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Network DEA based on DEA-ratio

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Abstract

Data envelopment analysis (DEA) is a technique to measure the performance of decision-making units (DMUs). Conventional DEA treats DMUs as black boxes and the internal structure of DMUs is ignored. Two-stage DEA models are special case network DEA models that explore the internal structures of DMUs. Most often, one output cannot be produced by certain input data and/or the data may be expressed as ratio output/input. In these cases, traditional two-stage DEA models can no longer be used. To deal with these situations, we applied DEA-Ratio (DEA-R) to evaluate two-stage DMUs instead of traditional DEA. To this end, we developed two novel DEA-R models, namely, range directional DEA-R (RDD-R) and (weighted) Tchebycheff norm DEA-R (TND-R). The validity and reliability of our proposed approaches are shown by some examples. The Taiwanese non-life insurance companies are revisited using these proposed approaches and the results from the proposed methods are compared with those from some other methods.

Keywords: Data envelopment analysis, DEA-R, Two-stage DEA

Introduction

Data envelopment analysis (DEA) is an approach used to measure the relative efficiency of a set of DMUs with multiple inputs and outputs, first introduced into the operations research and management science literature by Charnes et al. (1978). DEA has been used in various environments and numerous applications (Kou et al. 2021; Zha et al. 2020; Castelli et al. 2004 and references therein). The traditional DEA considers each DMUs as black boxes consuming some inputs to produce some outputs without regarding the internal structure. In real-world problems, DMUs may have a network or internal structures; see, for example Färe and Grosskopf (1996) (who introduced the concept of network DEA for the first time), Castelli et al. (2004), Tone and Tsutsui (2009) and Guo et al. (2017). Kao (2014) presented a review of network DEA. In some cases, DMUs may consist of two-stage network structures where the outputs of the first stage, known as intermediate measures/ outputs/ products, are inputs to the second stage. Many authors have studied two-stage network DEA. Kao and Hwang (2008) proposed a multiplicative efficiency aggregation approach in which the overall efficiency of the two-stage process is expressed as the product of the efficiency of two individual stages. Chen et al. (2009) revealed that Kao and Hwang's (2008) two-stage DEA model assumed constant returns to scale (CRS) and did not apply the variable returns to scale (VRS) assumption. So, they developed an additive efficiency decomposition approach in which the overall efficiency is expressed as a weighted average of the efficiency of the individual stages under VRS technology. Despotis et al. (2016) showed that the additive decomposition approach proposed by Chen et al. (2009) is biased toward the second stage and presents a composition approach to estimate unbiased efficiency scores for the individual stages.

Chen and Zhu (2004) offered a two-stage DEA model and indicated that the units with individual stages were efficient overall. However, using an example, they pointed out that overall efficient units do not necessarily indicate efficient performance in the two stages. Wang and Chin (2010) defined the overall efficiency of two-stage DMU under evaluation as the weighted harmonic mean of the efficiencies of the two-stage DMU in each stage. They also generalized the additive efficiency decomposition model of Chen et al. (2009) to taking into consideration the relative importance weights of two individual stages (see models (24), (26) and (28) of Wang and Chin 2010). Zha and Liang (2010) presented a method for studying a two-stage production process in which initial inputs of a two-stage DMU are freely allocated in both stages. However, their method does not allow for the existence of shared intermediate products or additional direct inputs to be used in the second stage. Yu and Shi (2014) improved the method of Zha and Liang (2010) and proposed a two-stage DEA parametric model where part of the outputs of the first stage are used as inputs in the second stage and additional inputs are allocated in the second stage but, does not allow for final outputs to be produced directly in the first stage or shared inputs. To overcome the aforementioned problems, Izadikhah and Farzipoor (2016) considered a two-stage DEA model in which initial inputs of a two-stage DMU are freely allocated in both stages and additional direct inputs are used in the second stage. Lozano et al. (2013) proposed a directional distance approach to deal with undesirable outputs. Lozano (2015a, 2015b) proposed slack-based measures (SBM) models for general network structures in which the exogenous inputs and outputs are considered at the system level instead of the process level. This model also relaxed the constraints for both the fixed-link and the free-link cases, thus enhanced the discriminating power of the model.

DEA models usually deal with data as absolute numeric values, while in the real world there are cases where data are ratios, for example, in efficiency measurement of financial institutions where financial ratios are included as output variables. There are two types of ratio data. In the first category, the DMU_j is considered as follows: $DMU_j = (X_j^V, X_j^R, Y_j^V, Y_j^R)$ in which X_j^V and Y_j^V are the input and output components with absolute numeric values and are non-ratio, respectively. X_j^R and Y_j^R are the input and output components with ratio scores, respectively. Therefore, in the first category, some input and output components are ratios and others are non-ratio measures. Emrouznejad and Amin (2009), Olesen et al. (2015), Olesen et al. (2017) and Hatami-Marbini and Toloo (2019) are examples of the first category. In the second category, the inputs and outputs of the DMU_j are as follows: $DMU_j = (X_j^V, Y_j^V)$ where X_j^V and Y_j^V are non-ratio input and output components which have absolute values. But, the ratio $\frac{Y_j^V}{X_j^V}$ or

$\frac{X_j^V}{Y_j^V}$ are defined. Despic et al. (2007), Liu et al. (2011), Gerami et al. (2020) and Gerami et al. (2020) are examples of the second category.

Emrouznejad and Amin (2009) proposed DEA models for dealing with ratio data. In turn, Olesen et al. (2015) showed that the use of ratio inputs and outputs in the variable returns-to-scale (VRS) and constant returns-to-scale (CRS) models generally violates the stated production assumptions. They developed new Ratio-CRS (R-CRS) and Ratio-VRS (R-VRS) models that allow the incorporation of ratio inputs and outputs. In the following, Olesen et al. (2017) developed radial and non-radial models under variable and constant returns-to-scale technologies. Hatami-Marbini and Toloo (2019) showed several problems in both solutions extended by Emrouznejad and Amin (2009) and introduced modified envelopment and multiplier DEA models for measuring performance to avoid the problems associated with Emrouznejad and Amin (2009). Despic et al. (2007) combined DEA methodology and Ratio Analysis (Fernandez-Castro and Smith 1994) and introduced the DEA-Ratio (DEA-R) model. In DEA-R all possible ratios "output/input" are treated as outputs within the standard DEA model. The DEA-R is an approach to apply expert's opinions in performance evaluationn of DMUs. For example, if a certain output cannot be produced by a certain input, then, the corresponding ratio "output/input" can be deleted from the model. Also, if the ratio data is important to managers, traditional two-stage DEA models are not applicable. Tohidnia and Tohidi (2019) applied DEA-R for evaluating the efficiency and productivity change of DMUs over time. For more discussions on the advantage of DEA-R see Mozaffari et al. (2020), Ostovan et al. (2020), Kamyab et al. (2021), Sotiros et al. (2019), Sexton and Lewis (2003), Zhang et al. (2015) and Gerami and Mozaffari (2013).

Considering the advantage of DEA-R models over the CCR-based models, we evaluate the efficiency of two-stage DMUs by DEA-R models. To this end, we combine DEA-R and two-stage DEA methodologies and propose two DEA ratio models to evaluate overall ratio efficiency and individual stage ratio efficiencies of two-stage DMUs. These two models are called Range Directional DEA-R (RDD-R) and (weighted) Tchebycheff norm DEA-R (TND-R) models. We state and prove some facts about these two models. In the TND-R model, the Decision Maker can impose the preference of each stage over the other stages and project inefficient divided data on the Pareto front by selecting convenience weights. Additionally, RDD-R model is translation invariant and unit invariant and projects all divided DMUs under evaluation on the strong frontier of production possibility set, under some conditions (see proposition 6. Moreover, we compare the RDD-R and TND-R models with the proposed models in Chen et al. (2009), Despotis et al. (2016) (models (21) and (24) of Despotis et al. 2016) and Wang and Chin (2010) (models (24), (26) and (28) with ($\lambda_1 = \frac{2}{5}, \lambda_2 = \frac{3}{5}$)) by some examples in order to show the validity and reliability of the proposed RDD-R and TND-R models and "two-stage DEA based on DEA-R."

This paper is organized as follows. "Literature review" section presents the literature review on two-stage DEA and ratio analysis. "Proposed methodology" section develops two models based on DEA-R to evaluate overall ratio efficiency and individual stages

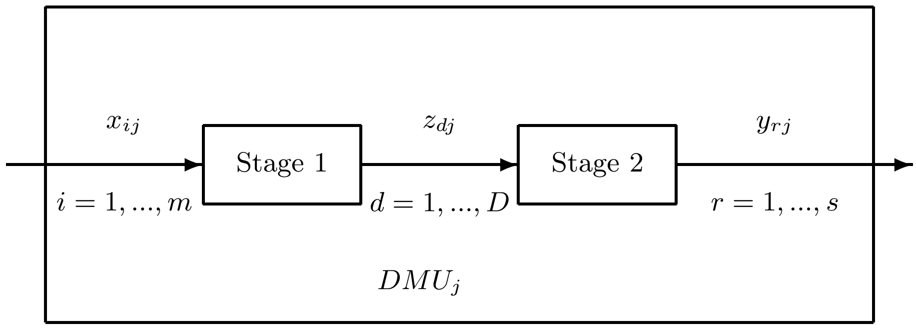

Fig. 1 A two-stage DMU

ratio efficiency of each two-stage DMUs under evaluation. Finally, "Numerical examples" section concludes the paper.

Literature review

Two-stage DEA

Färe and Whittaker (1995) and Färe and Grosskopf (1996) used an input oriented two-stage network DEA model to measure relative efficiencies in dairy production processes. Seiford and Zhu (1999) divided production process into independent sub-processes and calculated the efficiencies of the first stage, the second stage and the overall efficiency via three independent DEA models. Färe and Grosskopf (2000) presented a network DEA model for assessing Swedish Institute for Health Economics. Wang et al. (1997) and Noulas et al. (2001) proposed applications of two-stage DEA to non-life insurance policies and information technology, respectively. For an in-depth review of the multistage DEA model, see Castelli et al. (2004). Figure 1 indicates a simple two-stage production process where the first stage uses inputs x_i, $(i = 1, \ldots, m)$ to produce outputs z_d, $(d = 1, \ldots, D)$ and then stage 2 uses these z_d as inputs to produce final outputs y_r, $(r = 1, \ldots, s)$. In fact, the intermediate measures z_d are outputs of stage 1 and inputs of stage 2. The first and the second stage efficiencies of DMU_o, $(o = 1, \ldots, n)$, are defined as $\theta_o^1 = \frac{\sum_{d=1}^{D} w_d^1 z_{do}}{\sum_{r=1}^{s} v_i x_{io}}$ and $\theta_o^2 = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d^2 z_{do}}$, respectively, where v_i $(i = 1, \ldots, m)$ and w_d^1 $(d = 1, \ldots, D)$ are the input and output weights in the first stage and u_r $(r = 1, \ldots, s)$ and w_d^2 $(d = 1, \ldots, D)$ are the input and output weights in the second stage. Kao and Hwang (2008) defined the overall efficiency of DMU_o as the product of the two individual efficiencies, namely, $\theta_o = \theta_o^1 \times \theta_o^2$. Chen et al. (2009) found that the approach of Kao and Hwang (2008) cannot be extended to the VRS assumption (Banker et al. 1984) because, $\theta_o = \left[\frac{\sum_{d=1}^{D} w_d^1 z_{do} - w_o}{\sum_{r=1}^{s} v_i x_{io}}\right] \times \left[\frac{\sum_{r=1}^{s} u_r y_{ro} - u_o}{\sum_{d=1}^{D} w_d^2 z_{do}}\right]$ could not be converted into a linear form under the condition of $w_d^1 = w_d^2$. They therefore stated the overall efficiency of DMU_o as a weighted sum of the efficiencies for the individual stages and proposed the following two-stage DEA model under CRS assumption (model (11) of Chen et al. 2009):

$$
\begin{aligned}
\theta_o^* = \max\ & \sum_{d=1}^{D} w_d^1 z_{do} + \sum_{r=1}^{s} u_r y_{ro} \\
s.t.\ & \sum_{d=1}^{D} w_d^1 z_{do} + \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{d=1}^{D} w_d^1 z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1, \ldots, n \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d^1 z_{dj} \le 0 \quad j = 1, \ldots, n \\
& w_d^1 \ge 0 \qquad d = 1, \ldots, D \\
& v_i \ge 0 \qquad i = 1, \ldots, m \\
& u_r \ge 0 \qquad r = 1, \ldots, s
\end{aligned}
\tag{1}
$$

Once the overall efficiency is obtained, the efficiency scores for the two individual stages of DMU_o can be determined. If the first stage efficiency is prior to the second stage, θ_o^{1*} can be determined by solving the following LP model (model (18) of Chen et al. 2009):

$$
\begin{aligned}
\theta_o^{1*} = \max\ & \sum_{d=1}^{D} w_d^1 z_{do} \\
s.t.\ & \sum_{i=1}^{m} v_i x_{io} = 1 \\
& (1-\theta_o^*) \sum_{d=1}^{D} w_d^1 z_{do} + \sum_{r=1}^{s} u_r y_{ro} = \theta_o^* \quad j = 1, \ldots, n \\
& \sum_{d=1}^{D} w_d^1 z_{dj} + \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j = 1, \ldots, n \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d^1 z_{dj} \le 0 \\
& w_d^1 \ge 0 \qquad d = 1, \ldots, D \\
& u_r \ge 0 \qquad r = 1, \ldots, s \\
& v_i \ge 0 \qquad i = 1, \ldots, m
\end{aligned}
\tag{2}
$$

The efficiency for the second stage is then calculated as $\theta_o^2 = \frac{\theta_o^* - w_1^* . \theta_o^{1*}}{w_2^*}$, Also, by assuming pre-emptive priority for stage 2, θ_o^{2*} can be determined by solving the following LP model (model (19) of Chen et al. 2009):

$$
\begin{aligned}
\theta_o^{2*} = \max\ & \sum_{r=1}^{s} u_r y_{ro} \\
s.t.\ & \sum_{d=1}^{D} w_d^1 z_{do} = 1 \\
& \theta_o^* \sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r y_{ro} = 1 - \theta_o^* \\
& \sum_{d=1}^{D} w_d^1 z_{dj} + \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j = 1, \ldots, n \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d^1 z_{dj} \le 0 \\
& w_d^1 \ge 0 \qquad d = 1, \ldots, D \\
& u_r \ge 0 \qquad r = 1, \ldots, s \\
& v_i \ge 0 \qquad i = 1, \ldots, m
\end{aligned}
\tag{3}
$$

The efficiency for the first stage is then calculated as $\theta_o^1 = \frac{\theta_o^* - w_2^* . \theta_o^{2*}}{w_1^*}$, where

$$w_1^* = \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{r=1}^{s} u_r y_{ro} + \sum_{d=1}^{D} w_d z_{do}}$$
$$w_2^* = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{r=1}^{s} u_r y_{ro} + \sum_{d=1}^{D} w_d z_{do}}$$

by way of (1). If $\theta_o^1 = \theta_o^{1*}$ or $\theta_o^2 = \theta_o^{2*}$, then this indicates that we have a unique efficiency decomposition. Despotis et al. (2016) modeled the following single-objective program under the constant returns-to-scale (CRS) assumption; for assessments of the efficiencies of two individual stages and overall efficiency of the evaluated DMU_o (Model (21) of Despotis et al. 2016):

$$\begin{aligned} \min \ & \sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r y_{ro} \\ s.t. \ & \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad i = 1, ..., m, j = 1, ..., n \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0 \quad r = 1, ..., s, j = 1, ..., n \\ & \sum_{d=1}^{D} w_d z_{do} = 1 \\ & w_d \geq 0 \qquad d = 1, ..., D \\ & v_i \geq 0 \qquad i = 1, ..., m \\ & u_r \geq 0 \qquad r = 1, ..., s \end{aligned} \tag{4}$$

Once an optimal solution (v^*, w^*, u^*) of the model (4) is obtained, the efficiency scores for DMU_o under assessments in the first and second stages are respectively:

$$\begin{aligned} \hat{e}_o^1 &= \frac{\sum_{d=1}^{D} w_d^* z_{do}}{\sum_{i=1}^{m} v_i^* x_{io}} = \frac{1}{\sum_{i=1}^{m} v_i^* x_{io}} \\ \hat{e}_o^2 &= \frac{\sum_{r=1}^{s} u_r^* y_{ro}}{\sum_{d=1}^{D} w_d^* z_{do}} = \sum_{r=1}^{s} u_r^* y_{ro} \end{aligned} \tag{5}$$

Also, the overall efficiency of DMU_o is calculated as follows:

$$\hat{e}_o^0 = \frac{\hat{e}_o^1 + \hat{e}_o^2}{2} \tag{6}$$

Moreover, they employed the unweighted Tchebycheff norm (L_∞ norm) to formulate following model (Model (24) in Despotis et al. 2016):

$$
\begin{aligned}
&\min\ \delta \\
&s.t.\ \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad i = 1,\ldots,m, j = 1,\ldots,n \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad r = 1,\ldots,s, j = 1,\ldots,n \\
&\quad \sum_{i=1}^{m} v_i x_{io} - \delta \le E_o^1 \\
&\quad \sum_{r=1}^{s} u_r y_{ro} + \delta \ge E_o^2 \\
&\quad \sum_{d=1}^{D} w_d z_{do} = 1 \\
&\quad w_d \ge 0 \qquad\qquad d = 1,\ldots,D \\
&\quad v_i \ge 0 \qquad\qquad i = 1,\ldots,m \\
&\quad u_r \ge 0 \qquad\qquad r = 1,\ldots,s \\
&\quad \delta \ge 0
\end{aligned}
\tag{7}
$$

in which E_o^1 and E_o^2 indicate the efficiency scores of stage 1 and stage 2 and are calculated by model (20) of Despotis et al. (2016).

Once an optimal solution (v^*, w^*, u^*) of the model (7) is obtained, the individual efficiency scores and overall efficiency for DMU_o under assessment are obtained by (5) and (6), respectively.

Wang and Chin (2010) generalized two-stage DEA models proposed by Chen et al. (2009). They assigned specific weights $\lambda_1 > 0$ and $\lambda_2 > 0$, with $\lambda_1 + \lambda_2 = 1$, to each individual stage to reflect its relative importance in the whole process. They formulated the overall efficiency of DMU_o as follows (see model (24) of Wang and Chin 2010):

$$
\begin{aligned}
\theta_o^* = \max\ & \lambda_1 \Big(\sum_{d=1}^{D} w_d z_{do}\Big) + \lambda_2 \Big(\sum_{r=1}^{s} u_r y_{ro}\Big) \\
s.t.\ & \lambda_1 \sum_{i=1}^{m} v_i x_{io} + \lambda_2 \sum_{d=1}^{D} w_d z_{do} = 1 \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j = 1,\ldots,n \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \qquad j = 1,\ldots,n \\
& w_d \ge 0 \qquad\qquad d = 1,\ldots,D \\
& v_i \ge 0 \qquad\qquad i = 1,\ldots,m \\
& u_r \ge 0 \qquad\qquad r = 1,\ldots,s
\end{aligned}
\tag{8}
$$

Once the overall efficiency θ_o^* is obtained, θ_o^{1*} and θ_o^{2*} can then be determined by solving following LP models (see models (26) and (28) of Wang and Chin 2010):

$$
\begin{aligned}
\theta_o^{1*} = \max\ & \sum_{d=1}^{D} w_d z_{do} \\
s.t.\ & \sum_{i=1}^{m} v_i x_{io} = 1 \\
& (\lambda_1 - \lambda_2\theta_o^*) \sum_{d=1}^{D} w_d z_{do} + \lambda_1 u^1 + \lambda_2 \sum_{r=1}^{s} u_r y_{ro} + \lambda_2 u^2 = \lambda_1 \theta_o^* \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 && j = 1, ..., n \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0 && j = 1, ..., n \\
& w_d \ge 0 && d = 1, ..., D \\
& v_i \ge 0 && i = 1, ..., m \\
& u_r \ge 0 && r = 1, ..., s
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
\theta_o^{2*} = \max\ & \sum_{r=1}^{s} u_r y_{ro} \\
s.t.\ & \sum_{d=1}^{D} w_d z_{do} = 1 \\
& \lambda_2 \sum_{r=1}^{s} u_r y_{ro} + \lambda_2 u^2 - \lambda_1\theta_o^* \sum_{i=1}^{m} v_i x_{io} + \lambda_1 u^1 = \lambda_2 \theta_o^* - \lambda_1 \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 && j = 1, ..., n \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0 && j = 1, ..., n \\
& w_d \ge 0 && d = 1, ..., D \\
& v_i \ge 0 && i = 1, ..., m \\
& u_r \ge 0 && r = 1, ..., s
\end{aligned}
\tag{10}
$$

Ratio analysis

Consider a set of $\boldsymbol{n}$ DMUs which is associated with $\boldsymbol{m}$ inputs and $\boldsymbol{s}$ outputs. Particularly, each $DMU_j = (X_j, Y_j)\,(j \in J = \{1, \ldots, n\})$ consumes amount $x_{ij}(> 0)$ of input $\boldsymbol{i}$ and produces amount $y_{rj}(> 0)$ of output $\boldsymbol{r}$. Corresponding to any DMU_j, output-input ratio vector V_j is defined as follows:

$$V_j = \frac{Y_j}{X_j} = \left(\frac{y_{1j}}{x_{1j}}, \ldots, \frac{y_{sj}}{x_{1j}}, \frac{y_{1j}}{x_{2j}}, \ldots, \frac{y_{sj}}{x_{2j}}, \ldots, \frac{y_{1j}}{x_{mj}}, \ldots, \frac{y_{sj}}{x_{mj}} \right)$$

V_j can be considered as outputs vector i.e., the more $\frac{y_{rj}}{x_{ij}}$, the better $\frac{y_{rj}}{x_{ij}}$. Let $V = \bigcup_{j=1}^{n} V_j$ be a group of output-input ratio. Then, the smallest closed convex and free-disposal attainable set[1] that contains the observations can be expressed as follows: (see Liu et al. 2011 for detail)

[1] See Liu et al. (2011) for definition of attainable set.

$$P=\left\{\frac{y_r}{x_i}|\sum_{j=1}^{n}\lambda_j\frac{y_{rj}}{x_{ij}}\geq\frac{y_r}{x_i},\sum_{j=1}^{n}\lambda_j=1,\lambda_j\geq 0,i=1,\ldots,m,r=1,\ldots,s\right\}.$$

Fernandez-Castro and Smith (1994) calculated the ratio efficiency of DMU_o by solving the following output-oriented model, for the first time:

$$\begin{array}{ll}\max & \varphi_R\\ s.t. & \varphi_R\frac{y_{ro}}{x_{io}}\in P\ \ i=1,\ldots,m,r=1,\ldots,s\end{array}$$

or equivalently:

$$\begin{array}{ll}\max & \varphi_R\\ s.t. & \sum_{j=1}^{n}\lambda_j\frac{y_{rj}}{x_{ij}}\geq\varphi_R\frac{y_{ro}}{x_{io}}\ \ i=1,...,m,r=1,...,s\\ & \sum_{j=1}^{n}\lambda_j=1\\ & \lambda_j\geq 0\end{array}\tag{11}$$

Proposition 1 *At optimality of model* (11), $\varphi_R^*\geq 1$.[2]

Wu and Liang (2005) *used ratio analysis to develop an aggregated ratio model to evaluate* DMU_o *as follows:*

$$\begin{array}{lll}\psi^*= & \max & \sum_{i=1}^{m}\sum_{r=1}^{s}w_{ir}\frac{y_{ro}}{x_{io}}\\ & s.t. & \sum_{i=1}^{m}\sum_{r=1}^{s}w_{ir}\frac{y_{rj}}{x_{ij}}\leq 1\ j=1,...,n\\ & & w_{ir}\geq 0\qquad\qquad i=1,...,m,r=1,...,s\end{array}\tag{12}$$

Definition 1 Wu and Liang (2005) DMU_o is ratio efficient if and only if $\psi^*=1$.

Despic et al. (2007) referred to the model (12) as the DEA-R model and showed that this model is equivalent to the output-oriented DEA-R model (13):

$$\begin{array}{ll}\min & \Delta\\ s.t. & \sum_{i=1}^{m}\sum_{r=1}^{s}w_{ir}\frac{\frac{y_{rj}}{x_{ij}}}{\frac{y_{ro}}{x_{io}}}\leq\Delta\ j=1,...,n\\ & \sum_{i=1}^{m}\sum_{r=1}^{s}w_{ir}=1\\ & w_{ir}\geq 0\qquad\qquad i=1,...,m,r=1,...,s\end{array}\tag{13}$$

Mozaffari et al. (2014) investigated the relationship between the DEA-R models (13) and (12) and proved that $\psi^*=\frac{1}{\Delta^*}$ on optimality of the models (12) and (13) (also see Despic et al. 2007). Moreover, they proved that the model (11) is dual of the model (13) and therefore, $\varphi_R^*=\Delta^*$. So, we have the following property:

[2] *Superscript '*' indicates optimality.*

Proposition 2 *At optimality of models* (11) *and* (12), $\varphi_R^* = \frac{1}{\psi^*}$.

Definition 2 Mozaffari et al. (2014) DMU_o is DEA-R efficient if and only if the optimal objective function value of model (13), $\Delta^* = 1$. In view of the above discussion, the ratio efficiency of DMU_o under evaluation can be obtained using the models (11) or (12) or (13).

The issue of underestimation of efficiency and pseudo-inefficiency are two other problems that the traditional DEA models cannot overcome. Underestimation of efficiency occurs when input and output weights are computed as zero. Therefore, the importance of that variable is not considered in the efficiency score of the DMUs. In this case, the efficiency scores of DMUs are not calculated correctly. Different works have been reported in this field (see for instance Gerami et al. 2020, 2020). DEA-R models prevent the underestimation of efficiency and pseudo-inefficiency (Wei et al. 2011a, b, c). Therefore, they correctly calculate the efficiency score of DMUs. Moreover, in some cases, the available data are presented as ratio output/input. Also, it may occur that in producing an output only a subset of inputs is being used. In modeling such restrictions, traditional DEA is not applicable, and using the DEA-R approach much easier than other proposed approaches (see Tohidnia and Tohidi 2019 for detail). The flexibility of DEA-R in modeling such restrictions gave us the motivation to use DEA-R for evaluating two-stage DMUs. To this end, we combined DEA-R and network DEA to propose two new models for evaluating two-stage DMUs. In general, the proposed ratio network DEA models can be considered as a combination of the two-stage DEA efficiency model and ratio analysis, and thus, they will be more preferred by experts who are familiar with ratio analysis.

Proposed methodology

As referred to in the "Introduction" section, DEA-R is a suitable approach to incorporate expert"s opinions. Moreover, when just ratio data as "output/input" are available, DEA-R is applicable. This shows the importance of applying DEA-R to the two-stage DEA. Because of these facts, we evaluated the efficiencies of two-stage DMUs using the DEA-R model (11) instead of conventional DEA models and proposed two models which we call (1)the Range Directional DEA-R (RDD-R) model and (2) (weighted) Tchebycheff norm DEA-R (TND-R) model for measuring the individual stage ratio efficiencies and overall ratio efficiency of a two-stage DMU under evaluation. Our proposed models obtain these ratio efficiencies by solving only one model. This is the other advantage of our proposed models in comparison with some other existing two-stage models.

Let $\left\{DMU_j = (X_j, Z_j, Y_j) | j = 1, \ldots, n\right\}$ be a group of two-stage data; in which, $X_j = (x_{1j}, \ldots, x_{mj})$, $Z_j = (z_{1j}, \ldots, z_{Dj})$ and $Y_j = (y_{1j}, \ldots, y_{sj})$, we define divided data $\left\{DMU'_j = \left(\frac{Z_j}{X_j}, \frac{Y_j}{Z_j}\right) | j = 1, \ldots, n\right\}$ where $\frac{Z_j}{X_j} = \left(\frac{z_{1j}}{x_{1j}}, \ldots, \frac{z_{Dj}}{x_{1j}}, \ldots, \frac{z_{1j}}{x_{mj}}, \ldots, \frac{z_{Dj}}{x_{mj}}\right)$, $\frac{Y_j}{Z_j} = \left(\frac{y_{1j}}{z_{1j}}, \ldots, \frac{y_{sj}}{z_{1j}}, \ldots, \frac{y_{1j}}{z_{Dj}}, \ldots, \frac{y_{sj}}{z_{Dj}}\right)$, for each $i = 1, \ldots, m$, $d = 1, \ldots, D$ and $r = 1, \ldots, s$.

Again, we assume that the more $\frac{z_{dj}}{x_{ij}}$ and $\frac{y_{rj}}{z_{dj}}$ are, the more they are considered as outputs. So, we have a set of DMUs with $D \times m + s \times D$ outputs without any explicit inputs.

Similar to the attainable set P, the following attainable set defines a bounded closed convex and free-disposal set that contains the observations:

$$P' = \left\{ (W,V) \middle| \sum_{j=1}^{n} \lambda_j \frac{Z_j}{X_j} \geq W,\ \sum_{j=1}^{n} \mu_j \frac{Y_j}{Z_j} \geq V,\ \sum_{j=1}^{n} \lambda_j = 1, \right.$$
$$\left. \sum_{j=1}^{n} \mu_j = 1,\ \lambda_j,\ \mu_j \geq 0, W \in \mathbb{R}^{D\times m}, V \in \mathbb{R}^{D\times s} \right\} \subseteq \mathbb{R}^{D\times m + D\times s}.$$

The inequalities of P' are corresponding to stage 1 efficiency and stage 2 efficiency, respectively. In the sequel, we develop two models to estimate the overall ratio efficiency and individual stage ratio efficiencies of two-stage DMUs.

Tchebycheff norm DEA-R (TND-R) model

The weighted Tchebycheff norm turned out to be very useful in generating non-dominated solutions (Pareto optimal solutions) in multiple objective programs (see Kou et al. 2014; Kasimbeyli 2010). The distance between two points $X = (x_1, x_2, \ldots, x_n)$ and $X' = (x'_1, x'_2, \ldots, x'_n)$ with L_∞ norm is given by:

$$\begin{aligned} &\| X - X' \|_\infty \\ &= \max\{|x_1 - x'_1|, |x_2 - x'_2|, \ldots, |x_n - x'_n|\} \end{aligned}$$

The evaluation of the efficiency value of DMU with L_∞-Norm (Tchebycheff norm) was introduced by Tavares and Antunes (2001). They presented the following model named TCH in DEA for evaluation of the performance of DMU_k:

$$\begin{aligned} &\max\ \ U_o \\ &s.t.\ \ U_o + \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, \quad i = 1, \ldots, m \\ &\qquad U_o - \sum_{j=1}^{n} \lambda_j y_{rj} \leq -y_{ro}, \ r = 1, \ldots, s \\ &\qquad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, \qquad i = 1, \ldots, m \\ &\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \qquad r = 1, \ldots, s \\ &\qquad \sum_{j=1}^{n} \lambda_j = 1 \\ &\qquad \lambda_j,\ U_o \geq 0, \qquad j = 1, \ldots, n \end{aligned} \tag{14}$$

where $U_o \in [0, +\infty]$ is the efficiency value. Also, DMU_o is efficient in the Tavares model (14) if and only if in optimal solution $U_o^* = 0$, otherwise it is inefficient. With the introducing efficiency index as $\theta_o^* = \frac{1}{1+U_o^*}$, $\theta_o^* \in (0, 1]$, DMU_o is efficient in the Tavares model (14) if and only if in optimal solution $\theta_o^* = 1$.

The objective function of model (14) minimizes the distance between DMU_o and its projected point on frontier efficiency. Using the weighted Tchebycheff norm, the model (14) can be written as the following weighted programming problem:

$$
\begin{aligned}
\max \ & U_o \\
s.t. \ & U_o \mathbf{W}_1 + \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io}, \quad i = 1, ..., m \\
& U_o \mathbf{W}_2 - \sum_{j=1}^{n} \lambda_j y_{rj} \le -y_{ro}, \ r = 1, ..., s \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io}, \qquad\qquad i = 1, ..., m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \qquad\qquad r = 1, ..., s \\
& \sum_{j=1}^{n} \lambda_j = 1, \ \ \lambda_j, U_o \ge 0, \quad j = 1, ..., n
\end{aligned}
\tag{15}
$$

where $\mathbf{W}_1 \in \mathbb{R}^m_{\ge 0}$, $\mathbf{W}_2 \in \mathbb{R}^s_{\ge 0}$ are Decision Maker preference and $\mathbf{1W}_1 + \mathbf{1W}_2 = 1$. Now, motivated by the weighted model (15), we introduce (weighted) Tchebycheff norm DEA-R (TND-R) models for evaluating two-stage $DMU_o = (X_o, Z_o, Y_o), o \in \{1, 2, \ldots, n\}$, as follows:

$$
\begin{aligned}
\max \ & \alpha_1 + \alpha_2 \\
s.t. \ & \alpha_1 \mathbf{W}_1 + \frac{z_{do}}{x_{io}} \le \sum_{j=1}^{n} \lambda_j \frac{z_{dj}}{x_{ij}}, \quad i = 1, ..., m, d = 1, ..., D \\
& \alpha_2 \mathbf{W}_2 + \frac{y_{ro}}{z_{do}} \le \sum_{j=1}^{n} \mu_j \frac{y_{rj}}{z_{dj}}, \quad r = 1, ..., s, d = 1, ..., D \\
& \sum_{j=1}^{n} \mu_j \frac{z_{dj}}{x_{ij}} \ge \frac{z_{do}}{x_{io}}, \quad i = 1, ..., m, d = 1, ..., D \\
& \sum_{j=1}^{n} \lambda_j \frac{y_{rj}}{z_{dj}} \ge \frac{y_{ro}}{z_{do}}, \quad r = 1, ..., s, d = 1, ..., D \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \sum_{j=1}^{n} \mu_j = 1 \\
& \lambda_j, \ \alpha_1, \ \alpha_2 \ge 0, \quad j = 1, ..., n
\end{aligned}
\tag{16}
$$

where $\mathbf{W}_1 \in \mathbb{R}^m_{\ge 0}$, $\mathbf{W}_2 \in \mathbb{R}^s_{\ge 0}$ with $\mathbf{1W}_1 + \mathbf{1W}_2 = 1$, are weights addressing total preference over the two stages and are determined by the Decision Maker.

Suppose that (α_1^*, α_2^*) are the values of the objective function of model (16) at optimality. The stage ratio efficiencies and overall ratio efficiency scores are defined as $F = \frac{1}{1+\alpha_1^*}$, $S = \frac{1}{1+\alpha_2^*}$ and $O = F \times S$, respectively.

Proposition 3 *If divided DMU* $\left(\frac{Z_o}{X_o}, \frac{Y_o}{Z_o}\right)$ *lies on the strong frontier of the PPS* P' then, $DMU_o = (X_o, Z_o, Y_o)$ *is ratio-efficient overall and individual stages are ratio-efficient and vice versa.*

Definition 3 Suppose that (α_1^*, α_2^*) are the values of the objective function of model (16) at optimality. Then,

(a1) if $O = 1$, DMU_o is said to be overall ratio efficient.
(a2) if $F = 1$, DMU_o is said to be ratio efficient in stage 1.
(a3) if $S = 1$, DMU_o is said to be ratio efficient in stage 2.
(a4) if $O < 1$, DMU_o is said to be overall ratio inefficient.
(a5) if $F < 1$, DMU_o is said to be ratio inefficient in stage 1.
(a6) if $S < 1$, DMU_o is said to be ratio inefficient in stage 2.

Range directional DEA-R (RDD-R) model

Silva Portela et al. (2004) proposed range directional measure for handling the negative data based on the directional distance function approach which provides an efficiency score that results from the comparison of the DMU under evaluation with the so-called ideal point. Motivated by their approach, this paper defines Range Directional DEA-R (RDD-R) model as follows. Let $\{Y_j | j = 1, \ldots, n\}$, in which $Y_j = (y_{1j}, \ldots, lety_{sj})$, be a group of data in $\mathbb{R}^s$. The range directional model without explicit inputs for assessing $DMU_o = Y_o$ can be defined as follows:

$$\begin{array}{ll} \max & \beta_o \\ s.t. & Y_o + \beta_o R_o^+ \in P \\ \beta_o \geq 0 & \end{array}$$

or equivalently:

$$\begin{array}{ll} \max & \beta_o \\ s.t. & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \beta_o R_{ro}^+, \ r = 1, ..., s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j, \beta_o \geq 0 \end{array} \tag{17}$$

where $\mathbf{R}_o^+ = (R_{1o}^+, \ldots, R_{so}^+)^t$ is the vector between ideal point $Y_M = \left(\max_{j=1,\ldots,n} \{y_{1j}\}, \ldots, \max_{j=1,\ldots,n} \{y_{sj}\} \right)^t$ and $DMU_o = Y_o = (y_{1o}, \ldots, y_{so})$ under evaluation, that is, $\mathbf{R}_o^+ = Y_M - Y_o$ and $R_{ro}^+ = \max_{j=1,\ldots,n} \{y_{rj}\} - y_{ro}, r = 1, \ldots, s$.

Proposition 4 *Model (17) is always bounded, that is,* $0 \leq \beta_o^* \leq 1$. *DMU_o is efficient if* $\beta_o^* = 0$. *Therefore, the RDD-efficiency measure of DMU_o is given by* $\rho^* = 1 - \beta_o^*$.

By model (17), $DMU_o = (y_{1o}, y_{2o}, \ldots, y_{so})$ *is projected onto the frontier of P along the direction of the vector* $\mathbf{R}_o^+ \, at Y_o + \beta_o^* R_{ro}^+$.

Model (17) also has following important characteristic that can be used to handle negative data, that is:

Proposition 5 *Model (17) is translation invariant and unit invariant.*

Proof The proof is trivial. □

The following proposition discusses the conditions that DMU_o under evaluation is projected onto the strong frontier of the attainable set P via Range Directional model (17).

Proposition 6 *Suppose that the ideal point Y_M is constructed by s(=the dimension of R^s_+) extreme DMUs. If the vector of Y_M along with the vectors of these extreme efficient DMUs constitute an affine independent set then, DMU_o is projected onto the strong frontier (hyperplane) of attainable set P via model* (17).

Proof It is enough to show that the first r constraints of model (17) are as equality at optimality. Without loss of generality, suppose that Y_M is constructed by s extreme DMUs $Y_1, Y_2, \ldots, Y_s$. By affinity independence of the vectors $Y_1, Y_2, \ldots, Y_s$ and Y_M, the set $\{Y_1 - Y_M, Y_2 - Y_M, \ldots, Y_s - Y_M\}$ constitutes a basis for vector space R^s_+. Therefore, there exist the constants $\bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_s$ so that, $\sum_{j=1}^{s} \bar{\lambda}_j (Y_M - Y_j) = Y_M - Y_o, \bar{\lambda}_j \geq 0$. (Note that defining Y_M, the vector $Y_M - Y_o$ lies in the positive linear combination of the vectors $Y_M - Y_1, Y_M - Y_2, \ldots, Y_M - Y_s$). Therefore,
$\sum_{j=1}^{s} \bar{\lambda}_j (Y_M - Y_j) = Y_M - Y_0, \bar{\lambda}_j \geq 0, j = 1, \ldots, s$. This implies
$\sum_{j=1}^{s} \bar{\mu}_j Y_j = Y_o + \beta_o R_o^+ (Y_M - Y_o), \sum_{j=1}^{s} \bar{\mu}_j = 1$, where $\beta_o = 1 - \frac{1}{\sum_{j=1}^{s} \bar{\lambda}_j}, \bar{\mu}_j = \frac{\bar{\lambda}_j}{\sum_{j=1}^{s} \bar{\lambda}_j}$ and $R_o^+ = Y_M - Y_o$. So,

$$\sum_{j=1}^{s} \bar{\mu}_j Y_j = Y_o + \beta_o R_o^+,$$
$$\sum_{j=1}^{s} \bar{\mu}_j = 1,$$
$$\bar{\mu}_j \geq 0, j = 1, ..., s$$

thus, $(\beta_o, \bar{\mu}_j, j = 1, \ldots, s)$ is an optimal solution of (17). This completes the proof. □

Note: By proposition 6, if attainable set P contains strong hyperplane(s) then, DMU_o is projected onto them, by model (17).

Now, inspired by model (17), Range Directional DEA-R (RDD-R) models for evaluation two-stage $DMU_o = (X_o, Z_o, Y_o)$ is introduced as follows:

$$\begin{aligned} &\max \quad \beta_1 + \beta_2 \\ &s.t. \quad (\tfrac{Z_o}{X_o}, \tfrac{Y_o}{Z_o}) + (\beta_1 \frac{1}{\|\mathbf{R}_o^{1+}\|} \mathbf{R}_o^{1+}, \beta_2 \frac{1}{\|\mathbf{R}_o^{2+}\|} \mathbf{R}_o^{2+}) \in P' \ , \end{aligned} \tag{18}$$

where $\mathbf{R}_o^{1+} = \frac{Z_M}{X_M} - \frac{Z_o}{X_o}$ and $\mathbf{R}_o^{2+} = \frac{Y_M}{Z_M} - \frac{Y_o}{Z_o}$,

$$\frac{Z_M}{X_M} = \left(\frac{z_{1M}}{x_{1M}}, ..., \frac{z_{DM}}{x_{1M}}, ..., \frac{z_{1M}}{x_{mM}}, ..., \frac{z_{DM}}{x_{mM}}\right), \quad \frac{z_{dM}}{x_{iM}} = \max_{j=1,...,n} \{\frac{z_{dj}}{x_{ij}}\}$$
$$\frac{Y_M}{Z_M} = \left(\frac{y_{1M}}{z_{1M}}, ..., \frac{y_{DM}}{z_{1M}}, ..., \frac{y_{1M}}{z_{mM}}, ..., \frac{y_{DM}}{z_{mM}}\right), \quad \frac{y_{rM}}{z_{dM}} = \max_{j=1,...,n} \{\frac{y_{rj}}{z_{dj}}\}$$

for each $i = 1, \ldots, m$, $d = 1, \ldots, D$ and $r = 1, \ldots, s$. The notation $\|R\|$ denotes the norm of vector R.

The RDD-R model (18) projects the vector $\frac{Z_o}{X_o}$ and $\frac{Y_o}{Z_o}$ on the frontier of attainable set P' in direction of $\mathbf{R}_o^{1+}$ and $\mathbf{R}_o^{2+}$ and measure the ratio-efficiency of stage 1 and stage 2 of the two-stage DMU DMU_o, respectively. Model (18) can be written as following single model:

$$
\begin{aligned}
\max \quad & \beta_1 + \beta_2 \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j \frac{z_{dj}}{x_{ij}} \geq \frac{z_{do}}{x_{io}} + \beta_1 \frac{1}{\|\mathbf{R}_o^{1+}\|} R_{dio}^{1+} \ \ d = 1, ..., D, i = 1, ..., m, \\
& \sum_{j=1}^{n} \eta_j \frac{y_{rj}}{z_{dj}} \geq \frac{y_{ro}}{z_{do}} + \beta_2 \frac{1}{\|\mathbf{R}_o^{2+}\|} R_{rdo}^{2+} \ \ r = 1, ..., s, d = 1, ..., D, \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \sum_{j=1}^{n} \mu_j = 1 \\
& \lambda_j, \ \beta_1, \ \eta_j, \ \beta_2 \geq 0
\end{aligned}
\tag{19}
$$

where $R_{dio}^{1+} = \frac{z_{dM}}{x_{iM}} - \frac{z_{do}}{x_{io}}$ and $R_{rdo}^{2+} = \frac{y_{rM}}{z_{dM}} - \frac{y_{ro}}{z_{do}}$.

Proposition 7 *The RDD-R model* (19)*is always bounded that is,* $0 \leq \beta_1^* \leq 1$ *and* $0 \leq \beta_2^* \leq 1$.[3]

Two-stage $DMU_o = (X_o, Y_o, Z_o)$ is ratio-efficient if $\beta^* = 0$. Therefore, the RDD-ratio efficiency measure of DMU_o is given by $\rho^* = 1 - \beta^*$.

So, $\rho_1^* = 1 - \beta_1^*$, $\rho_2^* = 1 - \beta_2^*$ and $\rho_o^* = \rho_1^* \rho_2^*$ is said to be the RDD-ratio efficiency score of stage 1, RDD-ratio efficiency score of stage 2 and overall ratio-efficiency score of the two-stage DMU_o, respectively.

Definition 4 Suppose that (β_1^*, β_2^*) is the optimal solution of the model (19). Then,

(a1) if $\rho_o^* = 1$, DMU_o is said to be overall ratio efficient.
(a2) if $\rho_1^* = 1$, DMU_o is said to be ratio efficient in stage 1.
(a3) if $\rho_2^* = 1$, DMU_o is said to be ratio efficient in stage 2.
(a4) if $\rho_o^* < 1$, DMU_o is said to be overall ratio inefficient.
(a5) if $\rho_1^* < 1$, DMU_o is said to be ratio inefficient in stage 1.
(a6) if $\rho_2^* < 1$, DMU_o is said to be ratio inefficient in stages 2.

The following theorem states that individual stage efficiencies in the TND-R model (16) are equivalent to individual stage efficiencies in the RDD-R model (19):

Theorem 1 *At optimality of models* (16) *and* (19), $\alpha_1^* = 0$ $(\alpha_2^* = 0)$ *if and only if* $\beta_1^* = 0$ $(\beta_2^* = 0)$.

[3] (*) *is used for the optimal solution.*

Table 1 Example 1. Data set for DMUs (Extracted from Izadikhah and Farzipoor 2016)

DMU	Input(X)	Intermediate measure (Z)	Output (Y)	$\left(\frac{Z}{X}, \frac{Y}{X}\right)$
A	2	1.5	1.5	$A'\left(\frac{3}{4}, 1\right)$
B	4	4	5	$B'\left(1, \frac{5}{4}\right)$
C	5	5	6	$C'\left(1, \frac{6}{5}\right)$

Proof The proof is straightforward.

The insurance industry extends the productivities and services by providing safety and confidence. These companies have positive effects on the growth of the economy of a country. Fecher et al. (1993) was the first to conduct a study that applies DEA in evaluating the performance of insurance firms. Thereafter, many researchers have studied the insurance business by using the DEA techniques (see Eling and Jia 2019; Tone et al. 2019; An et al. 2020, for instance). Kaffash et al. (2019) found that from 1992 to 2018 there were 132 studies on the insurance sector. Studies reviewed in their paper applied DEA to calculate the efficiency of firms from various backgrounds, with multiple inputs and outputs. There are other studies in the literature (Borges et al. 2008; Cummins and Weiss 2013; Ilyas Ashiq 2019; Mandal 2014), which have calculated the efficiency of insurance companies by adopting DEA as a tool for efficiency measurement. Hwang and Kao (2006) were the first to employ Network-DEA to evaluate the performance of non-life insurance firms in Taiwan. Thereafter, many authors have investigated the performance of insurance firms using the two-stage DEA (see Kao 2014; Chen et al. 2009; Guo et al. 2017; An et al. 2020 and references therein). Most of the prior research used more traditional DEA models. A few recent studies have adopted new DEA models such as the Network-DEA. However, to the best of our knowledge, there are only two studies that used the Network-DEA and DEA-R simultaneously in the insurance industry (see Gerami et al. 2020; Ostovan et al. 2020). In the next section, we apply the proposed methods to evaluate the performance of non-life insurance firms in Taiwan also studied in Kao and Hwang (2008). □

Numerical examples

Example 1 In this example we evaluate two-stage DMUs that have been previously evaluated by Izadikhah et al. (2018). Consider 3 two-stage DMUs with each DMU consuming a single input (X) to produce a single output (Z), in stage 1 and a single input (Z) to produce a single output (Y), in stage 2. Data set for these DMUs are given in Table 1. The last column of Table 1 represents divided DMUs corresponding to the two-stage DMUs. Evidently, $(W_1, W_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$.

Figure 2 depicts efficient frontier of PPS constructed by divided DMUs. Ratio-efficiency scores of the proposed TND-R and RDD-R models and efficiency scores of the Izadikhah's method (Izadikhah et al. 2018) are reported in Table 2. Given Fig. 2 and the

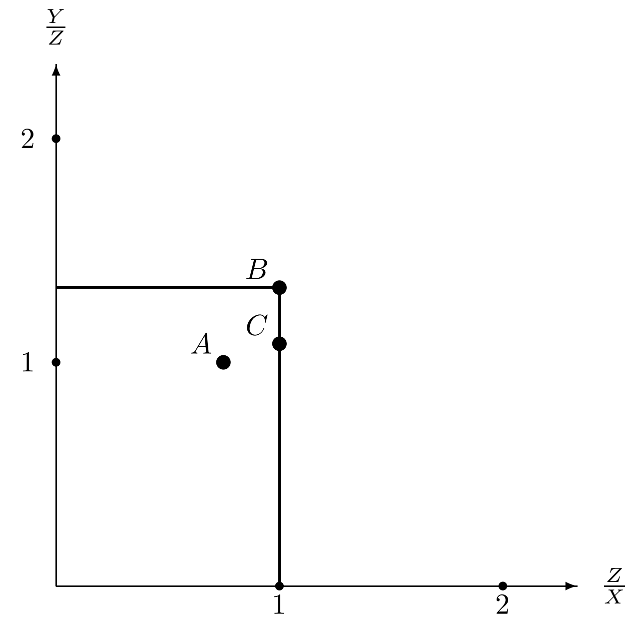

Fig. 2 PPS constructed by divided DMUs

Table 2 Numerical comparison between the TND-R and RDD-R methods and Izadikhah et al. (2018)

DMU	RDD-R method			TND-R method			Izadikhah et al. (2018)		
	Stage1	Stage 2	Overall	Stage1	Stage 2	Overall	Stage1	Stage 2	Overall
A	0.75	0.75	0.5625	0.8	0.8	0.640	0.75	0.80	0.60
B	1	1	1	1	1	1	1	1	1
C	1	0.95	0.95	1	0.952	0.952	1	0.96	0.96

obtained efficiency scores, only DMU B is individual stages ratio-efficient and overall ratio-efficient by our proposed methods and individual stages efficient and overall efficient by Izadikhah's method (Izadikhah et al. 2018). More importantly, these results show that proposed methods evaluate two-stage DMUs in constant return to scale technology.

In view of Fig. 2 and the results of Table 2, the proposed TND-R and RDD-R models are capable to compare the first (second) stage of each two-stage DMUs with that of others and ranking them. For example, in the first stage, two-stage DMU $A = (2, 1.5, 1.5)$ produces 1.5 output using 2 input and two-stage DMU $B = (4, 4, 5)$ produces 4 output using 4 input. So, ratio-efficiency of stage 1 of two-stage DMU B must be better (more) than that of two-stage DMU A. Table 2 confirms this fact. Also, the proposed models are capable to compare the overall efficiencies of two-stage DMUs and ranking them correctly. For example, two-stage DMU $A = (2, 1.5, 1.5)$ produces 1.5 output using 2 input and two-stage DMU $B = (4, 4, 5)$ produces 5 output using 4 input. It is reasonable to expected that overall efficiency of DMU B is better (more) than that of DMU A. Table 2 confirms this fact.

Table 3 Example 2. Data set for 10 two-stage DMUs

DMU	X_1	X_2	Z_1	Z_2	Y_1	Y_2	$\frac{Z}{X}$	$\frac{Y}{Z}$
1	2	3	4	9	16	18	$\left(2, \frac{4}{3}, \frac{9}{2}, 3\right)$	$\left(4, \frac{16}{9}, \frac{9}{2}, 2\right)$
2	8	9	2	3	1	1	$\left(\frac{1}{4}, \frac{2}{9}, \frac{3}{8}, \frac{1}{3}\right)$	$\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}\right)$
3	3	1	9	4	3	2	$\left(3, 9, \frac{4}{3}, 4\right)$	$\left(\frac{1}{3}, \frac{3}{4}, \frac{2}{9}, \frac{1}{2}\right)$
4	1	1	9	7	1	1	(9, 9, 7, 7)	$\left(\frac{1}{9}, \frac{1}{7}, \frac{1}{9}, \frac{1}{7}\right)$
5	1	1	1	1	1	1	(1, 1, 1, 1)	(1, 1, 1, 1)
6	2	2	3	3	2	2	$\left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right)$	$\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$
7	2	2	4	4	2	2	(2, 2, 2, 2)	$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$
8	6	6	3	3	6	6	$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$	(2, 2, 2, 2)
9	6	6	3	3	3	3	$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$	(1, 1, 1, 1)
10	2	1	9	4	3	2	$\left(\frac{9}{2}, 9, 2, 4\right)$	$\left(\frac{1}{3}, \frac{3}{4}, \frac{2}{9}, \frac{1}{2}\right)$

Table 4 Example 2. Results from RDD-R model (19) and TND-R model (16)

	TND-R model (16)			ranking			RDD-R model (19)			ranking		
DMU$_o$	**F**	**S**	**O**	**F**	**S**	**O**	**F**	**S**	**O**	**F**	**S**	**O**
1.	0.048	1.000	0.048	2	1	3	0.0806	1.000	0.0806	2	1	3
2.	0.018	0.070	0.001	7	6	9	0.0604	0.1619	0.0098	7	6	9
3.	1.000	0.091	0.091	1	3	1	1.000	0.1626	0.1626	1	5	5
4.	1.000	0.063	0.063	1	7	2	1.000	0.1473	0.1473	1	7	2
5.	0.020	0.111	0.0023	5	2	5	0.066	0.1998	0.0132	5	2	5
6.	0.022	0.086	0.0019	4	4	7	0.071	0.1755	0.0124	4	3	7
7.	0.024	0.077	0.0018	3	5	8	0.076	0.1656	0.0126	3	4	6
8.	0.019	1.000	0.019	6	1	4	0.062	1.000	0.0620	6	1	4
9.	0.019	0.111	0.0021	6	2	6	0.062	0.1998	0.0123	6	2	8
10.	1.000	0.091	0.091	1	3	1	1.000	0.1626	0.1626	1	5	1

"F","S" and "O" denote the first stage, the second stage and the overall stage of DMU_o, respectively

Example 2 Consider 10 two-stage DMUs with each DMUs consuming two inputs (X_1, X_2) to produce a two output (Z_1, Z_2) in stage 1 and consuming inputs (Z_1, Z_2) to produce two outputs (Y_1, Y_2) in stage 2. Data set for these DMUs are given in Table 3. Applying TND-R model (16) (with equal weights), RDD-R model (19) and model (21) in Despotis et al. (2016) to each divided $DMU_k, k = 1, \ldots, 10$, produces the results reported in Tables 4 and 5.

Similar to the example 1, this example shows the validity of proposed approaches. For example, two-stage DMU6 and DMU7 produce outputs (3, 3) and (4, 4), respectively, using the same inputs (2, 2) in stage 1. It is reasonable to expect that the rank of DMU7 must be better than that of DMU6. Table 4 confirms this fact. The ranking obtained for individual stages and overall system of all DMUs by TND-R, RDD-R models and Despotis et al. (2016) model (21) is shown in the 5th–7th columns and 11th–13th columns of Table 4 and 5th–7th columns of Table 5, respectively. More interestingly, the ranking results of our proposed methods and model (21) of Despotis et al. (2016) are very similar

Table 5 Results from Despotis et al. (2016) model (21)

	Despotis et al. (2016) model (21)			Ranking		
DMU_o	F	S	O	F	S	O
1.	0.642857	1.000	0.642857	2	1	1
2.	0.053571	0.1667	0.008928	7	5	7
3.	1.000	0.0833	0.083333	1	6	2
4.	1.000	0.071429	0.071429	1	7	3
5.	0.142857	0.5000	0.0714285	5	2	5
6.	0.214286	0.3333	0.0714286	4	3	4
7.	0.285714	0.2500	0.0714285	3	4	5
8.	0.071429	1.000	0.071429	6	1	3
9.	0.071429	0.5000	0.0357145	6	2	6
10.	1.000	0.0833	0.083333	1	6	2

to each other. Especially, the ranking results of stages 1 and 2 of the TND-R and RDD-R models and model (21) of Despotis et al. (2016) are almost the same. These results confirm the soundness of our model and its capacity of reliably solving the two-stage DEA problem.

Example 3 We here apply our new approaches to the 24 Taiwanese non-life insurance companies studied in Kao and Hwang (2008) where the inputs of stage 1 are Operation expenses (X1) and Insurance expenses (X2) and the output of stage 2 are Underwriting profit (Y1) and Investment profit (Y2). Also, the Direct written premiums (Z1) and Reinsurance premiums (Z2) are two intermediate measures, that is, the outputs of the stage 1 and the inputs to the stage 2. The data set appear in Table 6. In this example $m = s = D = 2$. Therefore, each divided data $DMU_j = \left(\frac{Z_j}{X_j}, \frac{Y_j}{Z_j}\right) \in \mathbb{R}^4 \times \mathbb{R}^4$. For such a two-stage structure and the data set, the efficiency results from RDD-R model (19) and TND-R model (16) are reported in Tables 7 and 8, respectively. As referred in Despic et al. (2007), DEA-R is comparable with conventional DEA; we compare the results obtained by our proposed RDD-R model (19) and TND-R model (16) and those by proposed methods in Chen et al. (2009) (models (18)–(19)), Despotis et al. (2016) (models (21) and (24) of Despotis et al. 2016) and Wang and Chin (2010) (models (19)–(21) and models (25), (27) and (29) with ($\lambda_1 = \frac{2}{5}, \lambda_2 = \frac{3}{5}$) of Wang and Chin 2010) and show there are some interesting relationship between them. The ranking results obtained from RDD-R model (19) for all the DMUs is shown in the 6th–8th columns of Table 7. Also, the ranking obtained from TND-R model (16) is shown in Table 8. For simplicity, all chosen Decision Maker's preference (weights) $W_j = (w_{1j}, w_{2j}, w_{3j}, w_{4j}),\ \ j = 1, 2$ in TND-R model (16) are given in Table 9. Table 10 shows the results obtained from Chen et al. (2009) (columns 2–5), Despotis et al. (2016) (columns 6–11) and Wang and Chin (2010) (columns 12–17). It is interesting to note that two-stage DMUs 9, 12, 15, 19, 24 are ratio-efficient in the first stage by our proposed models (for all weights in TND-R model) and all models in Table 10 (except for stage 1 of DMU 15 in Despotis et al. (2016) model (24)). Moreover, two-stage DMUs 3, 5, 17, 22 are ratio-efficient in the second stage by our proposed models (for all weights in TND-R model) and all models in Table (10). The ranking results for individual stages of all two-stage DMUs are almost

Table 6 Example 3. Data set for 24 Taiwanese non-life insurance companies (Kao and Hwang 2008)

DMU_j		Operation Expenses (X_1)	Insurance Expenses (X_2)	Direct written Premiums (Z_1)	Reinsurance Premiums (Z_2)	Underwriting Profit (Y_1)	Investment Profit (Y_2)
1	Taiwan Fire	1,178,744	673,512	7,451,757	856,735	984,143	681,687
2	Chung Kuo	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3	Tai Ping	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4	China Mariners	601,320	594,259	3,174,851	371,863	248,709	177,331
5	Fubon	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6	Zurich	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
7	Taian	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
8	Ming Tai	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
9	Central	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
10	The First	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
11	Kuo Hua	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12	Union	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
13	Shingkong	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
14	South China	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
15	Cathay Century	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
16	Allianz President	1,211,716	415,071	5,606,013	402,881	854,054	197,947
17	Newa	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
18	AIU	757,515	547,997	3,631,484	995,620	692,731	163,927
19	North America	159,422	182,338	1,141,951	483,291	519,121	46,857
20	Federal	145,442	53,518	316,829	131,920	355,624	26,537
21	Royal Sunalliance	84,171	26,224	225,888	40,542	51,950	6491
22	Aisa	15,993	10,502	52,063	14,574	82,141	4181
23	AXA	54,693	28,408	245,910	49,864	0.1	18,980
24	Mitsui Sumitomo	163,297	235,094	476,419	644,816	142,370	16,976

the same by our proposed models and all models in Table (10). But, there are some significant difference between the ranking results of our proposed models and those of the proposed models in overall in Table (10). Especially, DMU 24 is ranked as 24*th* DMU by Despotis et al. (2016) model (21) and 4*th* DMU by TND-R model (16). It may be due to round-off error problems. Comparison of the results obtained from the traditional CCR model (Kao and Hwang 2008) from one side and our proposed models from the other side, shows the similarity between them. DMUs 9, 12, 19 and 24 are stage 1 efficient in the traditional CCR, TND-R and RDD-R models. Also, DMUs 3, 5, 17 and 22 are stage 2 efficient in the CCR, TND-R and RDD-R models. The above discussion shows the validity and reliability of the proposed RDD-R and TND-R models.

Table 7 Example 3. Results from RDD-R model (19)

DMU_o		ρ_1^*	ρ_2^*	ρ_o^*	Ranking		
					Stage1	Stage2	Overall
1	Taiwan Fire	0.619136	0.272993	0.169020	4	10	10
2	Chung Kuo	0.685853	0.229561	0.157445	3	12	13
3	Tai Ping	0.225553	1.000000	0.225553	19	1	4
4	China Mariners	0.169137	0.193285	0.032692	18	18	24
5	Fubon	0.345343	1.000000	0.345343	7	1	2
6	Zurich	0.759439	0.222488	0.168966	2	14	11
7	Taian	0.210330	0.302617	0.063649	14	7	19
8	Ming Tai	0.253918	0.291722	0.074073	11	8	16
9	Central	1.000000	0.222597	0.222597	1	13	5
10	The First	0.200523	0.338090	0.067795	16	4	18
11	Kuo Hua	0.313446	0.221123	0.069310	8	15	17
12	Union	1.000000	0.307677	0.307677	1	6	3
13	Shingkong	0.312041	0.320531	0.100019	9	5	15
14	South China	0.207822	0.282088	0.058624	15	9	20
15	Cathay Century	1.000000	0.380049	0.380049	1	3	1
16	Allianz President	0.529846	0.233189	0.123554	5	11	14
17	Newa	0.193807	1.000000	0.193807	17	1	7
18	AIU	0.242163	0.187372	0.045375	12	20	23
19	North America	1.000000	0.210062	0.210062	1	16	6
20	Federal	0.350592	0.505326	0.177163	6	2	9
21	Royal Sunalliance	0.238490	0.200527	0.047824	13	17	22
22	Aisa	0.167047	1.000000	0.167047	19	1	12
23	AXA	0.298417	0.195809	0.058433	10	19	21
24	Mitsui Sumitomo	1.000000	0.180817	0.180817	1	21	8

Conclusion

In network DEA, if all data are expressed as ratio output/input or if in the production process an input is not used for producing a particular output or if data ratio is important to managers, traditional two-stage DEA models can no longer be used. Flexibility of the DEA-R in incorporating the mentioned restrictions and expert opinions to the DEA models gave us the motivation to apply DEA-R for the evaluation of two-stage DMUs. Also, the main contribution of this paper is to evaluate the two-stage DMUs using DEA-R to overcome the mentioned restrictions and shortfalls. To this end, we propose two novel models namely, RDD-R and TND-R to calculate individual ratio efficiencies and overall ratio efficiency of two-stage DMUs. Moreover, a decision-maker can consider the preference of each stage over other stages by selecting weights in the TND-R model. Also, the RDD-R model projects all divided DMU_o under evaluation on the strong frontier of production possibility set (see proposition 6). We compared our proposed methods with some existing methodologies to show validity and reliability of the of proposed approaches and "two-stage DEA based on DEA-R." The results show that our approaches are similar to the existing approaches. We evaluated two-stage DMUs using DEA-R where all the outputs from the first stage are the only inputs to the second stage. Initial studies had shown that our approach can be applied with free-link, and fixed-link assumptions. Moreover, it can be extended to

Table 8 Example 3. Results from TND-R model (16)

DMU_o	(W_1^1, W_2^1) (F, S, O)	Ranking	(W_1^2, W_2^2) (F, S, O)	Ranking	(W_1^3, W_2^3) (F, S, O)	Ranking (F, S, O)
1. Taiwan Fire	(0.402, 0.758, 0.305)	(5, 4, 8)	(0.518, 0.111, 0.058)	(4, 4, 10)	(0.026, 0.834, 0.022)	(4, 4, 9)
2. Chung Kuo	(0.934, 0.713, 0.666)	(2, 7, 3)	(0.958, 0.090, 0.087)	(2, 7, 8)	(0.360, 0.799, 0.288)	(2, 7, 6)
3. Tai Ping	(0.085, 1.000, 0.085)	(16, 1, 15)	(0.129, 1.000, 0.129)	(15, 1, 5)	(0.004, 1.000, 0.0036)	(10, 1, 16)
4. China Mariners	(0.065, 0.615, 0.040)	(21, 12, 23)	(0.100, 0.060, 0.006)	(20, 11, 22)	(0.003, 0.718, 0.002)	(11, 12, 22)
5. Fubon	(0.122, 1.000, 0.122)	(11, 1, 11)	(0.182, 1.000, 0.182)	(10, 1, 1)	(0.006, 1.000, 0.006)	(8, 1, 11)
6. Zurich	(0.486, 0.603, 0.293)	(3, 14, 9)	(0.602, 0.057, 0.034)	(3, 12, 13)	(0.036, 0.708, 0.026)	(3, 14, 7)
7. Taian	(0.078, 0.640, 0.050)	(18, 9, 21)	(0.120, 0.066, 0.008)	(17, 9, 20)	(0.003, 0.740, 0.0028)	(11, 9, 21)
8. Ming Tai	(0.082, 0.624, 0.051)	(17, 11, 20)	(0.125, 0.062, 0.008)	(16, 10, 20)	(0.004, 0.726, 0.0029)	(10, 11, 20)
9. Central	(1.000, 0.552, 0.552)	(1, 20, 6)	(1.000, 0.047, 0.047)	(1, 17, 12)	(1.000, 0.663, 0.663)	(1, 20, 5)
10. The First	(0.115, 0.734, 0.085)	(12, 5, 15)	(0.172, 0.099, 0.017)	(11, 5, 15)	(0.005, 0.815, 0.0042)	(9, 5, 14)
11. Kuo Hua	(0.109, 0.516, 0.056)	(14, 21, 19)	(0.164, 0.041, 0.007)	(13, 18, 21)	(0.005, 0.630, 0.0031)	(9, 21, 18)
12. Union	(1.000, 0.793, 0.793)	(4, 3, 1)	(1.000, 0.133, 0.133)	(1, 3, 4)	(1.000, 0.860, 0.860)	(9, 3, 1)
13. Shingkong	(0.110, 0.590, 0.065)	(13, 17, 18)	(0.165, 0.054, 0.009)	(12, 14, 13)	(0.005, 0.697, 0.0032)	(9, 16, 17)
14. South China	(0.076, 0.639, 0.049)	(19, 10, 22)	(0.116, 0.066, 0.008)	(18, 9, 20)	(0.003, 0.739, 0.002)	(11, 10, 22)
15. Cathay Century	(1.000, 0.716, 0.716)	(1, 6, 2)	(1.000, 0.092, 0.092)	(1, 6, 7)	(1.000, 0.802, 0.802)	(1, 6, 2)
16. Allianz President	(0.196, 0.589, 0.115)	(7, 18, 12)	(0.280, 0.054, 0.015)	(6, 14, 16)	(0.010, 0.696, 0.007)	(5, 17, 10)
17. Newa	(0.070, 1.000, 0.070)	(20, 1, 17)	(0.108, 1.000, 0.108)	(19, 1, 6)	(0.003, 1.000, 0.003)	(11, 1, 19)
18. AIU	(0.149, 0.596, 0.089)	(9, 15, 14)	(0.219, 0.056, 0.012)	(8, 13, 17)	(0.007, 0.703, 0.005)	(7, 15, 12)
19. North America	(1.000, 0.613, 0.613)	(1, 13, 4)	(1.000, 0.060, 0.060)	(1, 11, 9)	(1.000, 0.717, 0.717)	(3, 13, 3)
20. Federal	(0.398, 0.904, 0.360)	(6, 2, 7)	(0.514, 0.273, 0.140)	(5, 2, 2)	(0.026, 0.938, 0.024)	(4, 2, 8)
21. Royal Sunalliance	(0.142, 0.558, 0.079)	(10, 19, 16)	(0.210, 0.048, 0.010)	(9, 16, 18)	(0.007, 0.669, 0.0044)	(7, 19, 13)
22. Aisa	(0.090, 1.000, 0.090)	(15, 1, 13)	(0.137, 1.000, 0.137)	(14, 1, 3)	(0.004, 1.000, 0.0039)	(10, 1, 15)
23. AXA	(0.182, 0.673, 0.123)	(8, 8, 10)	(0.263, 0.076, 0.020)	(7, 8, 14)	(0.009, 0.767, 0.007)	(6, 8, 10)
24. Mitsui Sumitomo	(1.000, 0.582, 0.582)	(1, 19, 5)	(1.000, 0.053, 0.053)	(1, 15, 11)	(1.000, 0.690, 0.690)	(1, 18, 4)

"F","S" and "O" denote the first stage efficiency score, the second stage efficiency score and overall efficiency score, respectively

Table 9 Example 3. Chosen weights for TND-R model (16)

i.	$\mathbf{W}_1^i$ $(w_{11}, w_{21}, w_{31}, w_{41})$	$\mathbf{W}_2^i$ $(w_{12}, w_{22}, w_{32}, w_{42})$
1.	(0.125, 0.125, 0.125, 0.125)	(0.125, 0.125, 0.125, 0.125)
2.	(0.2, 0.2, 0.2, 0.2)	(0.005, 0.005, 0.005, 0.005)
3.	(0.005, 0.005, 0.005, 0.005)	(0.2, 0.2 ,0.2, 0.2)

Table 10 Example 3. Results from Chen et al. (2009), Despotis et al. (2016) and Wang and Chin (2010)

	Chen et al. (2009)		**Despotis et al. (2016)**				**Wang and Chin (2010)**	$((\lambda_1, \lambda_2) = (\frac{2}{5}, \frac{3}{5}))$
	Models (11)–(13)	**Ranking**	**Model (21)**	**Ranking**	**Model (24)**	**Ranking**	**Models (24), (26),(28)**	**Ranking**
DMU_o	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**
1. Taiwan Fire	(0.993, 0.704, 0.849)	(3, 4, 3)	(0.9926, 0.7045, 0.6992)	(3, 5, 3)	(0.9848, 0.7054, 0.6947)	(4, 4, 3)	(0.9926, 0.7045, 0.8202)	(3, 4, 4)
2. Chung Kuo	(0.998, 0.626, 0.812)	(2,5,5)	(0.9985, 0.6257, 0.6248)	(2, 6, 5)	(0.9971, 0.6260, 0.6242)	(2,7,5)	(0.9985, 0.6257, 0.7750)	(2, 5, 6)
3. Tai Ping	(0.690, 1, 0.817)	(18,1,4)	(0.6900, 1, 0.690)	(19, 1, 4)	(0.6900, 1, 0.6900)	(17,1,4)	(0.6900, 1.000, 0.8477)	(16, 1, 3)
4. China Mariners	(0.724, 0.420, 0.596)	(16,11,16)	(0.7243, 0.4200, 0.3042)	(17, 12, 15)	(0.7181, 0.4202, 0.3018)	(16,12,15)	(0.7243, 0.4200, 0.5659)	(15, 12, 15)
5. Fubon	(0.831, 0.923, 0.873)	(9,2,2)	(0.8307, 0.9233, 0.7670)	(9, 3, 1)	(0.8011, 0.9457, 0.7577)	(11,2,2)	(0.8307, 0.9233, 0.8821)	(9, 2, 1)
6. Zurich	(0.961, 0.406, 0.689)	(4,13,11)	(0.9606, 0.4057, 0.3897)	(4, 14, 12)	(0.9619, 0.4037, 0.3883)	(5,15,12)	(0.9606, 0.4057, 0.6330)	(4, 15, 12)
7. Taian	(0.752, 0.352, 0.580)	(11,17,18)	(0.7521, 0.3522, 0.2649)	(12, 17, 18)	(0.6827, 0.4026, 0.2748)	(19,16,18)	(0.6706, 0.4124, 0.5411)	(17, 13, 18)
8. Ming Tai	(0.726, 0.378, 0.579)	(14,14,19)	(0.7256, 0.3780, 0.2743)	(15, 15, 16)	(0.6748, 0.4076, 0.2750)	(22,14,17)	(0.7256, 0.3780, 0.5445)	(14, 16, 17)
9. Central	(1, 0.223, 0.612)	(1,21,20)	(1, 0.2233, 0.2233)	(1, 22, 20)	(0.9437, 0.2323, 0.2192)	(6,21,20)	(1.000, 0.2233, 0.5340)	(1, 21, 20)
10. The First	(0.862, 0.541, 0.713)	(7,8,9)	(0.8615, 0.5408, 0.4660)	(7, 9, 9)	(0.7845, 0.5597, 0.4391)	(12,9,9)	(0.8615, 0.5408, 0.6807)	(7, 9, 9)
11. Kuo Hua	(0.729, 0.207, 0.509)	(13,22,24)	(0.7292, 0.2066, 0.1507)	(14, 23, 23)	(0.6899, 0.2276, 0.1570)	(18,22,23)	(0.7292, 0.2066, 0.4562)	(13, 22, 23)
12. Union	(1, 0.760, 0.880)	(1,3,1)	(1, 0.7596, 0.7596)	(1, 4, 2)	(1, 0.7596, 0.7596)	(1,3,1)	(1.000, 0.7596, 0.8557)	(1, 3, 2)
13. Shin-gkong	(0.811, 0.243, 0.557)	(10,20,21)	(0.8107,0.2431,0.1970)	(10, 21, 21)	(0.6794, 0.3052, 0.2073)	(20,19,21)	(0.8107, 0.2431, 0.4992)	(10, 20, 21)
14. South China	(0.725, 0.374, 0.577)	(15,15,20)	(0.7246, 0.3740, 0.2710)	(16, 10, 17)	(0.6777, 0.4222, 0.2861)	(21,11,16)	(0.6699, 0.4309, 0.5501)	(18, 11, 16)
15. Cathay Century	(1, 0.614, 0.807)	(1,6,9)	(1, 0.6138, 0.6138)	(1, 7, 6)	(0.9371, 0.6376, 0.5976)	(7,6,6)	(1.000, 0.6138, 0.7683)	(1, 6, 7)

Table 10 (continued)

	Chen et al. (2009)		Despotis et al. (2016)				Wang and Chin (2010)	$((\lambda_1, \lambda_2) = (\frac{2}{5}, \frac{3}{5}))$
	Models (11)–(13)	**Ranking**	**Model (21)**	**Ranking**	**Model (24)**	**Ranking**	**Models (24), (26),(28)**	**Ranking**
DMU_o	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**	**(F, S, O)**
16. Allianz President	(0.886, 0.362, 0.639)	(6, 16, 13)	(0.9072, 0.3356, 0.3044)	(6, 18, 14)	(0.8871, 0.3597, 0.3191)	(8,17,14)	(0.8856, 0.3615, 0.5866)	(6, 17, 14)
17. Newa	(0.723, 0.460, 0.613)	(17,10,14)	(0.7232, 0.4597, 0.3325)	(18, 11, 13)	(0.5668, 0.6183, 0.3504)	(24, 8, 13)	(0.6276, 0.5736, 0.6014)	(19, 8, 13)
18. AIU	(0.794, 0.326, 0.587)	(10,18,17)	(0.7935, 0.3262, 0.2588)	(11, 19, 19)	(0.7691, 0.3335, 0.2565)	(14, 18, 19)	(0.7935, 0.3262, 0.5396)	(11, 18, 19)
19. North America	(1, 0.411, 0.706)	(1,12,10)	(1, 0.4112, 0.4112)	(1, 13, 11)	(0.9962, 0.4120, 0.4104)	(3, 13, 11)	(1.000, 0.4112, 0.6467)	(1, 14, 11)
20. Federal	(0.933, 0.586, 0.765)	(5,7,7)	(0.9332, 0.5857, 0.5465)	(5, 2, 8)	(0.7712, 0.6763, 0.5216)	(13, 5, 8)	(0.9332, 0.5857, 0.7305)	(5, 7, 8)
21. Royal Sunalliance	(0.751, 0.262, 0.541)	(12,19,23)	(0.7505, 0.2623, 0.1969)	(13, 20, 22)	(0.7434, 0.2668, 0.1984)	(15,20,22)	(0.7321, 0.2743, 0.4925)	(12, 19, 22)
22. Aisa	(0.590, 1, 0.742)	(19,1,8)	(0.5895, 1, 0.5895)	(20, 1, 7)	(0.5895, 1, 0.5895)	(23, 1, 7)	(0.5895, 1.000, 0.7822)	(20, 1, 5)
23. AXA	(0.843, 0.499, 0.685)	(8,9,12)	(0.8426, 0.4989, 0.4203)	(8, 10, 10)	(0.8141, 0.5079, 0.4135)	(10, 10, 10)	(0.8426, 0.4989, 0.6507)	(8, 10, 10)
24. Mitsui Sumitomo	(1, 0.087, 0.544)	(1,23,22)	(1, 0.0870, 0.0870)	(1, 24, 24)	(0.8267, 0.1255, 0.1037)	(9, 23, 24)	(1.000, 0.0870, 0.4522)	(1, 23, 24)

"F","S" and "O" denote the first stage efficiency score, the second stage efficiency score and overall efficiency score, respectively

the more complicated two-stage DEA models. For example, it can be extended to two-stage DEA models with a shared resources and feedback and in case there are external inputs. This, however, needs further research and a deeper analysis.

Abbreviations
CCR:: Charnes, Cooper and Rhodes; DMU:: Decision making unit; DEA:: Data envelopment analysis; DEA-R:: DEA-ratio; RDD-R:: Range directional DEA-R; TND-R:: Tchebycheff norm DEA-R; CRS:: Constant returns to scale; VRS:: Variable returns to scale; SBM:: Slack-based measure; R-CRS:: Ratio-CRS; R-VRS:: Ratio-VRS; LP:: Linear programming.

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Declarations

Competing interests
The author declares that he has no competing interests.

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References
Allen R, Athanassopoulos A, Dyson RG, Thanassoulis E (1997) Weights restrictions and value judgments in data envelopment analysis: evolution development and future directions. Ann Oper Res 73:13–34
An Q, Ping W, Emrouznejad A, Junhua H (2020) Fixed cost allocation based on the principle of efficiency invariance in two-stage systems. Eur J Oper Res 283(2):662–675
Ang S, Chen CM (2016) Pitfalls of decomposition weights in the additive multi-stage DEA model. Omega 58:139–153
Banker RD, Charnes A, Cooper WW (1984) Some models for the estimation of technical and scale inefficiencies in data envelopment analysis. Manage Sci 4:1078–1092
Borges MR, Nektarios M, Barros CP (2008) Analysing the efficiency of the Greek life insurance industry. Eur Res Stud 11(3):35–52
Castelli L, Pesenti R, Ukovich W (2004) DEA-like models for the efficiency evaluation of hierarchically structured units. Eur J Oper Res 154(2):465–476
Charnes A, Cooper WW, Rhodes E (1978) Measuring the efficiency of decision-making units. Eur J Oper Res 2:429–444
Charnes A, Cooper WW, Golany B, Seiford LM, Stutz J (1985) Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions. J Econ 30:91–107
Chen Y, Cook WD, Li N, Zhu J (2009) Additive efficiency decomposition in two-stage DEA. Eur J Oper Res 196:1170–1176
Chen Y, Zhu J (2004) Measuring information technology's indirect impact on firm performance. Inf Technol Manage 5(1–2):9–22
Chen Y, Liang L, Zhu J (2009) Equivalence in two-stage DEA approaches. Eur J Oper Res 193:600–604
Cummins JD, Weiss MA (2013) Analyzing firm performance in the insurance industry using frontier efficiency and productivity methods. Handbook of Insurance, pp 795–861
Cook WD, Zhu J, Bi G, Yang F (2010) Network DEA: additive efficiency decomposition. Eur J Oper Res 207:1122–1129
Despic O, Despic M, Paradi JC (2007) DEA-R: ratio-based comparative efficiency model, its mathematical relation to DEA and its use in applications. J Prod Anal 28:33–44
Despotis DK, Koronakos G, Dimitris S (2016) Composition versus decomposition in two-stage network DEA: a reverse approach. J Prod Anal 45(1):71–87
Emrouznejad A, Amin GR (2009) DEA models for ratio data: convexity consideration. Appl Math Model 33(1):486–498
Fecher F, Kessler D, Perelman S, Pestieau P (1993) Productive performance of the French insurance industry. J Product Anal 4:77–93
Fernandez-Castro A, Smith P (1994) To wards a general non-parametric model of corporate performance. Omega 22(3):237–49
Färe R, Grosskopf S (1996) Productivity and intermediate products: a frontier approach. Econ Lett 50(1):65–70
Färe R, Grosskopf S (2000) Network DEA. Socio-Econ Plan Sci 34(1):34–49
Färe R, Whittaker G (1995) An intermediate input model of dairy production using complex survey data. J Agric Econ 46(2):201–213
Geoffrion AM (1964) Proper efficiency and theory of vector maximization. J Math Anal Appl 22:618–630
Gerami J, Mozaffari MR (2013) Measuring performance of network structure by DEA-R model. Adv Environ Biol 7(7):1224–1232
Gerami J, Mozaffari MR, Wanke PF (2020) A multi-criteria ratio-based approach for two-stage data envelopment analysis. Expert Syst Appl https://doi.org/10.1016/j.eswa.2020.113508
Gerami J, Kiani Mavi R, Frzipoor Saen R, Kiani Mavi N (2020) A novel network DEA model for evaluating hospital service supply chain performance. Ann Oper Res 295:843–880. https://doi.org/10.1007/s10479-020-03744-z
Guo Ch, Wei F, Ding T, Zhang L, Liang L (2017) Multistage network DEA: decomposition and aggregation weights of component performance. Comput Ind Eng 133:64–74
Guo C, Abbasi SR, Foroughi AA, Zhu J (2017) Decomposition weights and overall efficiency in two-stage additive network DEA. Eur J Oper Res 257:896–906
Hwang S-N, Kao C (2006) Measuring managerial efficiency in non-life insurance companies: an application of two-stage data envelopment analysis. Int J Manag 23(3):699–720
Hatami-Marbini A, Toloo M (2019) Data envelopment analysis models with ratio data: a revisit. Comput Ind Eng 133:331–338
Izadikhah M, Farzipoor Saen R (2016) Evaluating sustainability of supply chains by two-stage range directional measure in the presence of negative data. Transp Res Part D 49:110–126
Izadikhah M, Tavana M, Di Caprio D, Santos-Arteaga FJ (2018) A novel two-stage DEA production model with freely distributed initial inputs and shared intermediate outputs. Expert Syst Appl 99:213–230
Khalili M, Camanho AS, Portela MCAS, Alirezaee MR (2010) The measurement of relative efficiency using data envelopment analysis with assurance regions that link inputs and outputs. Eur J Oper Res 203:761–770
Ostovan S, Mozaffari MR, Jamshidi A, Gerami J (2020) Evaluation of two-stage networks based on average efficiency using DEA and DEA-R with Fuzzy data. Int J Fuzzy Syst 22(1):1665–1678
Kao C, Hwang S-N (2008) Efficiency decomposition in two-stage data envelopment analysis: an application to non-life insurance companies in Taiwan. Eur J Oper Res 185(1):418–429

Kao C (2014) Network data envelopment analysis: a review. Eur J Oper Res 239:1–16
Kaffash S, Azizi R, Huang Y, Zhu J (2019) A survey of data envelopment analysis applications in the insurance industry 1993–2018. Eur J Oper Res 284(3):801–813
Kamyab P, Mozaffari MR, Gerami J, Wanke PF (2021) Two-stage incentives system for commercial banks based on centralized resource allocation model in DEA-R. Int J Product Perform Manag 70(2):427–458
Lim S, Zhu J (2016) A note on two-stage network DEA model: Frontier projection and duality. Eur J Oper Res 248:342–346
Liu WB, Zhang DQ, Meng W, Li XX (2011) A study of DEA models without explicit inputs. Omega 39:472–480
Lozano S, Gutiérrez E, Moreno P (2013) Network DEA approach to airports performance assessment considering undesirable outputs. Appl Math Model 37:1665–1676
Lozano S (2015) Alternative SBM model for network DEA. Comput Ind Eng 82:33–40
Lozano S (2015) A joint-inputs network DEA approach to production and pollution generating technologies. Expert Syst Appl 42(21):7960–7968
Ilyas AM (2019) An empirical investigation of efficiency and productivity in the Indian non-life insurance market. Benchmark Int J 26(7):2343–2371
Mozaffari MR, Gerami J, Jablonsky J (2014) Relationship between DEA models without explicit inputs and DEA-R models. CEJOR 22(1):1–12
Mozaffari MR, Dadkhah F, Jablonsky J, Wanke PF (2020) Finding efficient surfaces in DEA-R models. Appl Math Comput 386:125497
Noulas AG, Hatzigayios T, Lazaridis J, Lyroudi K (2001) Non-parametric production frontier approach to the study of efficiency of non-life insurance companies in Greece. J Financ Manag Anal Int Rev Finance 14(1):19–26
Olesen OB, Petersen NC, Podinovski VV (2015) Efficiency analysis with ratio measures. Eur J Oper Res 245(2):446–462
Olesen OB, Petersen NC, Podinovski VV (2017) Efficiency measures and computational approaches for data envelopment analysis models with ratio inputs and outputs. Eur J Oper Res 261:640–655
Podinovski VV, Athanassopoulos A (1998) Assessing the relative efficiency of decision making units using DEA models with weight restrictions. J Oper Res Soc 49:500–508
Seiford LM, Zhu J (1999) Profitability and marketability of the top 55 US commercial banks. Manag Sci 45:1270–1288
Seiford LM, Zhu J (1999) Profitability and marketability of the top 55 U.S. commercial banks. Manag Sci 45(9):1270–1288. https://doi.org/10.1287/mnsc.45.9.1270
Silva Portela MCA, Thanassoulis E, Simpson G (2004) Negative data in DEA: a directional distance function approach applied to bank branches. J Oper Res Soc 55:1111–1121
Song M, Wu J, Wang Y (2011) An extended aggregated ratio analysis in DEA. J Syst Sci Syst Eng 20(2):249–256
Sotiros D, Koronakos G, Despotis DK (2019) Dominance at the divisional efficiencies level in network DEA: the case of two-stage processes. Omega 85:144–155
Sexton T, Lewis H (2003) Two-stage DEA: an application to major league baseball. J Prod Anal 19(2–3):227–249
Tavares G, Antunes CH (2001) A Tchebycheff DEA model. In: Rutcor Research Report
Tohidi G, Razavyan S, Tohidnia S (2014) Profit malmquist index and its global form in the presence of the negative data in DEA. J Appl Math 1–8
Tohidnia S, Tohidi G (2019) Measuring productivity change in DEA-R: a ratiobased profit efficiency model. J Oper Res Soc 70(9):1511–1521
Tone K, Tsutsui M (2009) Network DEA: a slacks-based measure approach. Eur J Oper Res 197:243–252
Wang C, Gopal R, Zionts S (1997) Use of data envelopment analysis in assessing information technology impact on firm performance. Ann Oper Res 73:191–213. https://doi.org/10.1023/A:1018977111455
Wang YM, Chin KS (2010) Some alternative DEA models for two-stage process. Expert Syst Appl 37:8799–8808
Wei CK, Chen LC, Li RK, Tsai CH (2011) A study of developing an input oriented ratio-based comparative efficiency model. Expert Syst Appl 38:2473–2477
Wei CK, Chen LC, Li RK, Tsai CH (2011) Exploration of efficiency underestimation of CCR model: based on medical sectors with DEA-R model. Expert Syst Appl 38:3155–3160
Wei CK, Chen LC, Li RK, Tsai CH (2011) Using the DEA-R model in the hospital industry to study the pseudo-inefficiency problem. Expert Syst Appl 38:2172–2176
Wu D, Liang L (2005) Aggregated ratio analysis in DEA. Int J Inf Technol Decis Mak 4(3):369–384
Yu Y, Shi Q (2014) Two-stage DEA model with additional input in the second stage and part of intermediate products as final output. Expert Syst Appl 41(15):6570–6574
Zha Y, Liang L (2010) Two-stage cooperation model with input freely distributed among the stages. Eur J Oper Res 205(2):332–338
Zhang L, Hu H, Zhang D (2015) A credit risk assessment model based on SVM for small and medium enterprises in supply chain finance. Financ Innov 1(14):1–21
Tone K, Kweh QL, Lu W, Ting IWK (2019) Modeling investments in the dynamic network performance of insurance companies. Omega 88:237–247
Eling M, Jia R (2019) Efficiency and profitability in the global insurance industry. Pac Basin Finance J 57:101190
Kou G, Peng Y, Wang G (2014) Evaluation of clustering algorithms for financial risk analysis using MCDM methods. Inf Sci 275:1–12
Kasimbeyli R (2010) A nonlinear cone separation theorem and scalarization in nonconvex vector optimization. SIAM J Optim 20(3):1591–1619
Kou G, Olgu Akdeniz Ö, Dinçer H, Yäksel S (2021) Fintech investments in European banks: a hybrid IT2 fuzzy multidimensional decision-making approach. Financ Innov 7:39
Zha Q, Kou G, Zhang H et al (2020) Opinion dynamics in finance and business: a literature review and research opportunities. Financ Innov 6:44

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