in Fig. 1 seem to fluctuate randomly around zero, while the variance varies over time with the alternation of volatile and tranquil periods. Table 1 contains the descriptive statistics and stochastic properties for each series.

We see that the average stock market returns are slightly negative for all GCC countries. Moreover, we observe that the UAE faces the highest degree of risk as measured by the standard deviation (2.095%), followed by Saudi Arabia (1.840%) and Qatar (1.677%), while Bahrain experiences the lowest risk (1.335%), followed by Oman (1.396%), indicating that the OPEC stock markets are riskier than the non-OPEC stock markets. All series exhibit negative skewness and show excess kurtosis. Therefore, we can say that all indexes show a high asymmetry to the left. The observed asymmetry may indicate the presence of nonlinearities in the evolution process of returns. The

<sup>1</sup> To check the stationarity, we apply the unit root tests without and with structural breaks. We find evidence of stationarity. These results are not reported here but are available upon request.

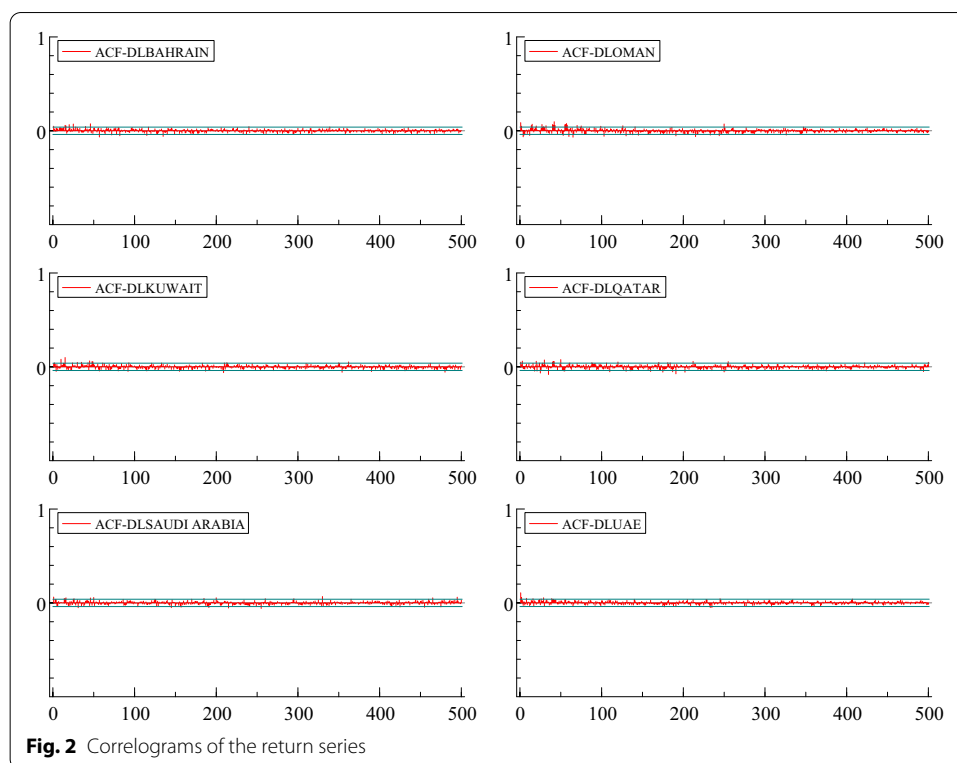
**Table 1** Descriptive statistics of return series

Countries	DLKuwait	DLQatar	DLSaudi Arabia	DLUAE	DLBahrain	DLOman
Mean (%)	−0.012	−0.001	−0.027	−0.048	−0.100	−0.020
Std Dev (%)	1.533	1.677	1.840	2.095	1.335	1.396
Skewness	−1.288	−1.018	−2.108	−0.908	−3.359	−1.612
Kurtosis	17.317	16.958	29.805	15.728	42.223	29.72
JB × 10 <sup>−4</sup>	1.709***	1.607***	5.945***	1.335***	28.686***	5.849***
Q(10)	26.240***	21.057***	31.776***	41.088***	32.782***	34.142***
ARCH (10)	29.556***	21.127***	13.238***	21.095***	12.498***	15.415***

JB is the statistic of Jarque and Bera test for normality, is the statistic of Ljung-Box test for serial correlation for order, and ARCH is the statistic of ARCH test for heteroskedasticity

\*\*\* denote significance at the 1% levels

departure from normality is confirmed by employing the Jarque–Bera test. This test strongly rejects the null hypothesis of normality for all series, which means that the minimum and maximum values have a greater deviation from the calculated mean.



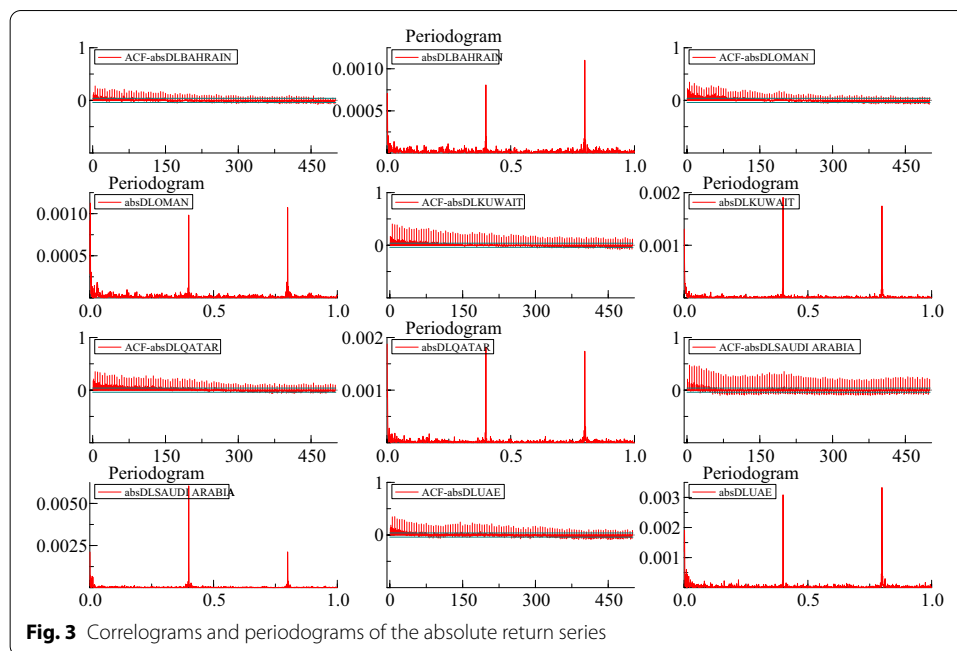
These findings clearly show that the probability of observing extremely negative and positive realizations for our return series is higher than that of a normal distribution. The Ljung-Box test shows significant evidence of the serial correlation for all series, and the Autoregressive Conditional Heteroscedasticity-Lagrange Multiplier (ARCH-LM) test indicates the presence of heteroskedasticity in all series (i.e., a rejection of the null hypothesis at the 1% level). Further, in the presence of heteroskedasticity, we try to identify an ARCH model type (Engle 1982) for each stock market return.

#### Long-range dependencies

The panel of Fig. 2 displays autocorrelation functions for daily returns. As shown in this Figure, the autocorrelation functions of all stock returns show fast decay at first lags but testify a hyperbolic decay for larger lags. Hence, daily returns seem to be autocorrelated. Moreover, the periodograms present a pole at zero frequency. Such features may suggest that the stock market return series display a long-term persistent structure.

Investigating the memory features of the volatility of stock market indices, we see that for the absolute return series,<sup>2</sup> the autocorrelation functions (Fig. 3) die off hyperbolically, whereas the periodograms reveal a strong explosion at the origin of the frequency interval. As shown in Fig. 3, spectral density, traced by the periodogram, shows several peaks at mid frequencies, which proves the presence of much seasonality. We can thus assert the existence of long-run dependencies in the volatility process of the stock market indexes. The findings outlined above suggest fitting long-memory processes in the

<sup>2</sup> We note that the absolute returns are considered as proxy of volatility.



levels and volatilities of the stock market returns, allowing the capturing of the specific dynamic structure of the first and second moments of the datasets.

## Results

This section estimates the following dual long (hyperbolic)-memory processes relying on the Quasi-Maximum Likelihood procedure. The choice of these models can be justified empirically by the analysis of the autocorrelation functions; they show that these functions decrease hyperbolically to zero as the lags increase and the associated spectral densities are unbounded, which may indicate the presence of long-memory behavior in the mean and variance. Additionally, these models constitute the most general class of the dual long-memory models described above and can capture long-range dependence in the mean and the volatility process.

Further, to test whether the process is a true long memory and not a short-memory process with level shifts, we use the test statistics of Perron and Qu (2010). Perron and Qu (2010) developed a simple test based on the log-periodogram estimator proposed by Geweke and Porter-Hudak (1983) and demonstrated that the test statistic follows a Gaussian process under the null hypothesis. They showed how the distribution of this estimator is highly dependent on the number of frequencies used, especially when the data generating process is a stationary short-memory process influenced by structural changes in level. This test is thus helpful to distinguish structural change from long-memory.

Due to the distributional properties of the stock returns, the Student's  $t$  distribution<sup>3</sup> is assumed for the innovations, as suggested by Bollerslev (1987). It is remarkable to note

<sup>3</sup> We have also estimated the models based on the skewed student distribution. Yet, the obtained results emphasize the superiority of the student's  $t$  distribution. For the sake of conciseness, we restrict ourselves to the latter distribution. Complete results are available upon request.

**Table 2** Estimates of ARFIMA-HYAPARCH model for each series

	Kuwait	Qatar	Saudi	UAE	Bahrain	Oman
$(p, d_m, q)$	$(1, d, 1)$	$(1, d, 1)$	$(0, d, 0)$	$(0, d, 1)$	$(0, d, 0)$	$(1, d, 2)$
$(P, d_v, Q)$	$(1, \delta, 1)$	$(1, \delta, 1)$	$(1, \delta, 1)$	$(1, \delta, 1)$	$(1, \delta, 1)$	$(1, \delta, 1)$
$\mu$	0.000 (0.389)	0.000 (0.744)				0.000 (0.572)
$d_m$	0.104 (3.974)***	0.137 (3.085)***	0.117 (4.783)***	0.096 (3.160)***	0.086 (4.670)***	0.129 (3.784)***
$\theta_1$	−0.208 (−1.742)*	−0.234 (−2.162)**				−0.216 (−1.901)**
$\theta_2$						
$\phi_1$	−0.208 (−1.742)*	−0.234 (−2.162)**		−0.216 (−2.127)**		
$\phi_2$	—	—				
$\omega$	0.000 (0.043)	0.001 (0.032)	0.000 (0.033)	0.000 (0.022)	0.001 (0.016)	0.003 (0.641)
$d_v$	0.425 (7.886)***	0.347 (4.551)***	0.348 (6.070)***	0.377 (5.547)***	0.435 (6.872)***	0.369 (8.342)***
$\text{Log}(\alpha)$	0.217 (3.763)***	0.197 (3.205)***	0.174 (3.879)***	0.238 (4.615)***	0.245 (3.982)***	0.245 (4.704)***
$\gamma$	−0.182 (−3.363)***	−0.197 (−3.678)***	−0.234 (−4.674)***	−0.228 (−3.233)***	−0.249 (−3.861)***	−0.145 (−4.389)***
$\delta$	1.972 (11.340)***	2.628 (10.467)***	2.274 (9.892)***	3.157 (12.093)***	2.728 (10.873)***	2.312 (12.520)***
$\beta_1$	0.677 (5.421)***	0.408 (4.521)***	0.546 (4.764)***	0.479 (4.983)***	0.578 (6.093)***	0.648 (6.169)***
$\varpi_1$	0.461 (3.125)***	0.293 (3.938)***	0.293 (3.887)***	0.369 (4.673)***	0.398 (3.659)***	0.301 (4.670)***
$\hat{\nu}$	5.131 (12.811)***	4.178 (13.901)***	5.675 (11.674)***	5.816 (10.632)***	4.755 (10.471)***	4.535 (11.150)***
Skw	0.254 (3.635)***	0.123 (1.768)*	0.124 (3.564)***	0.109 (1.757)*	0.234 (3.698)***	0.084 (1.212)
Ex. Kurt	3.999 (25.646)***	3.628 (25.984)***	3.857 (26.674)***	3.512 (23.982)***	3.853 (22.438)***	3.066 (23.404)***
$Q(20)$	22.559	22.601	22.082	21.672	20.874	18.359
$Q^2(20)$	10.365	11.113	10.874	10.359	10.743	9.105
$BDS(5)$	5.143	4.587	5.327	4.875	3.174	3.987
$\text{Log—}$	1039.039	1048.083	1046.675	1043.855	1047.972	1049.567
Likelihood						
Akaike	−0.072	−0.085	−0.075	−0.088	−0.082	−0.095

The values in parenthesis are the  $t$ -Student.  $\hat{\nu}$  is the degree of freedom of the student's  $t$  distribution. Skw is Skewness. Ex. Kurt is Excess of Kurtosis.  $Q(20)$  is the Ljung-Box statistic for serial correlation in the standardized residuals for order 20.  $Q^2(20)$  is the Ljung-Box statistic for serial correlation in the squared standardized residuals for order 20. \*, \*\*, and \*\*\* denote significance at the 10%, 5% and 1% levels respectively.

that lag order selection is a vital issue when specifying a dynamic model. To identify the truncation orders  $p, q, P$  and  $Q$  of the short-memory polynomials of the double long-memory adaptations, we use the Schwarz and Hannan-Quinn Information criteria.

### Modeling the GCC stock market returns

Estimation results (Tables 2 and 3) show that the fractional parameters in the mean equations of the three dual long-memory specifications are highly significant (at the 1% level). This confirms that the stock market returns exhibit a long-run dependence

**Table 3** Estimates of ARFIMA-G-GARCH model for each series

	Kuwait	Qatar	Saudi	UAE	Bahrain	Oman
$(p, d_m, q)$	$(1, d, 1)$	$(1, d, 1)$	$(0, d, 0)$	$(0, d, 1)$	$(0, d, 0)$	$(1, d, 2)$
$(P, d_v, Q)$	$(1, \delta, 1)$	$(1, \delta, 1)$	$(1, \delta, 1)$	$(1, \delta, 1)$	$(1, \delta, 1)$	$(1, \delta, 1)$
$\mu$						
$d_m$	0.101 (3.968)***	0.134 (3.185)***	0.119 (4.639)***	0.106 (3.148)***	0.089 (4.642)***	0.127 (3.689)***
$\theta_1$	-0.199 (-1.821)*	-0.231 (-2.145)**				-0.214 (-1.887)**
$\theta_2$						
$\phi_1$	-0.204 (-1.745)*	-0.238 (-2.169)**				
$\phi_2$	—	—				
$\omega$	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.00 (0.000)	0.000 (0.000)
$d_v$	0.422 (7.981)***	0.344 (4.673)***	0.352 (6.859)***	0.385 (5.763)***	0.433 (6.834)***	0.349 (8.573)***
$\lambda_v$	0.412 (5.872)***	0.414 (6.583)***	0.407 (5.347)***	0.410 (4.975)***	0.408 (5.352)***	0.403 (5.874)***
$\beta_1$	0.646 (5.674)***	0.412 (4.683)***	0.551 (4.784)***	0.476 (4.836)***	0.582 (6.682)***	0.674 (6.357)***
$\varpi_1$	0.458 (3.247)***	0.291 (3.952)***	0.287 (3.781)***	0.373 (4.836)***	0.392 (3.968)***	0.314 (4.792)***
$\hat{v}$	5.245 (13.451)***	4.137 (13.346)***	5.755 (12.673)***	5.349 (11.453)***	4.457 (10.785)***	4.561 (10.893)***
Skw	0.216 (3.768)***	0.112 (1.775)*	0.101 (3.786)***	0.099 (1.752)*	0.201 (3.584)***	0.075 (1.211)
Ex. Kurt	3.762 (22.345)***	3.638 (24.674)***	3.667 (24.673)***	3.458 (23.729)***	3.367 (20.649)***	3.231 (21.658)***
$Q(20)$	20.767	21.178	20.278	20.562	19.742	18.786
$Q^2(20)$	10.084	10.564	10.237	10.023	10.465	8.872
$BDS(5)$	4.676	4.214	4.773	4.266	3.547	3.371
$\text{Log-}$	1037.985	1046.353	1044.882	1042.773	1046.448	1048.992
Likelihood						
Akaike	-0.069	-0.082	-0.073	-0.085	-0.079	-0.092

See Table 2

structure.<sup>4</sup> Moreover, the retained adaptations yield almost the same value for the fractional differencing parameters  $d_m \in [0.086, 0, 137]$ .

On the other hand, the fractional orders of integration in the second conditional moments are statistically significant at the 1% level, which means that volatility is fractionally integrated. However, it should be stressed here that the three processes do not share the same degree of fractional integration in their scedastic functions. Estimates of the long-memory parameters vary from 0.347 for the HYGARCH adaptation to 0.348 for the FIAPARCH process and 0.362 for the HYAPARCH's case, whereas, interestingly, the FIGARCH model displays the highest persistence degree with a fractional differencing parameter of 0.425.

<sup>4</sup> The results of the Perron-Qu test (significance levels of the 95% intervals) show clearly that all series follow a long-memory process.



Parameters' estimates of the ARFIMA-FIGARCH specification show that the fractional order of integration  $d_v$  in the scedastic function is highly significant and statistically different from unity and zero (at the 1% significance level). This indicates that the impact of shocks to the conditional volatility flaunts a hyperbolic rate of decay instead of the conventional exponential decay inherent to the stable GARCH process or the infinite persistence pattern distinguishing IGARCH type models. The fractional differencing parameter  $d_v$  is highly significantly different from 0 and 1, rejecting the validity of the stable GARCH and the integrated GARCH (IGARCH) specifications.

Focusing on the estimates of the ARFIMA-HYGARCH process, we observe that the hyperbolic memory in variance, measured by the fractional order of integration  $d_v$ , is pronounced. The amplitude parameter  $\alpha$  is statistically different from 1, leading to the rejection of the FIGARCH alternative. A noteworthy feature here is the largest value of the  $\alpha$  parameter which exceeds 1 ( $\alpha > 1$ ). This suggests that the driving process of the stock returns is not covariance stationary.

It should be stressed that within the ARFIMA-HYGARCH framework, we can test for the restriction embodied in the ARFIMA-FIGARCH model, that is  $\alpha = 1$ , relying on a likelihood ratio type test. Formally, the likelihood ratio test is a statistical test used to compare the in-sample performance of nested models. The test statistic is asymptotically  $\chi^2$  distributed with degrees of freedom equal to the number of tested restrictions. If  $\ell_0$  denotes the log-likelihood value under the null hypothesis that the true model is FIGARCH, and  $\ell$  is the log-likelihood under the alternative that the true model is HYGARCH, the test statistic  $LR = 2(\ell - \ell_0)$  should follow a  $\chi^2$  distribution with a degree of freedom one (the number of restrictions) under the null hypothesis. In this case,  $LR$  rejects the constraint implied by the FIGARCH adaptation  $\alpha = 1$  at the 1% significance level ( $\chi^2_{(1)} = 6.634$ ), thus favoring the ARFIMA-HYGARCH specification over the ARFIMA-FIGARCH model.

The analysis of the estimation results of the ARFIMA-FIAPARCH parameterization calls for several observations: the power term  $\hat{\delta}$  is statistically different from two for the FIAPARCH parameterization, whereas the estimated asymmetry coefficient  $\hat{\gamma}$ , although small-valued, is significant and negative; this implies that positive shocks predict higher volatility than negative shocks. In other words, the negative sign of  $\hat{\gamma}$  suggests that “good news,” that is, an unanticipated stock market increase is more destabilizing than “bad news,” that is, an unanticipated stock market drop. A likelihood ratio test can be constructed in which the restricted case is the ARFIMA-FIGARCH specification (i.e.,  $\delta = 2$  and  $\gamma = 0$ ). The test statistic, which is asymptotically  $\chi^2$  distributed with two degrees of freedom (when the null hypothesis is true), yields a value of 21.056 and then rejects the constraints implied by the FIGARCH-type adaptation at the 1% significance level ( $\chi^2_{(2)} = 9.210$ ).

It is worth mentioning that the HYAPARCH specification adapts particularly well to the conditional variance since the Box-Pierce test statistics observed for the squared residuals are smaller than those obtained for the ARFIMA-FIGARCH, ARFIMA-HYGARCH, and ARFIMA-FIAPARCH specifications. Additionally, while assessing the adequacy of the dual long-memory models, we see that the stock market return series under the ARFIMA-FIAPARCH structure displays the highest log-likelihood value and the lowest Akaike Information Criteria among all the competing models. Therefore, the

ARFIMA-HYAPARCH structure prevails over all the competing models in capturing the dynamics that govern the first and second conditional moments of the analyzed series.

Furthermore, the residuals from the ARFIMA process are modeled using the G-GARCH model to estimate the seasonal long-memory behavior in the conditional variance. The spectral densities, represented by the periodogram (Fig. 3), are unbounded at equidistant frequencies, proving seasonality. The estimation results of the ARFIMA-G-GARCH model show special peaks at a frequency of  $\hat{\lambda}_\nu \in [0.403, 0.414]$  ( $\approx 1/2$  week) that correspond to semi-weekly cycles. Moreover, our empirical evidence suggests the same degree of persistence in the volatility of stock markets in the GCC countries.

Hence, from the various diagnostic statistics, the ARFIMA-HYAPARCH process and ARFIMA-G-GARCH adaptation appear to be the most satisfactory representation to describe the long-memory behavior of the stock market index in both its first and (power transformed or seasonal) second conditional moments.

For all countries, the empirical results indicate a significant change in the persistence structure, implying the presence of contagion effect. This long-range dependence may generate positive shocks in the global demand for industrial commodities that cause higher stock market prices or because stock market prices are positively related to the global business. Evidence regarding leverage effects implies that news in stock markets has an asymmetric impact on volatility. Particularly, bad news (negative shocks) gives rise compared to good news (positive shocks).

These results imply that the efficient stock market hypothesis should be rejected in the case of all countries. This result is consistent with Al Janabi et al. (2010) and Bley (2011), who found that the GCC stock markets are inefficient. The evince of persistence implies that stock market returns are predictable over the long term, and investors can anticipate their returns in a sufficiently long-term horizon. These findings are in line with Mimouni and Charfeddine (2016), Alqahtani et al. (2020), and Youssef and Mokni (2018).

### Predictive performances of the dual long-memory processes: a comparative study

In this section, we evaluate the out-of-sample forecasts of the stock market returns. For both the series, we use data until July 1, 2018 for estimation purposes and reserve those from July 2, 2018 for generating out-of-sample prediction values. The forecast horizons considered in this study correspond to five, ten, and fifteen steps in the future ( $s = 5, 10, 15$ ). To evaluate the accuracy of the forecasts, we apply three evaluation criteria: the mean square error, mean absolute error, and logarithmic loss function.

Additionally, we employ the Diebold–Mariano (DM) (1995) test to compare the predictive accuracy of two competing forecasts. The DM test uses a loss function associated with the forecast error of each forecast and tests the null hypothesis that the expected differential loss is zero, that is,  $E(D_t) = 0$ , where the loss differential  $D_t = h(e_{1t}) - h(e_{2t})$ . The two-loss functions are computed as follows:

$$h(e_{1t}) = h(\hat{r}_{1t} - r_{1t}) \quad \text{and} \quad h(e_{2t}) = h(\hat{r}_{2t} - r_{2t}), \quad (12)$$

where  $r_t$  is the actual value of the series and  $\hat{r}_{1t}$  and  $\hat{r}_{2t}$  are two predictions for  $r_t$ , where  $t = 1, 2, \dots, T$ . In most cases, the loss function is a square-error loss function or an absolute-error loss function. The hypotheses of interest are as follows:

$$h(e_{1t}) = h(\hat{r}_{1t} - r_{1t}) \quad \text{and} \quad \begin{cases} H_0 : E(h(e_{1,t-h}) - h(e_{2,t-h})) = 0 \\ H_1 : E(h(e_{1,t-h}) - h(e_{2,t-h})) \neq 0 \end{cases} \quad (13)$$

where  $h \geq 1$  is the forecast horizon.

The DM test has a standard normal limiting distribution under the null hypothesis; the relevant statistic of the test is given by

$$S_1 = \frac{\bar{D}}{\sqrt{\hat{V}(\bar{D})}} \rightarrow N(0, 1) \quad \text{and} \quad \bar{D} = \frac{\sum_{t=1}^n D_t}{n}, \quad (14)$$

where

$$\hat{V}(\bar{D}) = \frac{\hat{\kappa}_0 + 2 \sum_{k=1}^{n-1} \hat{\kappa}_k}{n} \quad \text{and} \quad \hat{\kappa}_k = \frac{\sum_{t=k+1}^n (D_t - \bar{D})(D_{t-k} - \bar{D})}{n}. \quad (15)$$

The DM test requires the loss differential to be covariance stationary. However, it may not be strictly necessary in some cases (Diebold 2015). Moreover, the DM statistic can be obtained by regressing the loss differential on a constant, using Newey-West standard errors. Nevertheless, the DM test is considerably found to be more versatile than the other tests of equal forecast accuracy.

Contrary to the widely used DM test, the Model Confidence Set (MCS), introduced by Hansen et al. (2003), allows for the comparison of multiple forecast models at once. This model selection method is an innovative way to manage the issue of selecting the best forecast models using an out-of-sample evaluation under a specified loss function. As defined in Hansen et al. (2011), an MCS, or  $M^*$ , is a subset of a collection of candidate models,  $M^0$ , consisting of superior forecast models for a given significance level. The set of superior forecast models is formally defined as:

$$M^* = \left\{ i \in M^0 : E(D_{ij,t}) \leq 0 \quad \forall j \in M^0 \right\}, \quad (16)$$

where  $E(D_{ij,t})$  is finite and does not depend on  $t$  for all  $i, j \in M^0$ , and  $D_{ij,t}$  is the loss differential. The purpose of this method is to determine the set of superior models, which can be done via a sequence of significance tests where the models that are found to be significantly inferior to other models of  $M^0$  are eliminated (Hansen et al. 2011). Henceforth, MCS can be viewed as a sequential DM test or the confidence interval of a parameter (Quaedvlieg 2021). Therefore, using a set of forecasting models rather than an individual model is interesting, as no generic model will consistently outperform other models in every conceivable scenario. For a level of significance fixed at each step, this test procedure is built using an algorithm that allows yielding  $p$ -values for all the forecast models under consideration. Hence, this method is different from other model selection criteria that consider a single model and disregard the surprisal of the underlying data.

Table 4 reports the out-of-sample forecast evaluation results of the ARFIMA-FIA-PARCH, ARFIMA-G-GARCH and the ARFIMA-HYAPARCH models using three

**Table 4** Out-of-sample forecasts of the stocks index

	Criterion	Kuwait	Qatar	Saudi	UAE	Bahrain	Oman
$s = 5$							
ARFIMA-FIAPARCH	MSE	0.024	0.017	0.013	0.026	0.023	0.010
	MAPE	0.156	0.125	0.097	0.146	0.131	0.094
	LL	22.035	11.831	11.640	6.638	4.469	9.803
	DM	1.956	1.568	1.672	1.564	1.434	1.726
ARFIMA-G-GARCH	MSE	0.022	0.015	0.012	0.024	0.022	0.009
	MAPE	0.148	0.121	0.094	0.143	0.132	0.086
	LL	20.578	11.348	10.754	5.675	4.258	9.382
	DM	1.898	1.502	1.583	1.497	1.379	1.682
ARFIMA-HYAPARCH	MSE	0.018*	0.015*	0.011*	0.023*	0.021*	0.008*
	MAPE	0.144	0.119	0.087	0.136	0.132	0.079
	LL	19.604*	10.807*	9.865*	5.175*	4.165*	8.657*
	DM	1.832	1.486	1.508	1.386	1.285	1.607
$s = 10$							
ARFIMA-FIAPARCH	MSE	0.028	0.022	0.021	0.018	0.019	0.017
	MAPE	0.149	0.131	0.128	0.123	0.125	0.118
	LL	18.638	11.469	10.584	5.443	4.467	8.326
	DM	2.135	1.793	1.874	1.789	1.768	1.941
ARFIMA-G-GARCH	MSE	0.022	0.017	0.013	0.021	0.018	0.011
	MAPE	0.143	0.128	0.107	0.120	0.122	0.098
	LL	17.982	11.095	10.048	5.175	4.165	7.981
	DM	2.026	1.681	1.759	1.679	1.698	1.882
ARFIMA-HYAPARCH	MSE	0.019*	0.010*	0.008*	0.023*	0.014*	0.007*
	MAPE	0.133	0.098	0.083	0.116	0.114	0.076
	LL*	17.342*	10.255*	9.282*	4.658*	4.025*	6.631*
	DM	1.997	1.605	1.688	1.613	1.594	1.771
$s = 15$							
ARFIMA-FIAPARCH	MSE	0.025	0.023	0.021	0.018	0.019	0.017
	MAPE	0.136	0.131	0.128	0.123	0.125	0.119
	LL	16.175	14.469	9.584	6.443	4.467	5.326
	DM	2.324	1.976	2.158	1.982	1.974	2.207
ARFIMA-G-GARCH	MSE	0.025*	0.016*	0.020*	0.015*	0.018	0.017
	MAPE	0.134	0.097	0.126	0.117	0.123	0.118
	LL	15.872*	11.456*	7.175*	5.855*	3.996	4.864
	DM	2.168	1.895	2.021	1.899	1.849	2.123
ARFIMA-HYAPARCH	MSE	0.022*	0.012*	0.023*	0.015*	0.016*	0.014*
	MAPE	0.131	0.087	0.116	0.102	0.121	0.112
	LL	15.340*	10.153*	5.175*	4.165*	2.467*	2.376*
	DM	2.006	1.823	1.996	1.875	1.7989	2.001

\* denote significance at the 5% levels

evaluation criteria ( $MSE$ ,  $MAPE$ , and  $LL$ )<sup>5</sup> for all series. The DM test uses the classical ARFIMA-FIARCH process as a benchmark for tests; this choice is due to in-sample variability of estimates. Additionally, reporting results of pairwise comparisons becomes rather laborious when the set of models ( $M$ ) increases since one must perform

<sup>5</sup> MSE: The Mean Square Error, MAPE: The Mean Absolute Prediction Error expressed as a percentage and LL: The Logarithmic Loss function.

$M(M - 1)/2$  tests. Additionally, we test the significance of forecast losses with MCS to determine the model's significance and predictive ability. This study uses the block bootstrap procedure and significance level  $\alpha = 5\%$  to determine the MCS  $p$ -values. Forecast models included in  $\hat{M}_{0.95}^*$  (95% MCS) are identified by an asterisk.

For all indexes, the ARFIMA-HYAPARCH specification outperforms all the other computing techniques and ARFIMA-HYAPGARCH models, thus yielding the most accurate stock market forecasts. The ARFIMA-HYAPARCH prediction errors are the smallest for all evaluation criteria and forecast horizons. Contrarily, the ARFIMA-FIAPARCH is the worst performing model in terms of out-of-sample predictive accuracy since it produces the largest forecast errors among the set of dual long-memory adaptations. Moreover, from Table 4, we observe that the ARFIMA-G-GARCH model outperforms the ARFIMA-FIAPARCH techniques. This model allows detecting and estimating the long-memory and seasonality. Nevertheless, from this table, ARFIMA-HYAPARCH seems to be the only model included in  $\hat{M}_{0.95}^*$  for all horizons under  $MSE$  and  $LL$ , whereas ARFIMA-FIAPARCH is the only model excluded from  $\hat{M}_{0.95}^*$  for all horizons under the  $MSE$  and  $LL$  criteria. Overall, the results of Table 4 seem to suggest that the ARFIMA-HYAPARCH model performs better compared to the other suggested models for almost all the horizons. However, it should be noted that the ARFIMA-G-GARCH model is a part of the 95% MCS for  $s = 15$  under  $MSE$  and  $LL$ . We can conclude that the ARFIMA-HYAPARCH tends to be the sole superior model for all horizons under  $MSE$  and  $LL$ , whereas the ARFIMA-G-GARCH model is also in  $\hat{M}_{0.95}^*$  for longer horizons under the same loss function. Generally, ARFIMA-HYAPARCH and ARFIMA-G-GARCH tend to be better models for long-term forecasting. In sum, depending on the loss functions chosen in our work, the results suggest that the ARFIMA-HYAPARCH model and the ARFIMA-G-GARCH model for longer horizons are the most preferred models for volatility forecasting since they are ranked highest in terms of their MCS  $p$ -value.

## Conclusions

This study attempted to investigate the presence of long-memory behavior in the returns and volatilities of the daily GCC stock market indexes. To combat this issue, we considered the applicability of three dual long-memory adaptations represented by the ARFIMA-FIGARCH, ARFIMA-HYGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYAPARCH models, and the ARFIMA-G-GARCH process. The empirical results demonstrate clear and consistent evidence for long-memory in all stock market indexes' first and second conditional moments. It should be stressed that the findings of long-range dependencies violate the weak form of market efficiency hypothesis and reject the martingale model, which states that conditioning on historical returns, future returns are unpredictable. The in-sample diagnostics and the out-of-sample predictive performances favor an ARFIMA-HYAPARCH modeling process. For all countries, the estimates of the power terms are highly and significantly different from two. This validates estimating an optimal power transformation within the model rather than the common use of a squared term for the variance equation.

The empirical results prove that the ARFIMA-HYAPARCH model is the most suitable forecasting technique. Further, it can produce smaller predicting errors than the

other computing techniques. It may be considered a powerful forecasting method, notably when we need higher forecasting accuracy. However, the estimation results of the asymmetry coefficients of the HYAPARCH parameterization diverge for the index series indicating different volatility structures. It would be worth noting that volatility dynamics are the focus of interest in the vast field of risk management and derivatives pricing. Otherwise, these markets present an opportunity for portfolio diversification at the regional level.

Our empirical investigation has important policy consequences for researchers, traders, and policymakers. The evidence of long-range serial dependence generates the possible predictability of returns and the volatility of most GCC stock markets, indicating the existence of market imperfections and behavioral biases. These obtained results reveal the inefficiency of the GCC stock markets, which can be explained by the differences in irrational comportment or some extreme economic and financial events. Therefore, the features of long-term persistence, asymmetry, and the powerful transformation of the conditional variance should be considered when calculating risk measures, deriving pricing formulas, handling short and long-term trading positions, or constructing hedging strategies.

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#### Author contributions

HB: co-Initiated the subject, analyzed the data in Matlab, contributed to the methodologies and interpretation, discussion of results and editing of the manuscripts. BS: Initiated the subject, review of literature and contributed to the discussion of the results. MBSZ: contributed to the revision of the economic methodology, to the development of the interpretations and the clarification of the analysis of the results. The authors read and approved the final manuscript.

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#### Availability of data and materials

All data are gathered via individual data channels such as Bloomberg. The models and data analysis are applied through computer programs, Oxmetrics and Matlab. All data and materials are available upon request.

#### Declarations

##### Competing interests

The authors declare that they have no competing interests.

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