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Overall profit Malmquist productivity index under data uncertainty

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Abstract
The calculation of the overall profit Malmquist productivity index (MPI) requires precise and accurate information on the input, output, input-output prices of each decision making unit (DMU). However, in many situations, some inputs and/or outputs and input-output prices are imprecise. As such, we consider the overall profit MPI problem when the input, output, and input-output prices are imprecise and vary over intervals, showing that method (MCM 54: 2827–2838, 2011) has some shortfalls. To remedy these shortfalls, we propose another method for measuring the overall profit MPI when the inputs, outputs, and price vectors vary over intervals. That is, to calculate the overall profit efficiency intervals, cone-ratio data envelopment analysis models can be applied to the incorporated information as weight restrictions. Further, we provide a new approach to calculating the upper bound of the overall profit efficiency of each DMU. A numerical example is provided for illustrating the proposed method.

Keywords: Data envelopment analysis, Imprecise data, Profit Malmquist productivity index

Introduction

The Malmquist productivity index (MPI) is one of the most popular approaches to measuring productivity changes over time and was introduced by Malmquist (1953). Under data envelopment analysis (DEA), productivity is defined as the ratio between efficiency and is measured by MPI for the same decision making unit (DMU) in two different periods. Caves et al. (1982a;1982b) proposed an MPI as the ratio of two input distance functions to calculate the relative performance of a DMU in different periods. Färe et al. (1994) extended the approach of Caves et al. (1982a) and constructed an MPI directly using input and output data as the geometric mean of the MPIs calculated in the two base periods. Using Farrell's (1957) methodology for the measurement of efficiency and that of Caves et al. (1982a) on the measurement of productivity, Färe et al. (1994) constructed an MPI directly from input and output data using DEA. However, this conventional profit MPI requires the input-output quantity and exact input-output prices to be available. However, in many situations, some inputs and/or outputs and input-output prices have imprecise data. Therefore, conventional profit MPI models are not suitable or applicable to measuring overall profit MPI.

Asmild et al. (2007) presented a framework in which DEA was used to measure the overall efficiencies of different behavioral objectives. Furthermore, they showed how this framework could be applied to assess the effectiveness of more general behavioral goals.

These objectives are revenue maximization, cost minimization, and profit maximization. Asmild et al. (2007) clarified the relationships between various cone-ratio DEA (CR-DEA) models and those used to measure overall efficiency. Aghayi et al. () evaluated the MPI of DMUs with desirable and undesirable interval outputs. To deal with data uncertainty, a fuzzy approach was proposed by Wanke et al. (2016), who also calculated the efficiency of banks. Mashayekhi and Omrani (2016) used a fuzzy approach for sorting genetic algorithms with uncertain data. Salehpour and Aghayi (2015) calculated revenue efficiency under price uncertainty to find a solution to minimizing the worst-case performance with uncertain data. Hatami-Marbini et al. (2018) developed an overarching evaluation process for estimating the RTS of DMUs under imprecise DEA (IDEA), where the input and output data lie within bounded intervals. For more on IDEA, see Shabani et al. (2019), Ebrahimi (2018), Fazelabdolabadi (2019), Toloo et al. (2018), Shokouhi et al. (2014), Hatami-Marbini et al. (2017), Ureña et al. (2019), Zhang et al. (2019), Kao et al. (2014) and the references therein.

The MPI computation using DEA with uncertain data has not been studied widely in the literature. For instance, Emrouznejad et al. (2011) studied the overall profit MPI using DEA with fuzzy and interval data. They extended the model (39) of Asmild et al. (2007) and proposed two methods for measuring the overall profit MPI when the input, output, and price vectors are fuzzy or vary over intervals (see Emrouznejad et al. (2011), models (3a) and (3b)). To the best of our knowledge, to compute the overall profit MPI of each DMU, the profit efficiency at time period t $[(t+1)$ must be computed using the technology and input-output prices at time period $t+1]$ (t) (see (Tohidi et al. 2010; Tohidi et al. 2014). However, in Emrouznejad et al.'s (2011) method, the input-output prices in periods t and $t+1$ are used simultaneously, meaning the results are not reasonable (see subsection 2.2. for details). As such, this paper overcomes the shortfall of Emrouznejad et al.'s model (3b) (Emrouznejad et al. 2011). Park (2001) introduced an approach to deal with IDEA involving variable input and output. He considered the multiplier and envelopment IDEA models (Cooper et al. 1999; Lee et al. 2002) and clarified the relationships between them. These models yield an upper and a lower bound on efficiency, respectively. Mostafaee and Saljooghi (2010) extended the classical cost efficiency models to include data uncertainty. However, Fang and Li (2012) showed that Mostafaee and Saljooghi's (2010) approach had some drawbacks. Then, Fang and Li (2013) extended Park's approach and presented an alternative IDEA method to calculate an upper and a lower bound of cost efficiency measurement in the presence of imprecise price inputs. Based on the studies of Park (2001) and Emrouznejad et al. (2011), this paper introduces alternative methods for measuring the overall profit MPI when the input, output, and input-output prices are uncertain and also measures the lower and upper bounds of overall profit MPIs. We show that the upper bound of the overall profit efficiency is obtained by incorporating uncertain data as interval data directly into the overall profit efficiency models. In addition, the lower bound of the overall profit efficiency is achieved by incorporating the same uncertain data as weight restrictions into a CR-DEA model. The main contributions of this paper are as follows: (a) we calculate the overall profit MPI assuming that input, output, and input-output prices are imprecise; (b) we characterized these imprecise data with interval methods; (c) we propose models to measure the overall profit efficiency in adjacent periods, which has a reasonable interpretation (see model (4)); (d) we establish new models to compute the upper bounds for the profit efficiency measures; (e) we extend the model using CR-DEA; and (f) we demonstrate the practical aspects of our model using a numerical example.

The remainder of this paper is organized as follows. "Overall profit efficiency and MPIs" section presents an overview of overall profit efficiency and Malmquist indices. "Main results" section proposes models to calculate the lower and upper bounds of the profit efficiency of each DMU within the period and for adjusted periods. In "Computational aspects" section, we develop new methods to calculate the upper bounds of the overall profit efficiency of each DMU. A numerical example is also provided in "Computational aspects" section. Finally, "Conclusions" section concludes the paper.

Overall profit efficiency and MPIs

MPIs measure the productivity change of a DMU between two different time periods. Färe et al. (1994, 1992) developed an input based non-parametric Malmquist index using DEA. This DEA-based Malmquist productivity can be extended to measure the productivity changes of DMUs over time. Here, we discuss the overall profit efficiency and overall profit MPIs.

Overall profit efficiency

Consider a set of n DMUs associated with m inputs and s outputs. Particularly, DMU_j $(j \in J = 1, ..., n)$ consumes amount x_{ij} of input i and produces amount y_{rj} of output r. Let $X_j = (x_{1j}, ..., x_{mj})$, where $X_j \geq 0$ & $X_j \neq 0$ and $Y_j = (y_{1j}, ..., y_{rj})$ and where $Y_j \geq 0$ & $Y_j \neq 0$. In addition, c and r are the input and output price vectors, respectively, for DMU_j $(j \in J = 1, ..., n)$, where $c \geq 0$ and $r \geq 0$, $c \neq 0$, and $r \neq 0$. Asmild (2007) presented the following model for measuring the overall profit efficiency of $DMU_o = (x_o, y_o)$, $(o = 1, ..., n)$:

$$\begin{array}{ll} \max & \frac{r_o^T y}{r_o^T y_o} - \frac{c_o^T x}{c_o^T x_o} \\ s.t. & -\sum_{j\in J} \lambda_j y_j + y \leq 0 \\ & \sum_{j\in J} \lambda_j x_j - x \leq 0 \\ & \sum_{j\in J} \lambda_j = 1, \\ & \lambda_j \geq 0 \end{array} \tag{1}$$

where x, y, and $\lambda_j, j \in J$ are variables and the objective function of this linear program is to maximize the difference between the revenue and cost ratios for a given price vector $p_o^T = \left(c_o^T, r_o^T\right)$ for the DMU_o under assessment. Superscript T stands for a transposed vector.

The following definition and theorem refer to (Toloo et al. 2008).

Definition 1 *DMU_o is overall profit efficient if in model (1),* $\frac{r_o^T y^*}{r_o^T y_o} - \frac{c_o^T x^*}{c_o^T x_o} = 0.$

Theorem 1 *For every optimal solution* (x^*, y^*, λ^*) *of (1), we have* $\frac{r_o^T y^*}{r_o^T y_o} - \frac{c_o^T x^*}{c_o^T x_o} \geq 0.$

Overall profit MPIs

Emrouznejad et al. (2011) used the following model to measure the overall profit efficiency in the adjacent period:

$$D_o^p\left(x_o^q,y_o^q|p,q=t,t+1,p\neq q\right)=\max\ \varphi-\theta$$
$$\begin{aligned} s.t.\quad & \varphi\left[\left(r_j^p\right)^T y_0^p\right]\leq\left(r_j^q\right)^T Y^q\lambda,\ \forall j,\\ & \theta\left[\left(c_j^p\right)^T x_0^p\right]\leq\left(c_j^q\right)^T X^q\lambda,\ \forall j,\\ & \lambda\geq 0, \end{aligned} \tag{2}$$

where X^p and Y^p are the input and output matrices of the observed data for period p, respectively. To the best of our knowledge, to compute the overall profit MPI of DMU_o, the profit efficiency of $DMU_o^p=\left(x_o^p,y_o^p\right)$ must be computed using the technology and input-output prices at period q, $(p,q=t,t+1,p\neq q)$ (see Tohidi et al. (2010; 2014). However, in (2), the input-output prices in period p and q are used simultaneously. To overcome this shortfall, this paper introduces variable returns to scale overall profit efficiency in the within and adjacent periods as (3) and (4), respectively:

$$B_o^p\left(x_o^p,y_o^p|p=t,t+1\right)=\max\ \frac{\left(r_o^p\right)^T y}{\left(r_o^p\right)^T y_o^p}-\frac{\left(c_o^p\right)^T x}{(c_o^p)^T x_o^p}$$
$$\begin{aligned} s.t.\quad & -\textstyle\sum_{j\in J}\lambda_j y_j^p+y\leq 0\\ & \textstyle\sum_{j\in J}\lambda_j x_j^p-x\leq 0\\ & \textstyle\sum_{j\in J}\lambda_j=1,\\ & \lambda\geq 0 \end{aligned} \tag{3}$$

$$D_o^q\left(x_o^p,y_o^p|p,q=t,t+1,p\neq q\right)=\max\ \frac{(r_o^q)^T y}{(r_o^q)^T y_o^p}-\frac{(c_o^q)^T x}{(c_o^q)^T x_o^p}$$
$$\begin{aligned} s.t.\quad & -\textstyle\sum_{j\in J}\lambda_j y_j^q+y\leq 0\\ & \textstyle\sum_{j\in J}\lambda_j x_j^q-x\leq 0\\ & \textstyle\sum_{j\in J}\lambda_j=1,\\ & \lambda\geq 0 \end{aligned} \tag{4}$$

where x_j^p and y_j^p are the input and output of DMU_j in period p, respectively.

Models (3) and (4) have clear interpretations. Model (3) calculates the profit efficiency of DMU_o^p using the technology and input-output prices in period p and model (4) calculates the profit efficiency of DMU_o^p using the technology and input-output prices in period q, $(p,q=t,t+1,p\neq q)$.

The following definitions and theorem refer to (Emrouznejad et al. 2011).

Theorem 2 *For every optimal solution* (x^*,y^*,λ^*) *of (3), we have* $\frac{\left(r_o^p\right)^T y^*}{\left(r_o^p\right)^T y_o^p}-\frac{\left(c_o^p\right)^T x^*}{\left(c_o^p\right)^T x_o^p}\geq 0.$

Proof Model (3) has a feasible solution $\lambda_o=1,\lambda_j=0,j\neq o$. Hence, the optimal objective, denoted by $\frac{\left(r_o^p\right)^T y^*}{\left(r_o^p\right)^T y_o^p}-\frac{\left(c_o^p\right)^T x^*}{\left(c_o^p\right)^T x_o^p}$, is greater than or equal to 0, i.e., $\frac{\left(r_o^p\right)^T y^*}{\left(r_o^p\right)^T y_o^p}-\frac{\left(c_o^p\right)^T x^*}{\left(c_o^p\right)^T x_o^p}\geq 0.$ □

Remarks 1 *The objective function values for model (4) can be less than or equal to zero.*

Definition 2 DMU_o *is overall efficient if* $D_o^{t+1}\left(x_o^{t+1},y_o^{t+1}\right)=0$ *and* $D_o^t\left(x_o^t,y_o^t\right)=0.$

Definition 3 *The efficiency scores of models (3) and (4) are respectively computed as follows:*

(i) If $\frac{\left(r_o^p\right)^T y^*}{\left(r_o^p\right)^T y_o^p} - \frac{\left(c_o^p\right)^T x^*}{\left(c_o^p\right)^T x_o^p} \geq 0$, then $\rho = \frac{1}{1+\frac{\left(r_o^p\right)^T y^*}{\left(r_o^p\right)^T y_o^p} - \frac{\left(c_o^p\right)^T x^*}{\left(c_o^p\right)^T x_o^p}}$.

(ii) If $\frac{\left(r_o^q\right)^T y^*}{\left(r_o^q\right)^T y_o^p} - \frac{\left(c_o^q\right)^T x^*}{\left(c_o^q\right)^T x_o^p} \leq 0$, then $\rho = 1 + \frac{\left(r_o^q\right)^T y^*}{\left(r_o^q\right)^T y_o^p} - \frac{\left(c_o^q\right)^T x^*}{\left(c_o^q\right)^T x_o^p}$.

Obviously, if $\rho = 1$ DMU_o is efficient and if $\rho < 1$, DMU_o is inefficient.

Definition 4 *The overall profit MPI of DMU_o is defined as follows:*

$$M_o = \sqrt{\frac{\rho_o^t\left(x_o^{t+1}, y_o^{t+1}\right)}{\rho_o^t\left(x_o^t, y_o^t\right)} \times \frac{\rho_o^{t+1}\left(x_o^{t+1}, y_o^{t+1}\right)}{\rho_o^{t+1}\left(x_o^t, y_o^t\right)}}.$$

Therefore, the following three conditions hold:

(i) $M_o > 1$, *increase productivity and observe progress;*
(ii) $M_o < 1$, *decrease productivity and observe regress; and*
(iii) $M_o = 1$, *no change in productivity at time $t+1$ compared to t.*

Main results

Here, we consider the overall profit efficiency and overall profit MPI of DMU_o, $o = 1, ..., n$ when input, output, and input-output prices are uncertain and also define an interval for the overall profit MPI of DMU_o. We reiterate that there are n DMUs under consideration.

Assume that $\left[x_{ij}^{pL}, x_{ij}^{pU}\right]$ and $\left[y_{kj}^{pL}, y_{kj}^{pU}\right]$ are the intervals of input i and output k of DMU_j, $(j \in J)$ in period p, respectively. Additionally, $\left[c_{io}^{pL}, c_{io}^{pU}\right]$, and $\left[r_{ko}^{pL}, r_{ko}^{pU}\right]$ are the intervals of the input-output prices of input i and output k of DMU_o, $o = 1, ..., n$ in period p, respectively. Models (3) and (4) can be extended to the overall profit efficiency models (5) and (6) with data uncertainty, respectively:

$$\begin{array}{lll}
\textit{Within}-\textit{period time} & & \\
B_o^p\left(x_o^p, y_o^p | p = t, t+1\right) = \max & \frac{\left(r_o^p\right)^T y}{\left(r_o^p\right)^T y_o^p} - \frac{\left(c_o^p\right)^T x}{\left(c_o^p\right)^T x_o^p} & \\
s.t. & -\sum_{j\in J} \lambda_j y_j^p + y \leq 0 & \\
 & \sum_{j\in J} \lambda_j x_j^p - x \leq 0 & \\
 & \sum_{j\in J} \lambda_j = 1 & \\
 & c_{io}^p \in \left[c_{io}^{pL}, c_{io}^{pU}\right], & i = 1, ..., m \\
 & r_{ko}^p \in \left[r_{ko}^{pL}, r_{ko}^{pU}\right], & k = 1, ..., s \\
 & x_{ij}^p \in \left[x_{ij}^{pL}, x_{ij}^{pU}\right], & i = 1, ..., m \\
 & y_{kj}^p \in \left[y_{kj}^{pL}, y_{kj}^{pU}\right], & k = 1, ..., s \\
 & \lambda_j \geq 0, & j \in J.
\end{array} \tag{5}$$

Adjacent-period time

$$
\begin{aligned}
D_o^q\left(x_o^p, y_o^p | p, q = t, t+1, p \neq q\right) = \max \quad & \frac{\left(r_o^q\right)^T y}{\left(r_o^q\right)^T y_o^p} - \frac{\left(c_o^q\right)^T x}{\left(c_o^q\right)^T x_o^p} \\
s.t. \quad & -\textstyle\sum_{j\in J} \lambda_j y_j^q + y \leq 0 \\
& \textstyle\sum_{j\in J} \lambda_j x_j^q - x \leq 0 \\
& \textstyle\sum_{j\in J} \lambda_j = 1 \\
& c_{io}^q \in \left[c_{io}^{qL}, c_{io}^{qU}\right], \qquad i = 1, ..., m \\
& r_{ko}^q \in \left[r_{ko}^{qL}, r_{ko}^{qU}\right], \qquad k = 1, ..., s \\
& x_{ij}^q \in \left[x_{ij}^{qL}, x_{ij}^{qU}\right], \qquad i = 1, ..., m \\
& y_{kj}^q \in \left[y_{kj}^{qL}, y_{kj}^{qU}\right], \qquad k = 1, ..., s \\
& x_{io}^p \in \left[x_{io}^{pL}, x_{io}^{pU}\right], \qquad i = 1, ..., m \\
& y_{ko}^p \in \left[y_{ko}^{pL}, y_{ko}^{pU}\right], \qquad k = 1, ..., s. \\
& \lambda_j \geq 0, \qquad j \in J
\end{aligned}
\tag{6}
$$

It can be observed that models (5) and (6) are nonlinear programming programs because of data uncertainty.

Of particular importance is how to solve the newly constructed profit efficiency models with data uncertainty in (5) and (6). To illustrate these issues, we introduce the following definitions, which are similar to those of Park (2001).

Definition 5 *(Potential profit efficiency in the within-period time) The DMU_o to be evaluated is potentially profit efficient in the within-period if and only if there exists at least one set of prices $c_o^p \in \left[c_o^{pL}, c_o^{pU}\right]$ and $r_o^p \in \left[r_o^{pL}, r_o^{pU}\right]$ and at least one set of input-output data satisfying $x_{ij}^p \in \left[x_{ij}^{pL}, x_{ij}^{pU}\right]$ and $y_{kj}^p \in \left[y_{kj}^{pL}, y_{kj}^{pU}\right]$ $(j \in J)$, so that $B_o^{p*} = 0$[1] in model (5).*

Definition 6 *(Perfect profit efficiency in the within-period time) The DMU_o to be evaluated is perfectly profit efficient in the within-period if and only if, for all $c_o^p \in \left[c_o^{pL}, c_o^{pU}\right]$ and $r_o^p \in \left[r_o^{pL}, r_o^{pU}\right]$ and all input-output data, $x_{ij}^p \in \left[x_{ij}^{pL}, x_{ij}^{pU}\right]$ and $y_{kj}^p \in \left[y_{kj}^{pL}, y_{kj}^{pU}\right]$ $(j \in J)$, $B_o^{p*} = 0$ are satisfied in model (5).*

Definition 7 *(Potential profit efficiency in the adjacent period) The DMU_o to be evaluated is potentially profit efficient in the adjacent period if and only if there exists at least one set of prices $c_o^q \in \left[c_o^{qL}, c_o^{qU}\right]$ and $r_o^q \in \left[r_o^{qL}, r_o^{qU}\right]$ and at least one set of input-output data satisfying $x_{ij}^q \in \left[x_{ij}^{qL}, x_{ij}^{qU}\right]$, $y_{kj}^q \in \left[y_{kj}^{qL}, y_{kj}^{qU}\right]$ $(j \in J)$, $x_{io}^p \in \left[x_{io}^{pL}, x_{io}^{pU}\right]$ and $y_{ko}^p \in \left[y_{ko}^{pL}, y_{ko}^{pU}\right]$ $(p, q = t, t+1, p \neq q)$, so that $D_o^{q*} = 0$ in model (6).*

Definition 8 *(Perfect profit efficiency in the adjacent period) The DMU_o to be evaluated is perfectly profit efficient in the adjacent-period time if and only if, for all $c_o^q \in \left[c_o^{qL}, c_o^{qU}\right]$ and $r_o^q \in \left[r_o^{qL}, r_o^{qU}\right]$ and all input-output data, $x_{ij}^q \in \left[x_{ij}^{qL}, x_{ij}^{qU}\right]$, $y_{kj}^q \in \left[y_{kj}^{qL}, y_{kj}^{qU}\right]$ $(j \in J)$, $x_{io}^p \in \left[x_{io}^{pL}, x_{io}^{pU}\right]$ and $y_{ko}^p \in \left[y_{ko}^{pL}, y_{ko}^{pU}\right]$ $(p, q = t, t+1, p \neq q)$ are satisfied so that $D_o^{q*} = 0$ in model (6).*

[1]Superscript * indicates optimality.

In definitions 5 and 7, the profit efficiency of DMU_o is measured for *some* data, while definitions 6 and 8 refer to the profit efficiency of DMU_o for *all* data. Therefore, perfect profit efficiency is measured in a more rigid manner than potential profit efficiency. In the spirit of Park (2001), we can represent these definitions using the following mathematical formulations, where term UPEW-S (UPEW-P) refers to the uncertain profit efficiency of $DMU_o^p = (x_o^p, y_o^p)$ with the technology and prices at time p, the within period, for some (for perfect (all)). Additionally, term UPEA-S (UPEA-P) refers to uncertain profit efficiency of $DMU_o^p = (x_o^p, y_o^p)$ with technology and prices at time q, the adjacent period, for some (for perfect (all)) $(p, q = t, t+1,\ \ p \neq q)$:

$$
\begin{array}{lll}
 & & \text{The UPEW-S model:} \\
\overline{B}_o^p\left(x_o^p, y_o^p | p = t, t+1\right) = \max & & \dfrac{\left(r_o^p\right)^T y}{\left(r_o^p\right)^T y_o^p} - \dfrac{\left(c_o^p\right)^T x}{\left(c_o^p\right)^T x_o^p} \\
 & s.t. & -\sum_{j\in J} \lambda_j y_j^p + y \leq 0 \\
 & & \sum_{j\in J} \lambda_j x_j^p - x \leq 0 \\
 & & \sum_{j\in J} \lambda_j = 1 \\
 & & \text{for some}\{c_{io}^p \in \left[c_{io}^{pL}, c_{io}^{pU}\right],\ i = 1, \dots, m\} \\
 & & \text{for some}\{r_{ko}^p \in \left[r_{ko}^{pL}, r_{ko}^{pU}\right],\ k = 1, \dots, s\} \\
 & & \text{for some}\{x_{ij}^p \in \left[x_{ij}^{pL}, x_{ij}^{pU}\right],\ i = 1, \dots, m\} \\
 & & \text{for some}\{y_{kj}^p \in \left[y_{kj}^{pL}, y_{kj}^{pU}\right],\ k = 1, \dots, s\} \\
 & & \lambda_j \geq 0.
\end{array}
\tag{7}
$$

$$
\begin{array}{c}
\text{The UPEW-P model:} \\
\text{Model (7) with } \underline{B}_o^p \text{ in place of } \overline{B}_o^p \text{ and “for all”} \\
\text{in place of “for some.”}
\end{array}
\tag{8}
$$

$$
\begin{array}{lll}
 & & \text{The UPEA-S model:} \\
\overline{D}_o^q\left(x_o^p, y_o^p | p, q = t, t+1, p \neq q\right) = \max & & \dfrac{\left(r_o^q\right)^T y}{\left(r_o^q\right)^T y_o^p} - \dfrac{\left(c_o^q\right)^T x}{\left(c_o^q\right)^T x_o^p} \\
 & s.t. & -\sum_{j\in J} \lambda_j y_j^q + y \leq 0 \\
 & & \sum_{j\in J} \lambda_j x_j^q - x \leq 0 \\
 & & \sum_{j\in J} \lambda_j = 1 \\
 & & \text{for some}\left\{c_{io}^q \in [c_{io}^{qL}, c_{io}^{qU}],\quad i = 1, \dots, m\right\} \\
 & & \text{for some}\left\{r_{ko}^q \in [r_{ko}^{qL}, r_{ko}^{qU}],\quad k = 1, \dots, s\right\} \\
 & & \text{for some}\left\{x_{ij}^q \in [x_{ij}^{qL}, x_{ij}^{qU}],\quad i = 1, \dots, m\right\} \\
 & & \text{for some}\left\{y_{kj}^q \in [y_{kj}^{qL}, y_{kj}^{qU}],\quad k = 1, \dots, s\right\} \\
 & & \text{for some}\left\{x_{io}^p \in [x_{io}^{pL}, x_{io}^{pU}],\quad i = 1, \dots, m\right\} \\
 & & \text{for some}\left\{y_{ko}^p \in [y_{ko}^{pL}, y_{ko}^{pU}],\quad k = 1, \dots, s\right\} \\
 & & \lambda_j \geq 0.
\end{array}
\tag{9}
$$

$$
\begin{array}{c}
\text{The UPEA-P model:} \\
\text{Model (9) with } \underline{D}_o^q \text{ in place of } \overline{D}_o^q \text{ and “for all”} \\
\text{in place of “for some.”}
\end{array}
\tag{10}
$$

Clearly, $\overline{B}_o^p \geq \underline{B}_o^p$ $(\overline{D}_o^q \geq \underline{D}_o^q)$ because the feasible region of model (8) (10) is always contained within the feasible region of model (7) (9).

Using models (7) and (8), we can obtain the interval of profit efficiency of DMU_o in the within-period as $\left[\underline{B}_o^p, \overline{B}_o^p\right]$. Additionally, by models (9) and (10), the interval of profit efficiency of DMU_o in the adjacent period can be obtained as $\left[\underline{D}_o^q, \overline{D}_o^q\right]$. $\underline{B}_o^p$ $(\underline{D}_o^q)$ is the lower bound of the interval overall profit efficiency of DMU_o from the pessimistic viewpoint in the within period (adjacent period) and $\overline{B}_o^p$ $(\overline{D}_o^q)$ is the upper bound of the interval overall profit efficiency of DMU_o from the optimistic viewpoint in the within period (adjacent period).

Remarks 2 *It is clear that model (5) is equivalent to the UPEW-S model (7) and model (6) is equivalent to the UPEA-S model (9).*

Because of the notion of *maximization "for some" and "for all"* in the permissible data, the UPEW-S (7), UPEW-P (8), UPEA-S (9), and UPEA-P models (10) are equivalent to the two-level mathematical programs (11), (12), (13), and (14), respectively:

$$\begin{aligned}
&\overline{B}_o^p = \max_{\substack{c_{io}^p \in \left[c_{io}^{pL}, c_{io}^{pU}\right] \\ r_{ko}^p \in \left[r_{ko}^{pL}, r_{ko}^{pU}\right] \\ x_{ij}^p \in \left[x_{ij}^{pL}, x_{ij}^{pU}\right] \\ y_{kj}^p \in \left[y_{kj}^{pL}, y_{kj}^{pU}\right]}} \quad \max \quad \frac{\left(r_o^p\right)^T y}{\left(r_o^p\right)^T y_o^p} - \frac{\left(c_o^p\right)^T x}{\left(c_o^p\right)^T x_o^p} \\
&\qquad s.t. \quad -\textstyle\sum_{j\in J} \lambda_j y_j^p + y \leq 0 \\
&\qquad\qquad \textstyle\sum_{j\in J} \lambda_j x_j^p - x \leq 0 \\
&\qquad\qquad \textstyle\sum_{j\in J} \lambda_j = 1 \\
&\qquad\qquad \lambda_j \geq 0.
\end{aligned} \tag{11}$$

Model (11) but with "$\underline{B}_o^p = \min$" in place of "$\overline{B}_o^p = \max$". (12)

$$\begin{aligned}
&\overline{D}_o^p = \max_{\substack{c_{io}^q \in \left[c_{io}^{qL}, c_{io}^{qU}\right] \\ r_{ko}^q \in \left[r_{ko}^{qL}, r_{ko}^{qU}\right] \\ x_{ij}^q \in \left[x_{ij}^{qL}, x_{ij}^{qU}\right] \\ y_{kj}^q \in \left[y_{kj}^{qL}, y_{kj}^{qU}\right] \\ x_{io}^p \in \left[x_{io}^{pL}, x_{io}^{pU}\right] \\ y_{ko}^p \in \left[y_{ko}^{pL}, y_{ko}^{pU}\right]}} \quad \max \quad \frac{\left(r_o^q\right)^T y}{\left(r_o^q\right)^T y_o^p} - \frac{\left(c_o^q\right)^T x}{\left(c_o^q\right)^T x_o^p} \\
&\qquad s.t. \quad -\textstyle\sum_{j\in J} \lambda_j y_j^q + y \leq 0 \\
&\qquad\qquad \textstyle\sum_{j\in J} \lambda_j x_j^q - x \leq 0 \\
&\qquad\qquad \textstyle\sum_{j\in J} \lambda_j = 1 \\
&\qquad\qquad \lambda_j \geq 0.
\end{aligned} \tag{13}$$

Model (13) but with "$\underline{D}_o^p = \min$" in place of "$\overline{D}_o^p = \max$". (14)

In model (11), the inner program calculates the overall profit efficiency for each given set of $\left(x_o^p, y_o^p\right)$ for a price vector $\left(r_o^p, c_o^p\right)$ defined in the outer program, using the technology of period p, while the outer program determines the set of $\left(x_o^p, y_o^p\right)$ and price vector $\left(r_o^p, c_o^p\right)$ that generate the highest overall profit efficiency. Additionally, in model (13), the inner program calculates the overall profit efficiency for each given set of $\left(x_o^p, y_o^p\right)$ for a given price vector $\left(r_o^q, c_o^q\right)$ using the technology of period q $(p, q = t, t+1, p \neq q)$, defined in the outer program, while the outer program determines the set of $\left(x_o^p, y_o^p\right)$ and price vector $\left(r_o^q, c_o^q\right)$ that generate the highest overall profit efficiency. A similar explanation can be provided for models (12) and (14).

By duality, models (11)-(14) are equivalent to the following models:

$$
\begin{array}{ll}
\overline{B}_o^p = \max & \min \ \mu_o \\
c_{io}^p \in \left[c_{io}^{pL}, c_{io}^{pU}\right] & \\
r_{ko}^p \in \left[r_{ko}^{pL}, r_{ko}^{pU}\right] & \\
x_{ij}^p \in \left[x_{ij}^{pL}, x_{ij}^{pU}\right] & \\
y_{kj}^p \in \left[y_{kj}^{pL}, y_{kj}^{pU}\right] & \\
& \begin{array}{lll}
s.t. & \sum_{k=1}^{s} \mu_k^p y_{kj}^p - \sum_{i=1}^{m} v_i^p x_{ij}^p + \mu_o \geq 0, & j = 1, \ldots n \\
& \mu_k^p = \dfrac{r_{ko}^p}{\left(r_o^p\right)^T y_o^p}, & k = 1, \ldots, s \\
& v_i^p = \dfrac{c_{io}^p}{\left(c_o^p\right)^T x_o^p}, & i = 1, \ldots, m \\
& \mu_k^p \geq 0, & k = 1, \ldots, s \\
& v_i^p \geq 0, & i = 1, \ldots, m \\
& \mu_0 & free,
\end{array}
\end{array}
\tag{15}
$$

Model (15) but with "$\underline{B}_o^p = \min$" in place of "$\overline{B}_o^p = \max$". (16)

$$
\begin{array}{ll}
\overline{D}_o^p = \max & \min \ \mu_o \\
c_{io}^q \in \left[c_{io}^{qL}, c_{io}^{qU}\right] & \\
r_{ko}^q \in \left[r_{ko}^{qL}, r_{ko}^{qU}\right] & \\
x_{ij}^q \in \left[x_{ij}^{qL}, x_{ij}^{qU}\right] & \\
y_{kj}^q \in \left[y_{kj}^{qL}, y_{kj}^{qU}\right] & \\
x_{io}^p \in \left[x_{io}^{pL}, x_{io}^{pU}\right] & \\
y_{ko}^p \in \left[y_{ko}^{pL}, y_{ko}^{pU}\right] & \\
& \begin{array}{lll}
s.t. & \sum_{k=1}^{s} \mu_k^q y_{kj}^q - \sum_{i=1}^{m} v_i^q x_{ij}^q + \mu_o \geq 0, & j = 1, \ldots n \\
& \mu_k^q = \dfrac{r_{ko}^q}{\left(r_o^q\right)^T y_o^p}, & k = 1, \ldots, s \\
& v_i^q = \dfrac{c_{io}^q}{\left(c_o^q\right)^T x_o^p}, & i = 1, \ldots, m \\
& \mu_k^q \geq 0, & k = 1, \ldots, s \\
& v_i^q \geq 0, & i = 1, \ldots, m \\
& \mu_0 & free.
\end{array}
\end{array}
\tag{17}
$$

Model (17) but with "$\underline{D}_o^p = \min$" in place of "$\overline{D}_o^p = \max$". (18)

First, we proceed to models (16) and (18). Their inner and outer programs have the same objective of minimization. Therefore, they can be combined into a one-level model by considering all constraints of the two programs simultaneously. The one-level models equivalent to (16) and (18) are (19) and (20), respectively:

$$\begin{aligned}
&\underline{B}_o^p = \min\ \mu_o \\
&\quad s.t.\ \sum_{k=1}^{s}\mu_k^p y_{kj}^p - \sum_{i=1}^{m} v_i^p x_{ij}^p + \mu_o \geq 0, && j = 1,\dots n \\
&\quad \mu_k^p = \frac{r_{ko}^p}{\left(r_o^p\right)^T y_o^p}, && k = 1,\dots,s \\
&\quad v_i^p = \frac{c_{io}^p}{\left(c_o^p\right)^T x_o^p}, && i = 1,\dots,m \\
&\quad c_{io}^p \in \left[c_{io}^{pL}, c_{io}^{pU}\right] \\
&\quad r_{ko}^p \in \left[r_{ko}^{pL}, r_{ko}^{pU}\right] \\
&\quad x_{ij}^p \in \left[x_{ij}^{pL}, x_{ij}^{pU}\right] \\
&\quad y_{kj}^p \in \left[y_{kj}^{pL}, y_{kj}^{pU}\right] \\
&\quad \mu_k^p \geq 0, && k = 1,\dots,s \\
&\quad v_i^p \geq 0, && i = 1,\dots,m \\
&\quad \mu_o && \mathit{free} \\
&\quad \lambda_j \geq 0,
\end{aligned} \tag{19}$$

$$\begin{aligned}
&\underline{D}_o^p = \min\ \mu_o \\
&\quad s.t.\ \sum_{k=1}^{s}\mu_k^q y_{kj}^q - \sum_{i=1}^{m} v_i^q x_{ij}^q + \mu_o \geq 0, && j = 1,\dots n \\
&\quad \mu_k^q = \frac{r_{ko}^q}{(r_o^q)^T y_o^p}, && k = 1,\dots,s \\
&\quad v_i^q = \frac{c_{io}^q}{(c_o^q)^T x_o^p}, && i = 1,\dots,m \\
&\quad c_{io}^q \in \left[c_{io}^{qL}, c_{io}^{qU}\right] \\
&\quad r_{ko}^q \in \left[r_{ko}^{qL}, r_{ko}^{qU}\right] \\
&\quad x_{ij}^q \in \left[x_{ij}^{qL}, x_{ij}^{qU}\right] \\
&\quad y_{kj}^q \in \left[y_{kj}^{qL}, y_{kj}^{qU}\right] \\
&\quad x_{io}^p \in \left[x_{io}^{pL}, x_{io}^{pU}\right] \\
&\quad y_{ko}^p \in \left[y_{ko}^{pL}, y_{ko}^{pU}\right] \\
&\quad \mu_k^q \geq 0, && k = 1,\dots,s \\
&\quad v_i^q \geq 0, && i = 1,\dots,m \\
&\quad \mu_o && \mathit{free} \\
&\quad \lambda_j \geq 0.
\end{aligned} \tag{20}$$

Evidently the above models (19) and (20) are nonlinear programming problems and thus difficult to solve. To linearize model (19), we introduce variables z^p and τ^p, defined by:

$$z^p = \frac{1}{\left(c_o^p\right)^T x_o^p},$$

$$\tau^p = \frac{1}{\left(r_o^p\right)^T y_o^p},$$

so that

$$v_i^p = c_{io}^p z^p \iff c_{io}^{pL} z^p \leq v_i^p \leq c_{io}^{pU} z^p,$$

$$\mu_k^p = r_{ko}^p \tau^p \iff r_{ko}^{pL} \tau^p \leq \mu_k^p \leq r_{ko}^{pU} \tau^p,$$

$$\sum_{k=1}^{s} r_{ko}^p \tau^p y_{ko}^p = 1 \iff \sum_{k=1}^{s} \mu_k^p y_{ko}^p = 1,$$

$$\sum_{i=1}^{m} c_{io}^p z^p x_{io}^p = 1 \iff \sum_{i=1}^{m} v_i^p x_{io}^p = 1.$$

Similarly, to the linearization of model (20), we introduce variables z^q and τ^q, defined by:

$z^q = \frac{1}{\left(c_o^q\right)^T x_o^p}$,
$\tau^q = \frac{1}{\left(r_o^q\right)^T y_o^p}$,
so that
$v_i^q = c_{io}^q z^q \Longleftrightarrow c_{io}^{qL} z^q \le v_i^q \le c_{io}^{qU} z^q$,
$\mu_k^q = r_{ko}^q \tau^q \Longleftrightarrow r_{ko}^{qL} \tau^q \le \mu_k^q \le r_{ko}^{qU} \tau^q$,
$\sum_{k=1}^s r_{ko}^q \tau^q y_{ko}^p = 1 \Longleftrightarrow \sum_{k=1}^s \mu_k^q y_{ko}^p = 1$,
$\sum_{i=1}^m c_{io}^q z^q x_{io}^p = 1 \Longleftrightarrow \sum_{i=1}^m v_i^q x_{io}^p = 1$,
where $p, q = t, t+1, p \ne q$.

Using the above variable alterations, models (19) and (20) can be converted into the following programming problems, whose optimal objective values coincide with those of (19) and (20), respectively:

$$
\begin{aligned}
\underline{B}_o^p = \min \quad & \mu_o \\
s.t. \quad & \sum_{k=1}^s \mu_k^p y_{kj}^p - \sum_{i=1}^m v_i^p x_{ij}^p + \mu_o \ge 0, \; j = 1, \dots n \\
& \sum_{k=1}^s \mu_k^p y_{ko}^p = 1 \\
& \sum_{i=1}^m v_i^p x_{io}^p = 1 \\
& c_{io}^{pL} z^p \le v_i^p \le c_{io}^{pU} z^p \\
& r_{ko}^{pL} \tau^p \le \mu_k^p \le r_{ko}^{pU} \tau^p \\
& x_{ij}^p \in \left[x_{ij}^{pL}, x_{ij}^{pU}\right] \\
& y_{kj}^p \in \left[y_{kj}^{pL}, y_{kj}^{pU}\right] \\
& z^p \ge 0 \\
& \tau^p \ge 0 \\
& \mu_k^p \ge 0, \qquad k = 1, \dots, s \\
& v_i^p \ge 0, \qquad i = 1, \dots, m \\
& \mu_o \qquad \mathit{free} \\
& \lambda_j \ge 0,
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\underline{D}_o^p = \min \quad & \mu_o \\
s.t. \quad & \sum_{k=1}^s \mu_k^q y_{kj}^q - \sum_{i=1}^m v_i^q x_{ij}^q + \mu_o \ge 0, \; j = 1, \dots n \\
& \sum_{k=1}^s \mu_k^q y_{ko}^p = 1 \\
& \sum_{i=1}^m v_i^q x_{io}^p = 1 \\
& c_{io}^{qL} z^q \le v_i^q \le c_{io}^{qU} z^q \\
& r_{ko}^{qL} \tau^q \le \mu_k^q \le r_{ko}^{qU} \tau^q \\
& x_{ij}^q \in \left[x_{ij}^{qL}, x_{ij}^{qU}\right] \\
& y_{kj}^q \in \left[y_{kj}^{qL}, y_{kj}^{qU}\right] \\
& x_{io}^p \in \left[x_{io}^{pL}, x_{io}^{pU}\right] \\
& y_{ko}^p \in \left[y_{ko}^{pL}, y_{ko}^{pU}\right] \\
& z^q \ge 0 \\
& \tau^q \ge 0 \\
& \mu_k^q \ge 0, \qquad k = 1, \dots, s \\
& v_i^q \ge 0, \qquad i = 1, \dots, m \\
& \mu_o \qquad \mathit{free} \\
& \lambda_j \ge 0.
\end{aligned} \tag{22}
$$

Similar to Lemma (1) of (Podinovski 2001), we propose the following Lemma, which refers to the general weight bound problem:

Lemma 1 *Imposing the absolute bounds of $c_{io}^{pL} z^p \leq v_i^p \leq c_{io}^{pU} z^p, (i = 1, ..., m, z^p \geq 0)$ and $r_{ko}^{pL} \tau^p \leq \mu_k^p \leq r_{ko}^{pU} \tau^p, (k = 1, ..., s, \tau^p \geq 0)$ is equivalent to imposing bounds on the ratios of the weights of the following form:*

$$\frac{c_{io}^{zL}}{c_{ko}^{zU}} \leq \frac{v_i^z}{v_k^z} \leq \frac{c_{io}^{zU}}{c_{ko}^{zL}} \quad i, k = 1..., m, \quad k > i, z = p, q,$$
$$\frac{r_{lo}^{zL}}{r_{\kappa o}^{zL}} \leq \frac{\mu_l^z}{\mu_\kappa^z} \leq \frac{r_{lo}^{zU}}{r_{\kappa o}^{zL}} \quad l, \kappa = 1, ..., s, \quad \kappa > l, z = p, q.$$

Let $y_{kj} = \alpha_k y_{kj}^U + (1 - \alpha_k) y_{kj}^L$ and $x_{ij} = \beta_i x_{ij}^U + (1 - \beta_i) x_{ij}^L$ for some $\alpha_k \in [0, 1]$ and $\beta_i \in [0, 1]$. It is easy to show that the first three constraints of models (21) and (22) can be written as (23) and (24), respectively:

$$\begin{aligned}
&\sum_{k=1}^{s} \gamma_k \left(y_{kj}^{Up} - y_{kj}^{Lp}\right) + \sum_{k=1}^{s} \mu_k^p y_{kj}^{Lp} - \sum_{i=1}^{m} v_i^p x_{ij}^{Lp} - \sum_{i=1}^{m} \omega_i \left(x_{ij}^{Up} - x_{ij}^{Lp}\right) + \mu_o \geq 0, \quad j = 1, ...n \\
&\sum_{k=1}^{s} \gamma_k \left(y_{ko}^{Up} - y_{ko}^{Lp}\right) + \sum_{k=1}^{s} \mu_k^p y_{ko}^{Lp} = 1 \\
&\sum_{i=1}^{m} \omega_i \left(x_{io}^{Up} - x_{io}^{Lp}\right) + \sum_{i=1}^{m} v_i^p x_{io}^{Lp} = 1,
\end{aligned} \tag{23}$$

$$\begin{aligned}
&\sum_{k=1}^{s} \gamma_k \left(y_{kj}^{Uq} - y_{kj}^{Lq}\right) + \sum_{k=1}^{s} \mu_k^q y_{kj}^{Lq} - \sum_{i=1}^{m} v_i^q x_{ij}^{Lq} - \sum_{i=1}^{m} \omega_i \left(x_{ij}^{Uq} - x_{ij}^{Lq}\right) + \mu_o \geq 0, \quad j = 1, ...n \\
&\sum_{k=1}^{s} \gamma_k \left(y_{ko}^{Up} - y_{ko}^{Lp}\right) + \sum_{k=1}^{s} \mu_k^q y_{ko}^{Lp} = 1 \\
&\sum_{i=1}^{m} \omega_i \left(x_{io}^{Up} - x_{io}^{Lp}\right) + \sum_{i=1}^{m} v_i^q x_{io}^{Lo} = 1,
\end{aligned} \tag{24}$$

where $\gamma_k = \mu_k \alpha_k$ and $\omega_i = v_i \beta_i$ for each i and k.

By applying Lemma 1 and constraints (23) and (24) to models (21) and (22), we obtain the equivalent linear formulations of models (21) and (22), called the CR-DEA models, as follows:

$$\begin{aligned}
&\underline{B}_o^p = \min \ \mu_o \\
&\quad s.t. \quad \sum_{k=1}^{s} \gamma_k (y_{kj}^{Up} - y_{kj}^{Lp}) + \sum_{k=1}^{s} \mu_k^p y_{kj}^{Lp} - \sum_{i=1}^{m} v_i^p x_{ij}^{Lp} - \sum_{i=1}^{m} \omega_i (x_{ij}^{Up} - x_{ij}^{Lp}) + \mu_o \geq 0, \quad j = 1, ...n \\
&\quad\quad \sum_{k=1}^{s} \gamma_k (y_{ko}^{Up} - y_{ko}^{Lp}) + \sum_{k=1}^{s} \mu_k^p y_{ko}^{Lp} = 1 \\
&\quad\quad \sum_{i=1}^{m} \omega_i (x_{io}^{Up} - x_{io}^{Lp}) + \sum_{i=1}^{m} v_i^p x_{io}^{Lp} = 1 \\
&\quad\quad \frac{c_{io}^{pL}}{c_{ko}^{pU}} \leq \frac{v_i^p}{v_k^p} \leq \frac{c_{io}^{pU}}{c_{ko}^{pL}}, \qquad i, k = 1..., m, k > i \\
&\quad\quad \frac{r_{lo}^{pL}}{r_{\kappa o}^{pL}} \leq \frac{\mu_l^p}{\mu_\kappa^p} \leq \frac{r_{lo}^{pU}}{r_{\kappa o}^{pL}}, \qquad l, \kappa = 1, ..., s, \kappa > l
\end{aligned}$$

$$
\begin{aligned}
&0 \le \gamma_k \le \mu_k^p \\
&0 \le \omega_i \le v_i^p \\
&\mu_k^p \ge 0, && k = 1, \dots, s \\
&v_i^p \ge 0, && i = 1, \dots, m \\
&\mu_0, free && ,
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
\underline{D}_o^p = \min\ &\mu_o \\
s.t. &\sum_{k=1}^{s} \gamma_k \left(y_{kj}^{Uq} - y_{kj}^{Lq} \right) + \sum_{k=1}^{s} \mu_k^q y_{kj}^{Lq} - \sum_{i=1}^{m} v_i^q x_{ij}^{Lq} - \sum_{i=1}^{m} \omega_i \left(x_{ij}^{Uq} - x_{ij}^{Lq} \right) + \mu_o \ge 0, \quad j = 1, \dots n \\
&\sum_{k=1}^{s} \gamma_k \left(y_{ko}^{Up} - y_{ko}^{Lp} \right) + \sum_{k=1}^{s} \mu_k^q y_{ko}^{Lp} = 1 \\
&\sum_{i=1}^{m} \omega_i \left(x_{io}^{Up} - x_{io}^{Lp} \right) + \sum_{i=1}^{m} v_i^q x_{io}^{Lp} = 1 \\
&\frac{c_{io}^{qL}}{c_{ko}^{qU}} \le \frac{v_i^q}{v_k^q} \le \frac{c_{io}^{qU}}{c_{ko}^{qL}}, && i, k = 1 \dots, m, k > i \\
&\frac{r_{lo}^{qL}}{r_{\kappa o}^{qL}} \le \frac{\mu_l^q}{\mu_\kappa^q} \le \frac{r_{lo}^{qU}}{r_{\kappa o}^{qL}}, && l, \kappa = 1, \dots, s, \kappa > l \\
&0 \le \gamma_k \le \mu_k^q \\
&0 \le \omega_i \le v_i^q \\
&\mu_k^q \ge 0, && k = 1, \dots, s \\
&v_i^q \ge 0, && i = 1, \dots, m \\
&\mu_o, \quad free.
\end{aligned}
\tag{26}
$$

Now, we proceed to models (15) and (17). By applying the mentioned variable alterations to (15) and (17), we have:

$$
\begin{aligned}
\overline{B}_o^p = \max_{\substack{\frac{v_i^p}{v_k^p} \in \left[\frac{c_{io}^{pL}}{c_{ko}^{pU}}, \frac{c_{io}^{pU}}{c_{ko}^{pL}} \right] \\ \frac{\mu_l^p}{\mu_\kappa^p} \in \left[\frac{r_{lo}^{pL}}{r_{\kappa o}^{pL}}, \frac{r_{lo}^{pU}}{r_{\kappa o}^{pL}} \right] \\ x_{ij}^p \in \left[x_{ij}^{pL}, x_{ij}^{pU} \right] \\ y_{kj}^p \in \left[y_{kj}^{pL}, y_{kj}^{pU} \right]}} \quad &\min\ \mu_o \\
s.t.\quad &\textstyle\sum_{k=1}^{s} \mu_k^p y_{kj}^p - \sum_{i=1}^{m} v_i^p x_{ij}^p + \mu_o \ge 0, \ j = 1, \dots n \\
&\textstyle\sum_{k=1}^{s} \mu_k^p y_{ko}^p = 1 \\
&\textstyle\sum_{i=1}^{m} v_i^p x_{io}^p = 1 \\
&\mu_k^p \ge 0, && k = 1, \dots, s \\
&v_i^p \ge 0, && i = 1, \dots, m \\
&\mu_0 && free,
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
&\overline{D}_o^p = \max \qquad \min \ \mu_o \\
&\frac{\mu_l^q}{\mu_\kappa^q} \in \left[\frac{r_{lo}^{qL}}{r_{\kappa o}^{pL}}, \frac{r_{lo}^{qU}}{r_{\kappa o}^{qL}}\right] \\
&\frac{\mu_l^q}{\mu_\kappa^q} \in \left[\frac{r_{lo}^{qL}}{r_{\kappa o}^{qL}}, \frac{r_{lo}^{qU}}{r_{\kappa o}^{qL}}\right] \\
&x_{ij}^q \in [x_{ij}^{qL}, x_{ij}^{qU}] \\
&y_{kj}^q \in [y_{kj}^{qL}, y_{kj}^{qU}] \\
&x_{io}^p \in [x_{io}^{pL}, x_{io}^{pU}] \\
&y_{ko}^p \in [y_{ko}^{pL}, y_{ko}^{pU}]
\end{aligned}
$$

$$
\begin{aligned}
s.t. \quad & \sum_{k=1}^{s} \mu_k^q y_{kj}^q - \sum_{i=1}^{m} v_i^q x_{ij}^q + \mu_o \geq 0, && j = 1, \ldots n \\
& \sum_{k=1}^{s} \mu_k^q y_{ko}^p = 1 \\
& \sum_{i=1}^{m} v_i^q x_{io}^p = 1 \\
& \mu_k^q \geq 0, && k = 1, \ldots, s \\
& v_i^q \geq 0, && i = 1, \ldots, m \\
& \mu_0 && \textit{free.}
\end{aligned}
\tag{28}
$$

Models (27) and (28) are two-level models. Several authors have proposed methods for solving two-level programs (see, e.g., (Bialas and Karwan 1984; Vicente and Calamai 1994)). However, due to the special structure of models (27) and (28), we introduce another solution in "Computational aspects" section.

We summarize the facts in the above propositions as follows:

1. If we enclose uncertain data into the profit efficiency model (3) as within-period model (5), this model measures the upper bound on the profit efficiency of DMU_o in time periods t and $t+1$. If we enclose the same uncertain data into the CR-DEA DEA model in the form of weight restrictions as model (25), this model measures the lower bound on the profit efficiency of DMU_o in time periods t and $t+1$.
2. If we enclose uncertain data into the profit efficiency model (4) as adjacent-period model (6), this model measures the upper bound on the profit efficiency of DMU_o at time t $(t+1)$ relative to the frontier at time $t+1$ (t). If we enclose uncertain data into the CR-DEA DEA model in the form of weight restrictions as model (26), this model measures the lower bound on the profit efficiency of DMU_o at time t $(t+1)$ relative to the frontier at time $t+1$ (t).
3. Profit efficiency model (5), model UPEW-S (7), and two-level model (27) yield the same profit efficiency as potential profit efficiency or the upper bound of profit efficiency in the within period. Model (5) incorporates uncertain data directly into the envelopment model. Model (7) measures the same profit efficiency for the most optimistic viewpoint. A similar argument can be put forward for models (6), (9), and (28).
4. UPEW-P model (8), two-level model (12), and CR-DEA model (25) yield the same efficiency as perfect efficiency or the lower bound of the profit efficiency of DMU_o

at time periods t and $t+1$ (in the within period). Model (8) calculates the same profit efficiency of DMU_o from the pessimistic viewpoint. A similar argument can be put forward for model UPEA-P (10), two-level model (14), and CR-DEA model (26) (in the adjacent period).

Definition 9 *The lower and upper bounds of the overall profit MPIs are obtained as follows:*

$$\overline{M} = \sqrt{\frac{\overline{\rho}_t^{t+1}}{\underline{\rho}_t^{t}} \times \frac{\overline{\rho}_{t+1}^{t+1}}{\underline{\rho}_{t+1}^{t}}},$$

$\underline{M} = \sqrt{\frac{\underline{\rho}_t^{t+1}}{\overline{\rho}_t^{t}} \times \frac{\underline{\rho}_{t+1}^{t+1}}{\overline{\rho}_{t+1}^{t}}}$, *where* $\overline{\rho}_p^p \left(\underline{\rho}_p^p\right), p = t, t+1$ *represents the optimistic (pessimistic) efficiency in the within period and is computed by model (7)(8) and definition 3. Additionally,* $\overline{\rho}_q^p \left(\underline{\rho}_q^p\right), p, q = t, t+1, p \neq q$ *represents the optimistic (pessimistic) efficiency in the adjacent-period time, and is computed by model (9)(10) and definition 3.*

Theorem 3 *Any* $\underline{M} \leq M \leq \overline{M}$ *can be considered as the overall profit MPI for* DMU_o.

Proof See (Emrouznejad et al. 2011). □

Emrouznejad et al. (2011) divided the overall MPI of any DMU_o into six classes, as follows:

- No change in productivity class. This class includes all the DMUs with constant productivity, that is, $E_o = \{DMU_j : \underline{M}_j = \overline{M}_j = 1\}$.
- Fully increasing productivity class. This class includes all the DMUs with increasing productivity and observed progress under the pessimistic viewpoint, that is, $E^{++} = \{DMU_j : 1 < \underline{M}_j \leq \overline{M}_j\}$.
- Fully decreasing productivity class. This class includes all the DMUs with decreasing productivity and observed regress under the optimistic viewpoint, that is, $E^{--} = \{DMU_j : \underline{M}_j \leq \overline{M}_j < 1\}$.
- Partially increasing productivity class. This class includes all the DMUs with increasing productivity under the optimistic viewpoint and no change in productivity under the pessimistic viewpoint, that is, $E^{+} = \{DMU_j : \underline{M}_j = 1, \overline{M}_j > 1\}$.
- Partially decreasing productivity class. This class includes all the DMUs with decreasing productivity under the pessimistic viewpoint and no change in productivity under the optimistic viewpoint, that is, $E^{-} = \{DMU_j : \underline{M}_j < 1, \overline{M}_j = 1\}$.
- Partially increasing–decreasing productivity class. This class includes all the DMUs with increasing productivity under the optimistic viewpoint and decreasing productivity under the pessimistic viewpoint, that is, $E = \{DMU_j : \underline{M}_j < 1 < \overline{M}_j\}$.

Computational aspects

As mentioned in the previous section, we can use CR-DEA models (25) and (26) to obtain the lower bounds of the overall profit efficiency of DMU_o from the pessimistic viewpoint in the within $\left(\underline{B}_o^p\right)$ and adjacent periods $\left(\underline{D}_o^q\right)$, respectively. Regarding the upper bounds, models (27) and (28) are nonlinear two-level programs and cannot be converted to linear one-level programs. Therefore, we propose new methods to achieve the upper bounds as follows. We define $\theta(\nu, \mu) =$

$\min\left\{\mu_o \mid \mu_o \geq \sum_{i=1}^{m} v_i^p x_{ij}^p - \sum_{k=1}^{s} \mu_k^p y_{kj}^p, v_i^p, \mu_k^p \geq 0\right\}$. It can be shown that $\theta(v,\mu)$ is piecewise linear, piecewise continuous, and a convex function. Moreover, the feasible spaces of (27) and (28) are bounded. As such, models (27) and (28) have bounded optimal solutions that occur on boundary of the feasible spaces. Therefore, we have the following propositions:

Proposition 1 *The optimal objective value of (27) is equal to:*

$$\max\left\{\sum_{k=1}^{s}\gamma(j)_k^*\left(y_{kj}^{Up}-y_{kj}^{Lp}\right)+\sum_{k=1}^{s}\mu(j)_k^{p*}y_{kj}^{Lp}-\sum_{i=1}^{m}\nu(j)_i^{p*}x_{ij}^{Lp}-\sum_{i=1}^{m}\omega(j)_i^*\left(x_{ij}^{Up}-x_{ij}^{Lp}\right), j=1,\dots,n\right\}, \tag{29}$$

where $\left(\gamma(j)^*, \nu(j)^{p*}, \mu(j)^{p*}, \omega(j)^*\right)$ $(j=1,\dots,n)$ *are the optimal solutions of the following linear model:*

$$\begin{aligned}
\max\ &\left\{\sum_{k=1}^{s}\gamma_k\left(y_{kj}^{Up}-y_{kj}^{Lp}\right)+\sum_{k=1}^{s}\mu_k^p y_{kj}^{Lp}-\sum_{i=1}^{m}\nu_i^p x_{ij}^{Lp}-\sum_{i=1}^{m}\omega_i\left(x_{ij}^{Up}-x_{ij}^{Lp}\right)\right\}, \quad j=1,\dots,n\\
s.t.\ &\sum_{k=1}^{s}\gamma_k\left(y_{ko}^{Up}-y_{ko}^{Lp}\right)+\sum_{k=1}^{s}\mu_k^p y_{ko}^{Lp}=1\\
&\sum_{i=1}^{m}\omega_i\left(x_{io}^{Up}-x_{io}^{Lp}\right)+\sum_{i=1}^{m}\nu_i^p x_{io}^{Lp}=1\\
&\frac{c_{io}^{pL}}{c_{ko}^{pU}}\nu_k^p\leq\nu_i^p\leq\frac{c_{io}^{pU}}{c_{ko}^{pL}}\nu_k^p \qquad i,k=1\dots,m,k>i\\
&\frac{r_{lo}^{pL}}{r_{\kappa o}^{pU}}\mu_\kappa^p\leq\mu_l^p\leq\frac{r_{lo}^{pU}}{r_{\kappa o}^{pL}}\mu_\kappa^p \qquad l,\kappa=1,\dots,s,\kappa>l\\
&0\leq\gamma_k\leq\mu_k^p\\
&0\leq\omega_i\leq\nu_i^p\\
&\mu_k^p\geq 0, \qquad k=1,\dots,s\\
&\nu_i^p\geq 0, \qquad i=1,\dots,m.
\end{aligned} \tag{30}$$

Proposition 2 *The optimal objective value of (28) is equal to:*

$$\max\left\{\sum_{k=1}^{s}\gamma(j)_k^*\left(y_{kj}^{Uq}-y_{kj}^{Lq}\right)+\sum_{k=1}^{s}\mu(j)_k^{q*}y_{kj}^{Lq}-\sum_{i=1}^{m}\nu(j)_i^{q*}x_{ij}^{Lq}-\sum_{i=1}^{m}\omega(j)_i^*\left(x_{ij}^{Uq}-x_{ij}^{Lq}\right), j=1,\dots,n\right\}, \tag{31}$$

where $\left(\gamma(j)^*, \nu(j)^{q*}, \mu(j)^{q*}, \omega(j)^*\right)$ $(j=1,\dots,n)$ *are the optimal solutions of the following linear model:*

$$\max\ \left\{\sum_{k=1}^{s}\gamma_k\left(y_{kj}^{Uq}-y_{kj}^{Lq}\right)+\sum_{k=1}^{s}\mu_k^q y_{kj}^{Lq}-\sum_{i=1}^{m}\nu_i^q x_{ij}^{Lq}-\sum_{i=1}^{m}\omega_i\left(x_{ij}^{Uq}-x_{ij}^{Lq}\right)\right\}, \quad j=1,\dots n$$

Table 1 Input and output data for the five DMUs in Example 1 at times t and $t+1$. Extracted from Emrouznejad et al. (2011)

DMU_j	x_{1j}		x_{2j}		y_{1j}		y_{2j}	
	t	$t+1$	t	$t+1$	t	$t+1$	t	$t+1$
1	(12, 15)	(10, 14)	(0.21, 0.48)	(0.32, 0.5)	(138, 144)	(130, 140)	(21, 22)	(20, 23)
2	(10, 17)	(11, 15)	(0.1, 0.7)	(0.21, 0.4)	(143, 159)	(137, 150)	(28, 35)	(24, 30)
3	(4, 5)	(3, 7)	(0.16, 0.35)	(0.22, 0.42)	(157, 198)	(146, 160)	(21, 29)	(20, 30)
4	(19, 22)	(14, 23)	(0.12, 0.19)	(0.31, 0.39)	(158, 181)	(159, 170)	(21, 25)	(25, 32)
5	(14, 15)	(17, 18)	(0.06, 0.09)	(0.1, 0.17)	(157, 180)	(160, 189)	(28, 40)	(18, 35)

$$
\begin{aligned}
s.t.\ & \sum_{k=1}^{s} \gamma_k \left(y_{ko}^{Up} - y_{ko}^{Lp}\right) + \sum_{k=1}^{s} \mu_k^q y_{ko}^{Lp} = 1 \\
& \sum_{i=1}^{m} \omega_i \left(x_{io}^{Up} - x_{io}^{Lp}\right) + \sum_{i=1}^{m} v_i^q x_{io}^{Lp} = 1 \\
& \frac{c_{io}^{qL}}{c_{ko}^{qU}} v_k^q \le v_i^q \le \frac{c_{io}^{qU}}{c_{ko}^{qL}} v_k^q \qquad i,k = 1\ldots,m, k > i \\
& \frac{r_{lo}^{qL}}{r_{\kappa o}^{qU}} \mu_\kappa^q \le \mu_l^q \le \frac{r_{lo}^{qU}}{r_{\kappa o}^{qL}} \mu_\kappa^q \qquad l,\kappa = 1,\ldots,s, \kappa > l \\
& 0 \le \gamma_k \le \mu_k^q \\
& 0 \le \omega_i \le v_i^q \\
& \mu_k^q \ge 0, \qquad k = 1,\ldots,s \\
& v_i^q \ge 0, \qquad i = 1,\ldots,m \\
& \mu_0 \qquad free.
\end{aligned}
\tag{32}
$$

In model (30) (32) we maximize the linear objective functions individually and then calculate the highest using (29) (31).

Example. We consider five DMUs with two inputs and two outputs, as per Table 1. Table 2 shows the interval price vectors at time t and $t+1$. Using models (25) and (30), the interval of profit efficiency for DMU_o in the within period is $\left[\underline{B}_o^p, \overline{B}_o^p\right]$ and using models (26) and (32) the interval of profit efficiency for DMU_o in the adjacent period time is $\left[\underline{D}_o^q, \overline{D}_o^q\right]$. The overall profit MPIs are shown in Table 3. As per Table 3, DMU_4 is classified in the fully increasing productivity class, which is the

Table 2 Numerical example 1

DMU_j	$(\underline{r}_{1j}, \overline{r}_{1j})$		$(\underline{r}_{2j}, \overline{r}_{2j})$		$(\underline{c}_{1j}, \overline{c}_{1j})$		$(\underline{c}_{2j}, \overline{c}_{2j})$	
	t	$t+1$	t	$t+1$	t	$t+1$	t	$t+1$
1	[10, 12]	[11, 15]	[30, 35]	[23, 30]	[100, 110]	[110, 115]	[50, 55]	[40, 50]
2	[9, 10]	[8, 9]	[27, 28]	[24, 29]	[110, 115]	[114, 117]	[40, 44]	[42, 45]
3	[8, 9]	[5, 9]	[25, 27]	[25, 26]	[105, 110]	[102, 109]	[42, 45]	[42, 50]
4	[9, 11]	[7, 12]	[29, 31]	[23, 26]	[107, 115]	[114, 115]	[50, 57]	[49, 52]
5	[10, 11]	[10, 14]	[28, 31]	[24, 30]	[111, 117]	[110, 114]	[47, 62]	[42, 52]

The price vector data for the five DMUs in Example 1 at times t and $t+1$

Table 3 Numerical example 1

DMU_j	$(\underline{B}_j^t(x_j^t,y_j^t),\overline{B}_j^t(x_j^t,y_j^t))$	$(\underline{B}_j^{t+1}(x_j^{t+1},y_j^{t+1}),\overline{B}_j^{t+1}(x_j^{t+1},y_j^{t+1}))$	$(\underline{D}_j^t(x_j^{t+1},y_j^{t+1}),\overline{D}_j^t(x_j^{t+1},y_j^{t+1}))$	$(\underline{D}_j^{t+1}(x_j^t,y_j^t),\overline{D}_j^{t+1}(x_j^t,y_j^t))$	$\underline{M}_j$	$\overline{M}_j$
1	[0.733,1.404]	[0.493, 1.547]	[0.239,1.548]	[0.036,1.279]	0.459	1.887
2	[0.918,1.435]	[0.143,1.673]	[0.190,1.412]	[0.453,0.765]	0.1434	1.450
3	[0.687, 1.001]	[0.998,1.09]	[0.0381,1.565]	[0.503,1.674]	0.246	2.975
4	[0.322,1.376]	[0.383, 1.402]	[0.323,1.316]	[0.100,1.807]	1.665	1.809
5	[0.209,1.173]	[0.001,1.143]	[0.361,1.322]	[0.296,1.354]	0.217	1.614

The interval of profit efficiency and overall profit MPI of the five DMUs in Example 1 observed progress under the pessimistic viewpoint. Other DMUs are classified in the partially increasing–decreasing productivity class and they thus have increasing productivity under the optimistic viewpoint and decreasing productivity under the pessimistic viewpoint. DMU_3 has the highest productivity progress of 2.975 under the optimistic viewpoint and DMU_5 the highest productivity decrease of 0.217. According to the optimistic viewpoint, all DMUs can be ranked by their productivity progress in the order $DMU_3 \succ DMU_1 \succ DMU_4 \succ DMU_5 \succ DMU_2$. However, according to the pessimistic viewpoint, the productivity regress is in the order $DMU_4 \succ DMU_1 \succ DMU_3 \succ DMU_5 \succ DMU_2$. Obviously, the productivity increase ranking may differ from the productivity decrease one.

Conclusions

Conventional DEA can be used to compute the productivity changes of a DMU over time under the profit MPI model, provided that the input, output, input costs, and output prices are known and exact for each DMU. However, in many situations, some inputs and/or outputs and input-output prices are imprecise. However, the conventional profit MPI model is not suitable to deal with inexact prices. Emrouznejad et al. (2011) studied the overall profit MPI using DEA with imprecise data and proposed two novel methods for measuring overall profit MPI. In this paper, we showed their method has some shortfalls. To overcome these shortfalls, we reformulated the conventional profit MPI model as an IDEA model by incorporating the available information into profit efficiency models and the same information into CR-DEA models in the form of a cone-ratio weight restriction. Additionally, the lower bounds of profit efficiency were easily calculated by solving a linear one-level program. Regarding the upper bounds, we proposes a new approach of solving n linear programming problems for each bound. This is the penalty we pay to calculate the upper bound of the overall profit efficiency when the data are inexact. We also presented a numerical example to demonstrate the applicability of the proposed framework.

Abbreviations

CR-DEA: Cone-ratio DEA; DEA: Data envelopment analysis; DMU: Decision making unit; IDEA: Imprecise DEA; MPI: Malmquist productivity index; UPEA-P: Uncertain profit efficiency of DMU_o in the adjacent period for perfect (all); UPEA-S: Uncertain profit efficiency of DMU_o in the adjacent period for some; UPEW-P: Uncertain profit efficiency of DMU_o in the within period for perfect; UPEW-S: Uncertain profit efficiency of DMU_o in the within-period time for some

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Authors' contributions

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Competing interests

The author declares that he has no competing interests.

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