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Optimal portfolio selection with volatility information for a high frequency rebalancing algorithm

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Abstract

We propose a high-frequency rebalancing algorithm (HFRA) and compare its performance with periodic rebalancing (PR) and threshold rebalancing (TR) strategies. PR refers to the process of adjusting the relative weight of assets within portfolios at regular time intervals, whereas TR is a process of setting allocation limits for portfolios and rebalancing when portfolios exceed a specific percentage of deviation from the target allocation. The HFRA is constructed as an integration of pairs trading and a threshold-based rebalancing strategy, and the profitability of the HFRA is examined to determine the optimal portfolio size. The HFRA is applied to a dataset of real price series from cryptocurrency exchange markets across various trends and volatility regimes. Using cointegrated price data, it is shown that increasing the number of assets in a portfolio supports the profitability of the HFRA in an up-trend and reduces the potential loss of the HFRA in a down-trend in a high-volatility environment. For low-volatility regimes, although increasing portfolio size marginally enhances the HFRA's profitability, the profits of portfolios of varied sizes do not significantly differ. It is demonstrated that when volatility is relatively high and the trend is upward, the HFRA can yield a substantial return via portfolios of large sizes. Moreover, the profitability of the HFRA is compared with that of the PR and TR strategies for long-term application. The HFRA is more profitable than the PR and TR strategies. This achievement of the HFRA is also validated statistically using the Fisher–Pitman permutation test.

Keywords: Algorithmic trading, Pair trading, Rebalancing algorithm, Crypto-assets, Volatility, Cointegration

Introduction

The utilization of blockchain technology led to the emergence of a supra-governmental monetary system that operates independently of any central authority. Unlike traditional fiat currencies, cryptocurrencies are not controlled by entities such as central banks. Bitcoin, the first cryptocurrency introduced in 2008 by an anonymous author using the pseudonym Satoshi Nakamoto (2008), is a prime example of an encryption-based innovation. Since the introduction of Bitcoin in 2009, the rapid growth of financial technologies has led to the creation of a variety of crypto-assets such as stablecoins, utility tokens, and non-fungible tokens (Bains et al. 2022), and these crypto-assets have been widely utilized by financial investors owing to their advantageous properties. They enable processes that support financial transactions to become more efficient, transparent, fast, and flexible (Romero Ugarte 2018). In addition, crypto-assets have some advantages over conventional payment systems, such as peer-to-peer focus, user autonomy, discretion, and minimal transaction fees for international payments (Grobys et al. 2021). Through these features, crypto-assets, especially Bitcoin gained the interest of investors, government institutions, and academia.

As of 2023, a significant number (approximately 9000) of alternative cryptocurrencies, commonly known as altcoins, exist, with a total market cap of $1136 billion. Moreover, it is widely accepted that the top 20 cryptocurrencies have a substantial share, accounting for about 90% of the entire market (Statista 2023).

As cryptocurrency markets have high volatility, it is difficult to make a profit through crypto-asset trading (Cohen 2021), and most research in this field focuses on algorithms that accurately predict returns and volatility to achieve profitable investments in crypto-assets (Fang et al. 2022; Dong and Boutaba 2019; Kaya Soylu et al. 2020; Baur and Dimpfl 2021; Guindy 2021).

Over the past 2 decades, algorithmic trading (AT) has played an important role in financial markets. AT is a methodology for automatically executing orders using a defined set of instructions to realize a trade without human intervention (Chaboud et al. 2014; Cartea et al. 2015). Improvements in computer technologies has been accompanied by the widespread use of AT, mostly among institutional investors (Liang et al. 2020). Accordingly, AT reached 70% of the overall trading volume in some exchange markets (European Central Bank 2019). An extension of AT is high-frequency trading (HFT), which allows the execution of buy and sell orders at a very fast rate (Cartea and Jaimungal 2013; Vo and Yost-Bremm 2020; Jain et al. 2021; Virgilio 2022). Major features of HFTs are their high speed, turnover rates, and order-to-trade ratios. Trading strategies require sophisticated algorithms and technological tools. With the increasing use of artificial intelligence and machine learning (ML) techniques, there is growing interest in AT (Konrad and Philip 1994; Liu et al. 2021; Zhang et al. 2021; Bağcı 2021).

The literature on AT strategies for crypto-assets is growing. One of the first contributors to the literature is Madan et al. (2015). They used ML algorithms to predict Bitcoin price (Madan et al. 2015). In the last few years, numerous studies investigated gains from cryptocurrencies with ML techniques (Zbikowski 2016; Jiang and Liang 2017; McNally et al. 2018; Atsalakis et al. 2019). Ślepaczuk and Zenkova (2018) employed the support vector machine (SVM) model to build an AT strategy. They state that an equally weighted portfolio as a benchmark strategy outperforms all benchmark strategies as well as the SVM strategy (Ślepaczuk and Zenkova 2018). Recent research by Leung and Nguyen (2019) suggested that setting greater entry and exit levels leads to larger profits in the case of cointegrated portfolios involving four cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Bitcoin Cash (BCH), and Litecoin (LTC) (Leung and Nguyen 2019). Cohen (2021) attempted to determine an optimal trading strategy for Bitcoin. They used three popular algorithmic systems and demonstrated that the relative strength index yields poorer results than the buy-and-hold strategy. However, the moving average convergence diversion and pivot reversal strategies significantly outperform the buy-and-hold strategy. Moreover, their optimization process, which is based on direct transaction prices, produced even better results (Cohen 2021). The profitability of trading strategies designed using ML techniques was also explored in a study by Sebastião and Godinho (2021).

The pair-trading strategy has been widely adopted in AT. As previously reported, pair trading is based on the historical co-movement of the prices of the two securities. This strategy assumes that the spread between the two securities reverts to its historical mean. Hence, the linear combination of these two securities is a stationary process. Divergency in prices is utilized by opening a long position in an undervalued security and a short position in an overvalued security. Once the spread returns to its historical equilibrium, profits can be generated by closing the positions (Gatev et al. 2006). A comprehensive literature review by Krauss (2017) showed that the literature on pair-trading strategies can be divided into five approaches. The method introduced by Gatev et al. (2006) is a distance approach that uses nonparametric distance metrics to identify pair-trading opportunities. The remaining four methodologies are based on the cointegration approach relying on formal cointegration testing (Vidyamurthy 2004), the time series approach concentrating on finding optimal trading procedures (Elliott et al. 2005), the stochastic control approach (Jurek and Yang 2007) focusing on identifying optimal portfolio holdings, and other approaches, including further applicable pairs-trading schemes. In addition, methods such as the ML approach (Huck 2009), the copula approach (Rad et al. 2016), and the principal components analysis approach (Avellaneda and Lee 2010) make seminal contributions to the pair-trading literature (Krauss 2017). The cointegration relationship between the two assets signifies a long-term relationship. Taking advantage of this relationship, we can effectively model the co-movement of an asset pair and use it to implement a high-performance pair-trading strategy (Rad et al. 2016). The major benefit of the cointegration approach is that it provides long-term relationships between traded assets based on short-term deviations from this long-run equilibrium with the expectation of mean reversion.

Another important application of AT is portfolio selection and portfolio asset allocation (DeMiguel et al. 2009; Zilinskij 2015; Dayanandan and Lam 2015). Portfolio rebalancing strategies and algorithms are widely used to set portfolio's asset allocations. These strategies are categorized into three main groups: periodic rebalancing (PR), threshold (or tolerance-band) rebalancing (TR), and a combination of PR and TR (Zilinskij 2015; Zilbering et al. 2015). PR refers to the process of adjusting the relative weights of assets within a portfolio at regular intervals. This involves returning the portfolio's allocation to its original target or desired asset allocation. PR ensures the reinstatement of original asset allocation by selling overperforming assets and buying underperforming assets. By doing so, investors can maintain the desired level of risk and return on their portfolios. With PR, the portfolio weights of assets are rebalanced to an initially determined allocation at regular time intervals such as daily, monthly, or annual (Das et al. 2014; Costabile and Gaudenzi 2017). In the PR strategy, the only parameter considered is time, regardless of other market conditions. Portfolio allocation can also be rebalanced within the trading day, at hourly or per-minute timeframes if the considered stock markets are highly volatile. Using the TR strategy, the asset allocation in a portfolio is rebalanced when the weight of an asset in the portfolio exceeds a specific (maximum or minimum) limit, such as 2% or 5% (Bernoussi and Rockinger 2022). The threshold may depend on market-based parameters, complex models of return predictability, or risk considerations (Moallemi and Saglam 2015). Using a combination of periodic and threshold strategies, the assets in the portfolio are rebalanced in a specified time interval (monthly or annual) if and only if the portfolio's asset allocation deviates to a certain extent from the predetermined asset allocation (Zilbering et al. 2015). Rebalancing algorithms are based on these strategies and additional market conditions (e.g., trends, volatility, or transaction costs) to achieve an optimal portfolio allocation (Bernoussi and Rockinger 2022; Zhao et al. 2021). Many recent studies on rebalancing algorithms focus on transaction costs (Woodside-Oriakhi et al. 2013; Wang et al. 2014; Mittal and Mehlawat 2014; Mattei 2018), historical return data (Jung and Kim 2017; Zhao et al. 2021), risks and liquidity (Gupta et al. 2012; Mittal and Mehlawat 2014), and return skewness (DeMiguel et al. 2013). In addition to market variables, portfolio rebalancing studies are based on social parameters such as investor sentiment (Yu et al. 2022), real-world household portfolios (Horn and Oehler 2020), and the effects of inattention on portfolio choices (Rachedi 2018). Furthermore, innovative computational methods, including quantum computing (Hodson et al. 2019), fuzzy simulation (Gupta et al. 2012), genetic algorithms (Mittal and Mehlawat 2014), and reinforcement learning (Lim et al. 2021), have been utilized to achieve dynamic portfolio rebalancing (Moallemi and Saglam 2015; Jung and Kim 2017).

This study investigates the profitability of portfolios based on a cointegration approach with an AT strategy. To this end, a high-frequency rebalancing algorithm (HFRA), which integrates pairs trading and a threshold-based rebalancing strategy, is proposed and implemented on portfolios of various sizes under six different trend and volatility regimes. The impact of the trend and volatility of price series on the profitability of the proposed HFRA was examined using a real dataset (obtained from cryptocurrency exchange markets). The robustness of the HFRA algorithm is validated by long-term applications, and a comparison of the performance of the HFRA, PR, and TR strategies reveals that HFRA is more profitable than the PR and TR strategies. This result indicates that using HF data in the HFRA makes a substantial difference in profitability by capturing market spikes more accurately.

Today, exchange platforms such as Binance and Kucoin offer trading bots to investors to automatically use rebalancing strategies (Binance 2023; Kucoin 2023). To use these bots, investors must create a portfolio of their own accord and determine the rebalancing conditions. Therefore, the findings of this study, which examines the profitability of different portfolio choices and rebalancing strategies, may be of interest to both individual and institutional investors.

The contributions of this work are as follows. First, we integrate pairs trading with a high-frequency rebalancing algorithm and apply the algorithm to real price series from cryptocurrency exchange markets. Second, few studies focused on pair-trading strategies in the cryptocurrency market compared to other financial markets. Moreover, in contrast to previous studies on rebalancing strategies, we use a high-frequency (per minute) price series of crypto-assets, which allows for a more realistic approach for HFT algorithms. Using cointegrated price series, which enables the model and utilization of the co-movement of traded pairs, is another advantage of the HFRA strategy.

The rest of the paper is organized as follows. In the next section, the methodology and scheme of the proposed trading algorithms are explained, and details of the real dataset are expressed. The algorithms are then applied to real price series from exchange markets, and the obtained results are discussed and compared with those of previous studies. Finally, the main conclusions of this study are discussed.

Dataset and methods

In this study, a HFRA is proposed and applied to the real price series of various crypto-assets that are obtained from cryptocurrency exchange markets. Accordingly, we consider price series of multiple assets to apply the HFRA to portfolios of various sizes. Note that the HFRA is a market-independent strategy; thus, it can be applied to any stock market.

Real dataset

For real data applications, the historical data of the following 40 crypto-assets were obtained from the Binance Exchange for 2021 (Binance 2022): Aave (AAVE), Cardano (ADA), Algorand (ALGO), Cosmos (ATOM), Avalanche (AVAX), Axie Infinity (AXS), Bitcoin Cash (BCH), BNB (BNB), Chiliz (CHZ), Dash (DASH), Dogecoin (DOGE), Polkadot (DOT), MultiversX (Elrond) (EGLD), EOS (EOS), Ethereum Classic (ETC), Ethereum (ETH), Filecoin (FIL), Fantom (FTM), The Graph (GRT), Hedera Hashgraph (HBAR), MIOTA (IOTA), Chainlink (LINK), Litecoin (LTC), Decentraland (MANA), Polygon (MATIC), Maker (MKR), NEAR Protocol (NEAR), NEO (NEO), The Sandbox (SAND), Synthetix Network Token (SNX), Solana (SOL), Theta Token (THETA), Tron (TRX), Uniswap (UNI), VeChain (VET), Stellar Lumens (XLM), Monero (XMR), Ripple (XRP), Tezos (XTZ), and Zcash (ZEC). We have the price series of these assets on the Bitcoin markets; thus, we have a price series where the quote currency is BTC, for example, AAVE/BTC or ADA/BTC markets. These are crypto-assets with the largest market cap, according to https://coinmarketcap.com/ in December 2022, and are traded on the Binance cryptocurrency exchange market.

We have a per-minute price level for 12 months (in 2021) for all crypto-assets considered; thus, each price series includes a closing price of 43,200 ($30 \times 24 \times 60 = 43{,}200$) minutes (if the month contains 30 days). These real price series are categorized into six groups according to their volatility and trends.

- Up-trend with high volatility
- Up-trend with low volatility
- Down-trend with high volatility
- Down-trend with low volatility
- No-trend with high volatility
- No-trend with low volatility

To classify the series, their volatilities are calculated as follows.

Let P_t denote the asset price at time t. Then, $r_t = (P_t - P_{t-1})/(P_{t-1})$ represents simple daily returns on an asset. The standard deviation (σ) is the most common measure of volatility and is calculated as

$$\sigma = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}|\mathbf{r}_t - \mu|^2} \tag{1}$$

where μ is the mean value of $\mathbf{r}$. By multiplying the standard deviation by the square root of the number of periods (T), volatility (v) is

$$v = \sigma\sqrt{T}. \tag{2}$$

The volatility of the price series is computed using Matlab®, and they are classified as relatively high volatility if the volatility is larger than 50% (or 0.5) and relatively low volatility if the volatility is less than 50%. After the price series are categorized into six subgroups, the Johansen cointegration test is applied to each pair of price series (pairs) in the portfolios. Thus, we apply the HFRA to cointegrated price data for all trends and volatility regimes. The formulation of the Johansen cointegration test is outlined in the following subsection.

The Johansen test and vector error correction model (VECM)

The Johansen test analyzes whether two or more time series can form a cointegrating relationship, and its methodology is based on a vector autoregressive (VAR) model (Johansen 1991). The general form of the VAR model of order p with a constant term is

$$\mathbf{x}_t = \sum_{i=1}^{p}\mathbf{A}_i\mathbf{x}_{t-i} + \boldsymbol{\varepsilon}_t, \tag{3}$$

where $\mathbf{x}_t$ is an $m \times 1$ vector of variables integrated of order one, and $\boldsymbol{\varepsilon}_t$ is an $m \times 1$ vector of innovations.

By differencing the series Eq. (3) for $p > 1$, this VAR in levels can be transformed to a vector error correction model given by

$$\Delta\mathbf{x}_t = \boldsymbol{\Pi}\mathbf{x}_{t-1} + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\boldsymbol{\Delta}\mathbf{x}_{t-i} + \boldsymbol{\varepsilon}_t. \tag{4}$$

Here, $\boldsymbol{\Pi}$ is the coefficient matrix for the first lag and $\boldsymbol{\Gamma}$ are the matrices for each differenced lag. If $p = 1$, then the model is reduced to $\Delta\mathbf{x}_t = \boldsymbol{\Pi}\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t$. Matrix $\boldsymbol{\Pi}$ can be written in terms of the vector (or matrix) of the adjustment parameters $\boldsymbol{\alpha}$ and the vector (or matrix) of cointegrating vectors $\boldsymbol{\beta}$ as

$$\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}', \tag{5}$$

where $\boldsymbol{\beta}$ represents the cointegration vectors and $\boldsymbol{\alpha}$ represents the effect of each cointegrating vector on the $\Delta\mathbf{x}$ variables in the model. The rank of $\boldsymbol{\Pi}$ determines the number of independent rows in $\boldsymbol{\Pi}$ and the number of cointegration vectors.

The Johansen test's null hypothesis is H_0: $r = 0$ indicates no cointegration at all, against the alternative that H_1: $r > 0$ indicates a cointegration relationship within two or perhaps more time series. The Johansen testing procedure sequentially tests whether $r = 0, 1, ..., m - 1$, where m is the number of time series being tested.

Five different cases are used to estimate the Johansen test based on Eq. (4). In our case, we test the unrestricted trend, assuming that intercepts and linear trends are present in the cointegrating relations and deterministic quadratic trends are present in the levels of the data. Based on these assumptions, we can rewrite the VECM in Eq. (4) as

$$\Delta \mathbf{x}_t = \boldsymbol{\Gamma}_i \Delta \mathbf{x}_{t-i} + \boldsymbol{\alpha}(\boldsymbol{\beta}\mathbf{x}_{t-1} + \boldsymbol{\mu} + \boldsymbol{\rho} t) + \boldsymbol{\gamma} + \boldsymbol{\tau} t + \boldsymbol{\varepsilon}_t, \tag{6}$$

where $\boldsymbol{\mu}$ is an r-by-1 vector of constants (intercepts) in the cointegrating relations (Johansen 1995). $\boldsymbol{\rho}$ is an $r - by - 1$ vector of linear time trends in cointegrating relations. $\boldsymbol{\gamma}$ is an m-by-1 vector of constants (the deterministic linear trends in $\mathbf{x}_t$). $\boldsymbol{\tau}$ is an m-by-1 vector of linear time-trend values (deterministic quadratic trends in $\mathbf{x}_t$). $\boldsymbol{\varepsilon}_t$ is an m-by-1 vector of random Gaussian innovations, each with a mean of 0 and collectively an m-by-m covariance matrix, $\boldsymbol{\Sigma}$. For $t \neq s$, $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\varepsilon}_s$ are independent.

The outcomes of the Johansen cointegration test are calculated using the function *jcitest.m* provided in the Econometrics Toolbox of Matlab®.

The high frequency rebalancing algorithm (HFRA)

In this study, an algorithmic trading algorithm is constructed to integrate pair-trading and threshold-based rebalancing strategies. For the proposed HFRA, we assume that at the starting point of the trade, we have a portfolio with equally weighted asset allocations. We use an equally weighted asset allocation to simplify the comparison of the profitability of portfolios of various sizes. Thus, all assets in the portfolio have equal value (current price × quantity). For example, we have 50 P_1 with price 0.02 $(50 \times 0.02 = 1)$ and 25 P_2 with price 0.04 $(25 \times 0.04 = 1)$. After creating the portfolio, the HFRA is implemented as follows. If the parity (or price difference in percent) of the values of the two assets is higher than a threshold (T), the overvalued asset will be sold and the undervalued asset will be bought to equalize their value to the average value. The optimal threshold value changes according to market parameters (e.g., volatility, price level, or volume) and portfolio size, and in the final part of this study it is revealed that there is not a common optimal threshold value. Therefore, we calculate the average profits of the portfolios when the T value gradually increased from 1 to 20% with 1% step size. We determine this threshold interval (between 1 and 20%) for the markets considered by observing the maximum price jumps in the per-minute market data. As shown in Fig. 1, each asset is denoted by P_i where $i = 1, 2, 3, ..., n$, and the number of assets in the portfolio can be two (see panel (a)), three (see panel (b)), or more than three (see panel (c)).

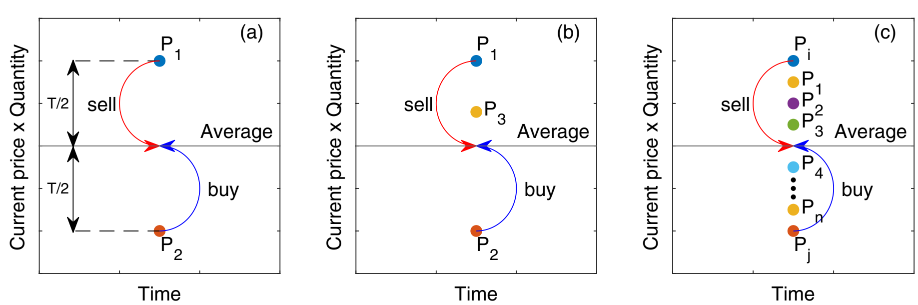

Fig. 1 HFRA diagram when the portfolio includes **a** two, **b** three, and **c** multiple (n) assets

Importantly, for real trading on exchange markets, traders must pay trading fees (or transaction costs) to execute buy and sell orders. If the threshold value is relatively small, the number of trades increases, and in this case, trading fees become a critical factor that significantly affects profitability (Lang et al. 2011; Woodside-Oriakhi et al. 2013). The trading fee is taken into account as 0.1% (i.e., the taker transaction fee on the Binance cryptocurrency exchange) in the pseudocodes below and the Matlab® script of the HFRA that is given in the Appendix (see Rebalancing_Algorithm.m).

Algorithm 1 presents the pseudocode that implements HFRA when the portfolio includes two assets (P_1 and P_2). The following definitions are used in the algorithm:

Current price of an asset (P_i): CP_{P_i}

Asset quantity (P_i): Q_{P_i}

Total value of an asset (P_i): TV_{P_i} and $TV_{P_i} = CP_{P_i} \times Q_{P_i}$

Threshold for ratio: T

Trading fee: TF

```
while (1) do
    TV_{P_1} = CP_{P_1} × Q_{P_1}
    TV_{P_2} = CP_{P_2} × Q_{P_2}
    if TV_{P_1}/TV_{P_2} > (1 + T) then
        Sell (1 + TF) × 0.5 × T × Q_{P_1} with price CP_{P_1}
        buy (1 − TF) × (0.5 × T × Q_{P_1})/CP_{P_2} quantity of P_2
    else if TV_{P_2}/TV_{P_1} > (1 + T) then
        Sell (1 + TF) × 0.5 × T × Q_{P_2} with price CP_{P_2}
        buy (1 − TF) × (0.5 × T × Q_{P_2})/CP_{P_1} quantity of P_1
    end if
end while
```

Algorithm 1 Pseudo code of the HFRA for portfolios with 2 assets

Thus, the Algorithm 1 equalizes the total value of 2 assets when the ratio is higher than a threshold value. When the number of assets in the portfolio exceeds two, Algorithm 2 will be implemented. We assume that the number of assets is n where $n \geq 2$.

```
while (1) do
    for i ← 1 to n do
        TV_{P_i} = CP_{P_i} × Q_{P_i}
        for j ← 1 to n do
            TV_{P_j} = CP_{P_j} × Q_{P_j}
            if TV_{P_i}/TV_{P_j} > (1 + T) then
                Sell (1 + TF) × 0.5 × T × Q_{P_i} with price CP_{P_i}
                buy (1 − TF) × (0.5 × T × Q_{P_i})/CP_{P_j} quantity of P_j
            end if
        end for
    end for
end while
```

Algorithm 2 Pseudo code of the HFRA for portfolios with multiple (n) assets

Clearly, Algorithm 1 is a special case of Algorithm 2, which can be obtained when n=2.

To explain the implementation of the HFRA explicitly, it is applied to two price series (assets), as shown in Fig. 2. These series show the per-minute closing price ratios for the AAVE/BTC and AVAX/BTC markets in the first week of 2021. Thus, we

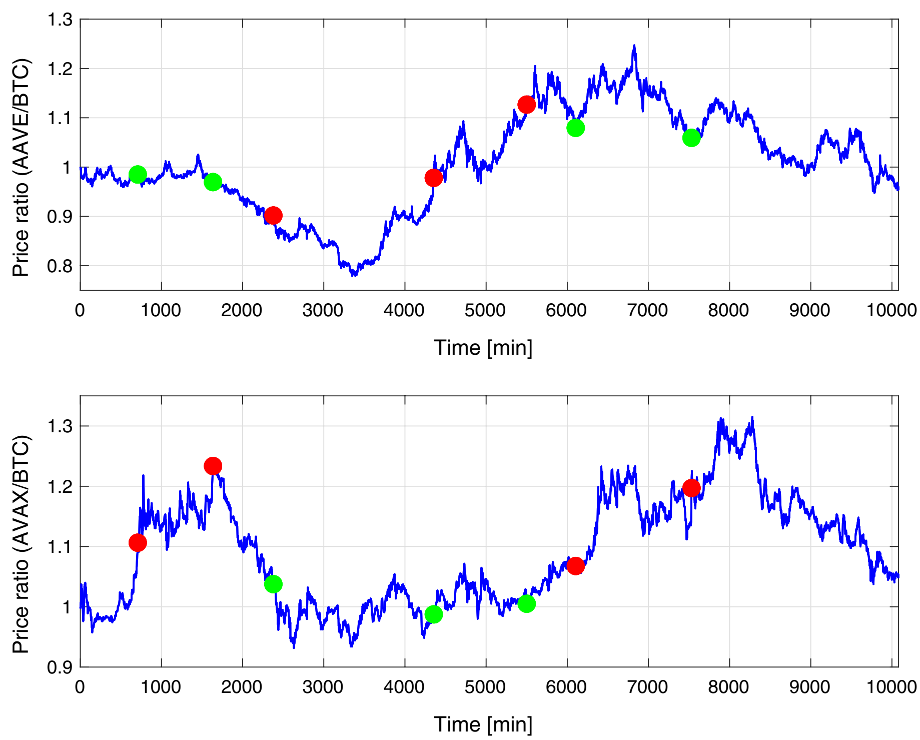

Fig. 2 Application of the HFRA to the AAVE/BTC and AVAX/BTC markets in the first week of 2021. Green and red marks denote executed buy and sell orders, respectively

have price ratios of 10,080 min that correspond to one week ($60 \times 24 \times 7 = 10{,}080$). In this simulation and in the remainder of this work, the initial values of the price series are normalized (or fixed) to 1 by dividing all elements of the series by the first element of the series. This adjustment does not change the volatility or trend regime of the price series and facilitates comparison of the price series in the portfolio. To have an equally weighted portfolio of two assets, we assume that we have two crypto-assets (AAVE and AVAX) in the portfolio with the same quantity of one, where the initial price ratio of both assets is set to one. The threshold value (T) is taken as 12%, and with a realistic approach, 0.1% of the trading fee is applied for each buy and sell order.

The HFRA trades 7 times, where the green and red marks indicate consecutively executed buy and sell orders, respectively. While the first asset (AAVE) is sold 3 times and bought 4 times, the second asset (AVAX) is sold 4 times and bought 3 times. At the end of this trade, there are 1.0284 AAVE and 0.9949 AVAX quantities and the final price ratios of AAVE/BTC and AVAX/BTC are 0.9542 and 1.0485, respectively. Thus, the total portfolio value is $1.0284 \times 0.9542 + 0.9949 \times 1.0485 = 2.0245$. This result reveals that the HFRA provides significant profit (1.225%) in a single week for the markets considered.

Assumptions and limitations

The profit of the HFRA depends on the threshold value T (corresponding to the profit margin), and there is no common optimal threshold for portfolios of varied sizes and different volatility regimes. Thus, we will increase the T value from 1 to 20% gradually with a 1% step size, and the profit of the HFRA will be calculated as the average of the obtained profits. If the maximum parity (or price difference in percentages) for the considered price series was less than 20%, the upper limit for the threshold was selected as the largest parity value. In other words, if there is no trade by the HFRA, the computed profit will not be considered in the calculation of the average profit.

Note here that we assume that the capital size is 1 BTC for each market; thus, the total capital for the HFRA will be the size of the portfolio (n) times 1 BTC. Observing order books on Binance cryptocurrency exchange markets, we see that the buy and sell amounts of current (bid-ask) price levels are larger than 1% of the daily volume of the market considered. Thus, to avoid price impacts on the markets considered for a real portfolio application in exchange markets, the capital for each asset should be less than ($1/T$) % of the market with the least daily volume. For instance, if the daily market volumes of P_1/BTC and P_2/BTC are 30 and 40 BTC, and the threshold (T) is chosen as 0.1 (or 10%), then the capital for each asset should be less than $(1/0.1) \times 30/100 = 3$ BTC. Otherwise, a potential trading loss can be realized because of the price impact. It is clear that decreasing the threshold value and/or considering high-volume markets reduces the risk of trading loss caused by the price impact. In this direction, for the analysis in this study, the assets in portfolios are selected from the 40 crypto-assets that have the largest market cap, and the threshold value is limited to 20%.

Results and discussion

In this section, we first apply HFRA to real price series (assets) under six different trend and volatility regimes (defined in the previous section), and then demonstrate the practicability of HFRA via real data implementations. For the real price series, the profit of the HFRA will be examined when the portfolio includes 2, 3, and 5 assets (or crypto-currencies).

As we noted in the previous section, to facilitate a comparison of the price series in the portfolio, the initial values of the price series are normalized (or fixed) to 1 by dividing all elements of the series by the first element of the series. This adjustment has no effect on the volatility, trend, or cointegration of the price series or the calculated profits of the portfolios.

Importantly, we apply the Johansen cointegration test to the full sample period (one month) of the price series for each portfolio. Although there is a cointegration relationship according to the full-sample period and the simple moving average approach for varied window sizes (daily, hourly, or per minute), there is no meaningful relationship when examining the cointegration of the series with the rolling and expanding window approaches (Leung and Nguyen 2019; Tadi and Kortchemski 2021). Therefore, we state only the statistics for the full-sample Johansen cointegration test in the remainder of the study. In addition, the HFRA is not a cointegration based pairs trading strategy, and the entry/exit threshold levels can be determined by market-based parameters or risk considerations (Moallemi and Saglam 2015); consequently, the HFRA does not require the calculation of a threshold value according to the cointegration statistics.

For real-data applications, the historical data of 40 crypto-assets (listed in Sec. 2) are utilized. Price series are categorized into six groups according to their volatility and trend regimes, and the Johansen cointegration test is applied to each pair of price series (pairs) in the portfolios.

Up-trend

In Fig. 3, we plot the series of per-minute closing prices of AAVE/BTC, MKR/BTC, SNX/BTC, SOL/BTC, and XLM/BTC markets for January 2021. These series show an up-trend with high volatility, and the price ratios vary between 0.779 and 3.158.

The volatility of the price series, p-values of the Johansen cointegration test for all pairs, and profit of the HFRA when the portfolio includes 2, 3, and 5 crypto-assets are shown in Tables 1, 2, 3, 4, and 5, respectively.

Table 1 shows that the volatility values of the series are larger than 0.50 (or 50%); thus, volatility is relatively high for the price series considered. All p-values in Table 2 are less than 0.05, indicating that all price series in the portfolio are cointegrated. The profits of the portfolios of the two crypto-assets are presented in Table 3, which shows that the profits are between 125.64 and 184.98%, and the average profit is 151.5%. Similarly, Table 4 displays the profits of the portfolios of the three crypto-assets, which are between 134.49 and 175.88%, with an average profit of 156.06%. Table 5 shows that the

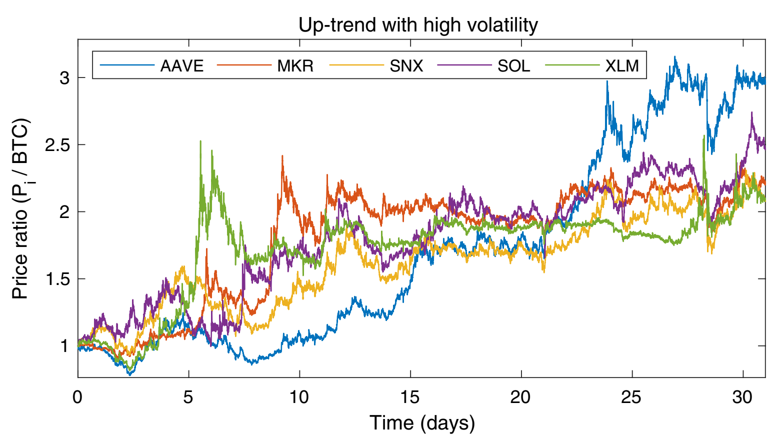

Fig. 3 Per-minute closing price ratio of the AAVE/BTC, MKR/BTC, SNX/BTC, SOL/BTC, and XLM/BTC markets in January 2021

Table 1 Price series volatility

AAVE	MKR	SNX	SOL	XLM
0.5622	0.6368	0.5852	0.7265	0.7113

Table 2 p values of the Johansen cointegration test (r_0, r_1) for all pairs

	XLM		SOL		SNX		MKR	
	r_0	r_1	r_0	r_1	r_0	r_1	r_0	r_1
AAVE	0.0146	0.0176	0.0051	0.0093	0.0136	0.0094	0.0483	0.0336
MKR	0.0245	0.0179	0.0013	0.0179	0.0154	0.0154		
SNX	0.0044	0.0022	0.0010	0.0010				
SOL	0.0046	0.0039						

Table 3 Profit of the HFRA with a portfolio of 2 crypto-assets

	XLM (%)	SOL (%)	SNX (%)	MKR (%)
AAVE	172.30	184.98	156.96	175.13
MKR	138.38	150.83	128.17	
SNX	125.64	135.73		
SOL	146.89			

Table 4 Profit of the HFRA with a portfolio of 3 crypto-assets

	XLM-SOL (%)	XLM-SNX (%)	SOL-SNX (%)	XLM-MKR (%)	SOL-MKR (%)	SNX-MKR (%)
AAVE	173.92	155.17	162.87	166.04	175.88	155.90
MKR	152.36	134.49	142.53			
SNX	141.45					

Table 5 Profit of the HFRA with a portfolio of 5 crypto-assets

AAVE–MKR–SNX–SOL–XLM	160.46%

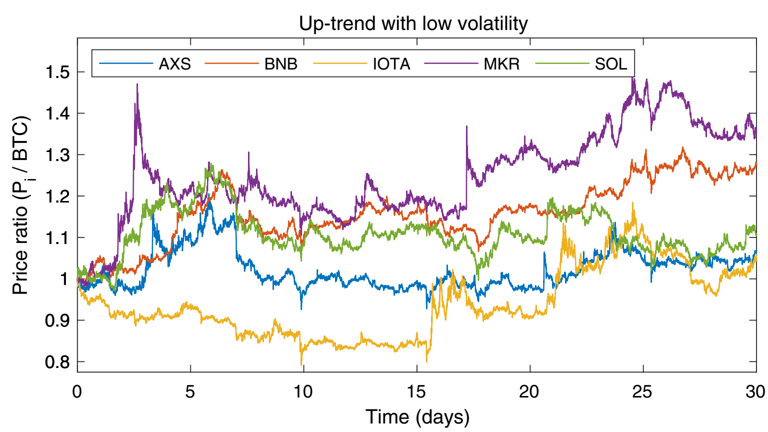

Fig. 4 Per-minute closing price ratio of the AXS/BTC, BNB/BTC, IOTA/BTC, MKR/BTC, and SOL/BTC markets in November 2021

profit of a portfolio with 5 crypto-assets is 160.46%. These results reveal that the average profit of portfolios increases as the number of crypto-assets in the portfolio increases if the asset prices are in an up-trend with relatively high volatility.

In Fig. 4, we plot the series of per-minute closing prices of AXS/BTC, BNB/BTC, IOTA/BTC, MKR/BTC, and SOL/BTC markets in November 2021. These series show an up-trend with relatively low volatility and the price ratio varies between 0.791 and 1.492.

The volatility of price series, p-values of Johansen cointegration test for all pairs, and profit of the HFRA when the portfolio is including 2, 3, and 5 crypto-assets are shown in Tables 6, 7, 8, 9, and 10, respectively.

Table 6 Price series volatility

AXS	BNB	IOTA	MKR	SOL
0.2913	0.1556	0.3119	0.3091	0.2399

Table 7 *p*-values of the Johansen cointegration test (r_0, r_1) for all pairs

	SOL		MKR		IOTA		BNB	
	r_0	r_1	r_0	r_1	r_0	r_1	r_0	r_1
AXS	0.0010	0.0044	0.0042	0.0051	0.0080	0.0084	0.0015	0.0029
BNB	0.0395	0.0070	0.0187	0.0082	0.0434	0.0156		
IOTA	0.0268	0.0085	0.0077	0.0103				
MKR	0.0176	0.0040						

Table 8 Profit of the HFRA with a portfolio of 2 crypto-assets

	SOL (%)	MKR (%)	IOTA (%)	BNB (%)
AXS	9.93	22.04	7.77	17.40
BNB	19.45	32.99	17.51	
IOTA	9.48	21.81		
MKR	23.94			

Table 9 Profit of the HFRA with a portfolio of 3 crypto-assets

	SOL-MKR (%)	SOL-IOTA (%)	MKR-IOTA (%)	SOL-BNB (%)	MKR-BNB (%)	IOTA-BNB (%)
AXS	18.73	9.31	17.47	15.67	24.36	14.89
BNB	25.76	15.96	24.27			
IOTA	18.68					

Table 10 Profit of the HFRA with a portfolio of 5 crypto-assets

AXS–BNB–IOTA–MKR–SOL	18.70%

Table 6 shows that the volatility values of the series are less than 0.50; thus, volatility is relatively low for the price series considered. All *p*-values in Table 7 are less than 0.05, indicating that all price series in the portfolio are cointegrated. The profits of the portfolios of the two crypto-assets are presented in Table 8, which shows that the profits are between 7.77 and 32.99%, and the average profit is 18.23%. Similarly, Table 9 displays the profits of the portfolios of the three crypto-assets, which are between 9.31 and 25.76%, with an average profit of 18.51%. Table 10 shows that the profit of a portfolio with 5 crypto-assets is 18.70%. These results reveal that although the average profit of portfolios increases as the number of crypto-assets in the portfolio increases, there is no significant difference between profits when asset prices are in an up-trend with relatively low volatility.

Down-trend

In Fig. 5, we plot the series of per-minute closing prices of AXS/BTC, CHZ/BTC, FIL/BTC, FTM/BTC, and GRT/BTC markets for May 2021. These series show a down-trend with relatively high volatility, and the price ratio varies between 0.46 and 1.45.

The volatility of price series, *p*-values of Johansen cointegration test for all pairs, and loss of the HFRA when the portfolio is including 2, 3, and 5 crypto-assets are shown in Tables 11, 12, 13, 14, and 15, respectively.

Table 11 shows that the volatility values of the series are greater than 0.50; thus, volatility is relatively high for the price series considered. *p*-values in Table 12 are less than 0.05 for all pairs; therefore, all the price series in the portfolio are cointegrated. The losses of the portfolios of the 2 crypto-assets are presented in Table 13, which shows that the losses are between −29.63 and −21.13%, and the average loss is −25.20%. Similarly, Table 14 displays the losses of the portfolios of 3 crypto-assets that are between −26.39 and −21.32%, and the average loss is −24.07%. Table 15 shows that the loss of the portfolio with 5 crypto-assets is −23.04%. These results reveal that average portfolio loss decreases as the number of crypto-assets in the portfolio increases.

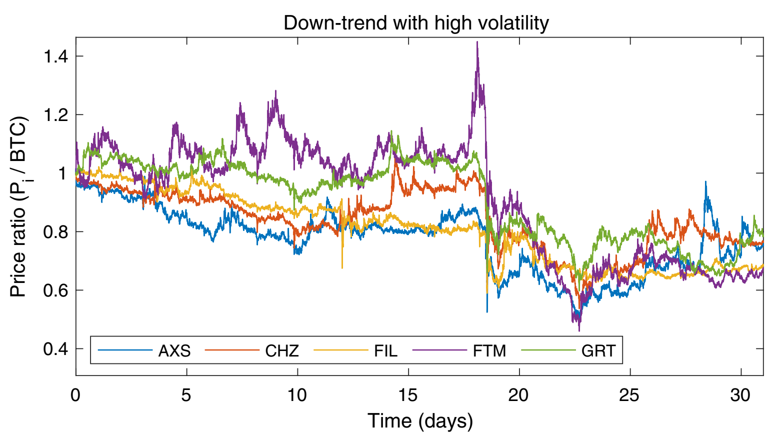

Fig. 5 Per-minute closing price ratio of AXS/BTC, CHZ/BTC, FIL/BTC, FTM/BTC, and GRT/BTC markets in May 2021

Table 11 Price series volatility

AXS	CHZ	FIL	FTM	GRT
0.6713	0.5606	0.5199	0.7834	0.5018

Table 12 *p*-values of the Johansen cointegration test (r_0, r_1) for all pairs

	GRT		FTM		FIL		CHZ	
	r_0	r_1	r_0	r_1	r_0	r_1	r_0	r_1
AXS	0.0043	0.0090	0.0103	0.0077	0.0010	0.0044	0.0010	0.0118
CHZ	0.0010	0.0115	0.0037	0.0102	0.0010	0.0046		
FIL	0.0010	0.0033	0.0010	0.0028				
FTM	0.0010	0.0077						

Table 13 Profit of the HFRA with a portfolio of 2 crypto-assets

	GRT (%)	FTM (%)	FIL (%)	CHZ (%)
AXS	− 22.93	− 26.78	− 26.77	− 22.78
CHZ	− 21.13	− 25.84	− 25.25	
FIL	− 25.41	− 29.63		
FTM	− 25.44			

Table 14 Profit of the HFRA with a portfolio of 3 crypto-assets

	GRT–FTM (%)	GRT–FIL (%)	FTM–FIL (%)	GRT–CHZ (%)	FTM–CHZ (%)	FIL–CHZ (%)
AXS	− 23.54	− 24.29	− 26.39	− 21.32	− 23.71	− 24.02
CHZ	− 23.06	− 23.12	− 25.54			
FIL	− 25.72					

Table 15 Profit of the HFRA with a portfolio of 5 crypto-assets

AXS–CHZ–FIL–FTM–GRT	-23.04%

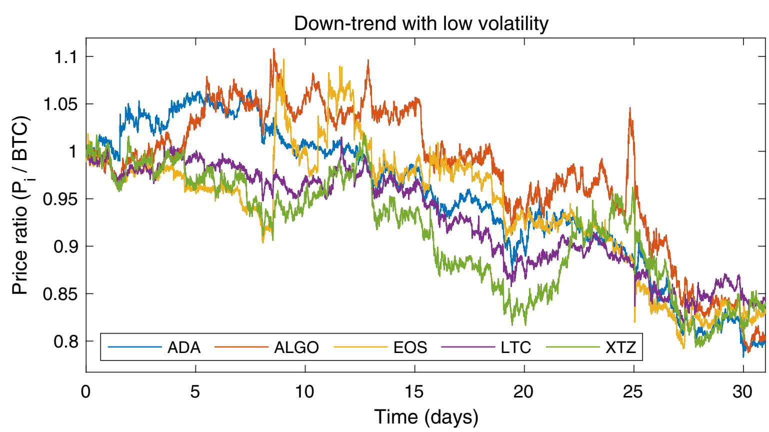

Fig. 6 Per-minute closing price ratio of the ADA/BTC, ALGO/BTC, EOS/BTC, LTC/BTC, and XTZ/BTC markets in July 2021

In Fig. 6, we plot the series of per-minute closing prices of ADA/BTC, ALGO/BTC, EOS/BTC, LTC/BTC, and XTZ/BTC markets for July 2021. These series are in a down-trend with relatively low volatility, and the price ratio varies between 0.78 and 1.11.

The volatility of price series, *p*-values of Johansen cointegration test for all pairs, and loss of the HFRA when the portfolio is including 2, 3, and 5 crypto-assets are shown in Tables 16, 17, 18, 19, and 20, respectively.

Table 16 shows that the volatility of the series are less than 0.50; thus, volatility is relatively low for the price series considered. *p*-values in Table 17 are less than 0.05 for all pairs; therefore, all the price series in the portfolio are cointegrated. The

Table 16 Price series volatility

ADA	ALGO	EOS	LTC	XTZ
0.1675	0.2298	0.2425	0.1577	0.2780

Table 17 *p*-values of the Johansen cointegration test (r_0, r_1) for all pairs

	XTZ		LTC		EOS		ALGO	
	r_0	r_1	r_0	r_1	r_0	r_1	r_0	r_1
ADA	0.0010	0.0010	0.0010	0.0010	0.0091	0.0058	0.0010	0.0123
ALGO	0.0097	0.0204	0.0015	0.0187	0.0447	0.0276		
EOS	0.0032	0.0046	0.0011	0.0070				
LTC	0.0010	0.0010						

Table 18 Profit of the HFRA with portfolio of 2 crypto-assets

	XTZ (%)	LTC (%)	EOS (%)	ALGO (%)
ADA	− 20.12	− 18.71	− 19.66	− 20.27
ALGO	− 19.18	− 17.70	− 18.53	
EOS	− 18.52	− 17.06		
LTC	− 17.62			

Table 19 Profit of the HFRA with a portfolio of 3 crypto-assets

	XTZ-LTC (%)	XTZ-EOS (%)	LTC-EOS (%)	XTZ-ALGO (%)	LTC-ALGO (%)	EOS-ALGO (%)
ADA	− 18.76	− 19.36	− 18.39	− 19.78	− 18.88	− 19.39
ALGO	− 18.07	− 18.66	− 17.72			
EOS	− 17.69					

Table 20 Profit of the HFRA with a portfolio of 5 crypto-assets

ADA–ALGO–EOS–LTC–XTZ	− 18.58%

losses of the portfolios of the two crypto-assets are presented in Table 18, which shows that the losses are between −20.27 and −17.06%, and the average loss is −18.74%. Similarly, Table 19 displays the losses of the portfolios of 3 crypto-assets that are between −19.78 and −17.69%, and the average loss is −18.67%. Table 20 shows that the loss of the portfolio with 5 crypto-assets is −18.58%. These results indicate that although the average loss of portfolios decreases slightly as the number of crypto-assets in the portfolio increases, there is no significant difference between the losses of portfolios of varied sizes.

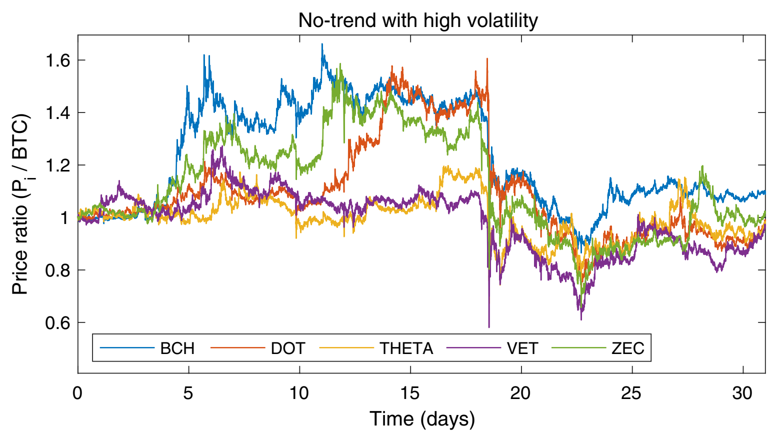

Fig. 7 Per-minute closing price ratio of the BCH/BTC, DOT/BTC, THETA/BTC, VET/BTC, and ZEC/BTC markets in May 2021

Table 21 Price series volatility

BCH	DOT	THETA	VET	ZEC
0.5444	0.5903	0.597	0.8015	0.5971

Table 22 *p*-values of the Johansen cointegration test (r_0, r_1) for all pairs

	ZEC		VET		THETA		DOT	
	r_0	r_1	r_0	r_1	r_0	r_1	r_0	r_1
BCH	0.0010	0.0331	0.0010	0.0283	0.0088	0.0287	0.0370	0.0161
DOT	0.0338	0.0410	0.0010	0.0248	0.0025	0.0406		
THETA	0.0061	0.0283	0.0010	0.0010				
VET	0.0010	0.0208						

Table 23 Profit of the HFRA with a portfolio of 2 crypto-assets

	ZEC (%)	VET (%)	THETA (%)	DOT (%)
BCH	6.58	0.27	4.76	1.90
DOT	− 1.18	− 7.86	− 3.06	
THETA	0.91	− 5.84		
VET	− 2.80			

No trend

In Fig. 7, we plot a series of per-minute closing prices for BCH/BTC, DOT/BTC, THETA/BTC, VET/BTC, and ZEC/BTC markets in May 2021. These series have relatively low volatility with no trend, and the price ratio varies between 0.58 and 1.66.

The volatility of price series, *p*-values of Johansen cointegration test for all pairs, and profit of the HFRA when the portfolio is including 2, 3, and 5 crypto-assets are shown in Tables 21, 22, 23, 24, and 25, respectively.

Table 24 Profit of the HFRA with a portfolio of 3 crypto-assets

	ZEC-VET (%)	ZEC-THETA (%)	VET-THETA (%)	ZEC-DOT (%)	VET-DOT (%)	THETA-DOT (%)
BCH	2.39	4.99	0.85	3.82	− 1.06	2.06
DOT	− 2.75	0.06	− 4.58			
THETA	− 1.49					

Table 25 Profit of the HFRA with a portfolio of 5 crypto-assets

BCH–DOT–THETA–VET–ZEC	1.70%

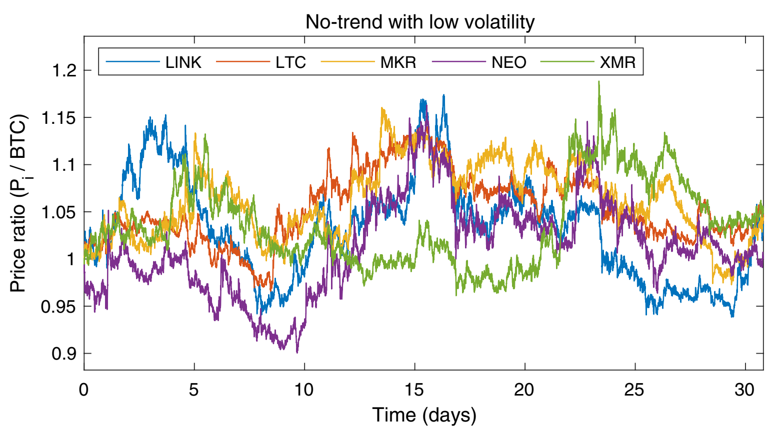

Fig. 8 Per-minute closing price ratio of the LINK/BTC, LTC/BTC, MKR/BTC, NEO/BTC, and XMR/BTC markets in August 2021

Table 21 shows that the volatility values of the series are larger than 0.50; thus, volatility is relatively high for the price series considered. All p-values in Table 22 are less than 0.05, indicating that all price series in the portfolio are cointegrated. The profits of the portfolios of the 2 crypto-assets are presented in Table 23, which shows that the profits are between −7.86 and 6.58%, and the average profit is −0.63%. Similarly, Table 24 reports that the profits of the portfolios of 3 crypto-assets that are between −4.58 and 4.99%, and the average profit is 0.43%. Table 25 shows that the profit of a portfolio with 5 crypto-assets is 1.70%. These results reveal that the average profit of portfolios increases as the number of crypto-assets in the portfolio increases if asset prices have no trend with relatively high volatility.

In Fig. 8, we plot the series of per-minute closing prices of the LINK/BTC, LTC/BTC, MKR/BTC, NEO/BTC, and XMR/BTC markets in August 2021. These series have relatively low volatility with no trends, and the price ratio varies between 0.58 and 1.66.

The volatility of price series, p-values of Johansen cointegration test for all pairs, and profit of the HFRA when the portfolio including 2, 3, and 5 crypto-assets are shown in Tables 26, 27, 28, 29, and 30, respectively.

Table 26 shows that the volatility values of the series are less than 0.50; thus, volatility is relatively low for the price series considered. All p-values in Table 27 are less

Table 26 Price series volatility

LINK	LTC	MKR	NEO	XMR
0.2297	0.1711	0.2342	0.2864	0.2445

Table 27 *p*-values of the Johansen cointegration test (r_0, r_1) for all pairs

	XMR		NEO		MKR		LTC	
	r_0	r_1	r_0	r_1	r_0	r_1	r_0	r_1
LINK	0.0208	0.0148	0.0010	0.0162	0.0112	0.0092	0.0319	0.0102
LTC	0.0102	0.0323	0.0010	0.0107	0.0024	0.0119		
MKR	0.0174	0.0055	0.0010	0.0096				
NEO	0.0267	0.0065						

Table 28 Profit of the HFRA with a portfolio of 2 crypto-assets

	XMR (%)	NEO (%)	MKR (%)	LTC (%)
LINK	1.95	− 0.31	− 0.40	0.85
LTC	4.15	2.06	1.72	
MKR	2.62	0.75		
NEO	3.31			

Table 29 Profit of the HFRA with a portfolio of 3 crypto-assets

	XMR–NEO (%)	XMR–MKR (%)	NEO–MKR (%)	XMR–LTC (%)	NEO–LTC (%)	MKR–LTC (%)
LINK	1.86	1.50	0.13	2.41	0.96	0.89
LTC	3.36	2.94	1.60			
MKR	2.36					

Table 30 Profit of the HFRA with a portfolio of 5 crypto-assets

LINK–LTC–MKR–NEO–XMR	1.88%

than 0.05, indicating that all price series in the portfolio are cointegrated. The profits of the portfolios with 2 crypto-assets are presented in Table 28, which shows that the profits are between −0.40 and 4.15%, and the average profit is 1.67%. Similarly, Table 29 displays the profits of the portfolios of 3 crypto-assets that are between 0.13 and 3.36%, with an average profit of 1.80%. Table 30 shows that the profit of a portfolio with 5 crypto-assets is 1.88%. These results reveal that the average profit of portfolios increases slightly as the number of crypto-assets in the portfolio increases if the asset prices have no trend with relatively low volatility.

Long-term application

To demonstrate the profitability of the HFRA on long-term implementations and compare the performance of the HFRA with the previous studies, we implement it for a

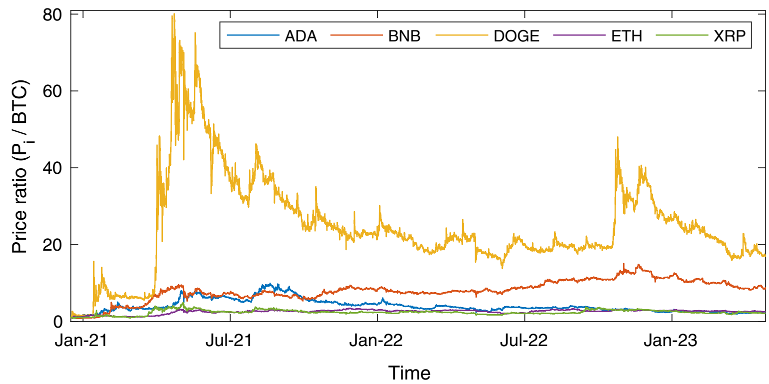

Fig. 9 Per-minute closing price ratio of the ADA/BTC, BNB/BTC, DOGE/BTC, ETH/BTC, and XRP/BTC markets between January 1, 2021 and April 30, 2023 (28 months)

Table 31 Price series volatility for 28 months

ADA	BNB	DOGE	ETH	XRP
1.3838	1.0604	6.2226	0.7720	1.5686

special portfolio of five crypto-assets with the largest market caps (ADA/BTC, BNB/BTC, DOGE/BTC, ETH/BTC, and XRP/BTC) for the last 28 months. We obtain the per-minute price level of these markets for 28 months (between January 1, 2021, and April 30, 2023) from Binance cryptocurrency exchange and plot the price ratios of these series in Fig. 9.

We compare the performance of the HFRA with the TR and PR strategies, which have been studied widely in the literature (DeMiguel et al. 2013; Das et al. 2014; Das and Goyal 2015; Zilinskij 2015; Costabile and Gaudenzi 2017; Symitsi and Chalvatzis 2019; Bernoussi and Rockinger 2022). As explained in the introduction, in the PR approach, assets are rebalanced periodically, such as daily, weekly, monthly, quarterly, or annually. In the TR approach, assets are rebalanced when the weight of an asset in the portfolio exceeds a specific (maximum or minimum) limit, such as 2% or 5%. This threshold can be determined using market-based parameters or risk considerations (Moallemi and Saglam 2015). In the next part of this study, we show that there is no predetermined (optimal) threshold value for the portfolio considered; thus, we compare the performance of HFRA and TR strategies by calculating the average profits for threshold values between 1 and 20%.

Table 31 shows the volatility of the five series between January 1, 2021, and April 30, 2023. The long-term volatility of the price series varied between 0.7720 (77.20%) and 6.2226 (622.26%).

In Table 32, the profitability of the HFRA, TR, and PR strategies are compared for an equally weighted portfolio of the five assets over 28 months. We used per-minute closing price data for the HFRA and the daily closing price for the TR algorithm. For the PR strategy, we selected daily, weekly, monthly, quarterly, and yearly intervals. Table 32 shows that the long-term profit of the HFRA outperforms those of the TR and PR strategies for all time intervals. The difference between the profits of the HFRA (1435.01%)

Table 32 Long-term profits of the HFRA, TR, and PR strategies

		PR				
HFRA	**TR**	**Daily**	**Weekly**	**Monthly**	**Quarterly**	**Annually**
1435.01%	935.04%	905.74%	815.56%	938.09%	934.11%	737.44%

and TR (935.04%) strategies indicates that utilizing high-frequency rebalancing (per minute of data) provides a remarkable advantage to the HFRA strategy. There is also no meaningful relationship between profit and the period of rebalancing (daily, weekly, monthly, quarterly, and annually) for the PR strategy.

To demonstrate the robustness of these results, we compared the daily, weekly, and monthly profits of the HFRA, TR, and PR strategies visually in Fig. 10. Figure 10 shows that the profit of the HFRA is higher than TR and PR strategies for all time periods considered.

To demonstrate the importance of high-frequency data, we apply the HFRA to the price data of various periods (1, 2, and 5 min) in Fig. 11 and compare the daily, weekly, and monthly profits. Using the per-minute closing price of the markets provides the best profitability (see the blue line in each panel), while the smallest profit is obtained using 5 min closing price of the markets (see the black dots in each panel). This reveals that the frequency of the price data plays a critical role in threshold-based rebalancing strategies, and as the frequency of the data increases, the profitability of the HFRA increases.

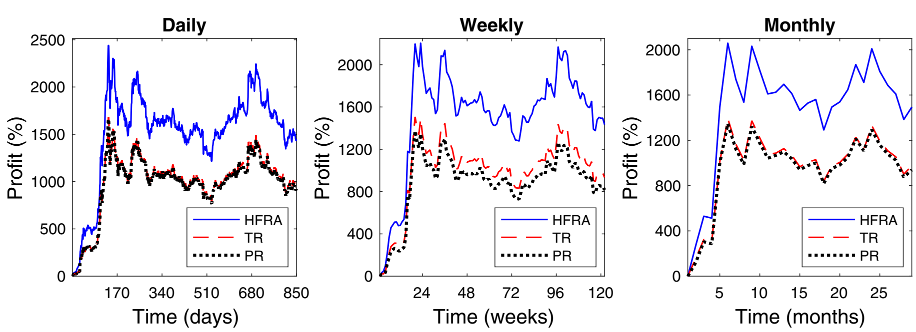

Fig. 10 Profit of the HFRA, TR, and PR strategies from a long-term application between January 1, 2021 and April 30, 2023 for various periods (daily, weekly, and monthly)

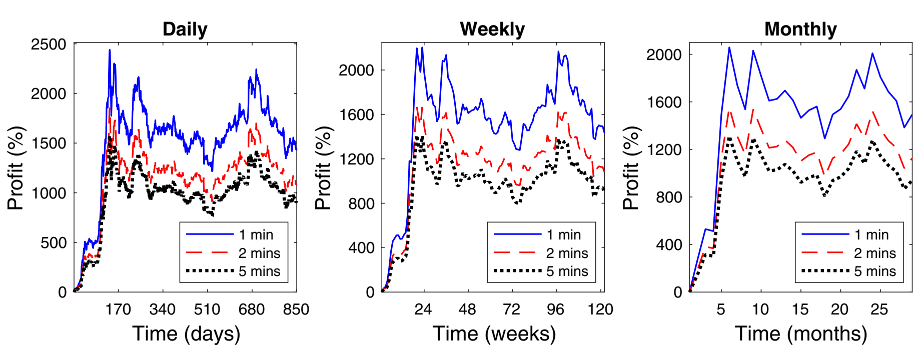

Fig. 11 Profit of the HFRA from a long-term application between January 1, 2021 and April 30, 2023 for various periods (daily, weekly, and monthly) when the price data period is 1, 2, and 5 min

In addition, we apply a permutation test to the long-term profits of HFRA, TR, and PR strategies to show that we obtain statistically significant higher returns using the HFRA. In Table 33, we report the one-sided p-values for the Fisher–Pitman permutation tests (Fisher 1935; Pitman 1937), which test for the difference between the means of the two independent samples. The one-sided null hypothesis states that the mean of the first distribution is not higher than that of the second.

The output in the first row of Table 33 shows that we can reject the null hypothesis that the return on the HFRA strategy is no higher than that on the TR strategy for all periods (daily, weekly, and monthly). In addition, in the second row, the null hypothesis that the return of the HFRA strategy is no higher than the return of the PR can be rejected, revealing that the HFRA strategy provides larger returns than the PR strategy for all periods. The final null hypothesis is that the return from the TR strategy is no higher than that from the PR strategy. As the p-values of the daily and monthly statistics were 0.0848 and 0.3776, respectively, and these values were greater than the significance level of 0.05, we failed to reject the null hypothesis. This result indicates that there is no significant difference between the returns of the two strategies. However, we find evidence that the TR strategy is characterized by greater returns than the PR strategy on a weekly basis.

Furthermore, the average number of trades for the HFRA and TR strategies are compared in Fig. 12 over 28 months. This analysis does not include the PR strategy because the number of trades is predetermined in PR, such as daily or weekly trades. In Fig. 12, the number of trades for each threshold value is calculated as the average number of trades during 28 months. For example, when the threshold is 4%, the average number of trades per month is 120 and 13 for the HFRA and TR strategies, respectively. The box plot inside shows a magnified portion of the original figure for threshold values between 10 and 20%. Figure 12 reveals that the number of trades, and consequently, the number of rebalances, for the HFRA is greater than the TR strategy for all threshold values, and as the threshold decreases, the difference between the number of trades increases.

that there is not an optimal threshold (or profit margin) value for the HFRA strategy.

Finally, we show that there is no optimal threshold (or profit margin) for the HFRA strategy. In Fig. 13a, the optimal threshold and corresponding maximum profit are plotted for 28 months between January 1, 2021, and April 30, 2023. Similarly, the relationship between the average volatility of the five crypto-assets and the threshold is shown in Fig. 13b. This analysis reveals no meaningful relationship between the threshold and volatility of the portfolio considered. Thus, there is no practical way to determine (or predict) the optimal threshold value using existing parameters. Therefore, the profit of

Table 33 p-Values of the permutation test for the long-term application of the HFRA, TR, and PR strategies

Strategies	One-sided p-Values		
	Daily	Weekly	Monthly
HFRA–TR	0	0	0
HFRA–PR	0	0	0
TR–PR	0.0848	0.0026	0.3776

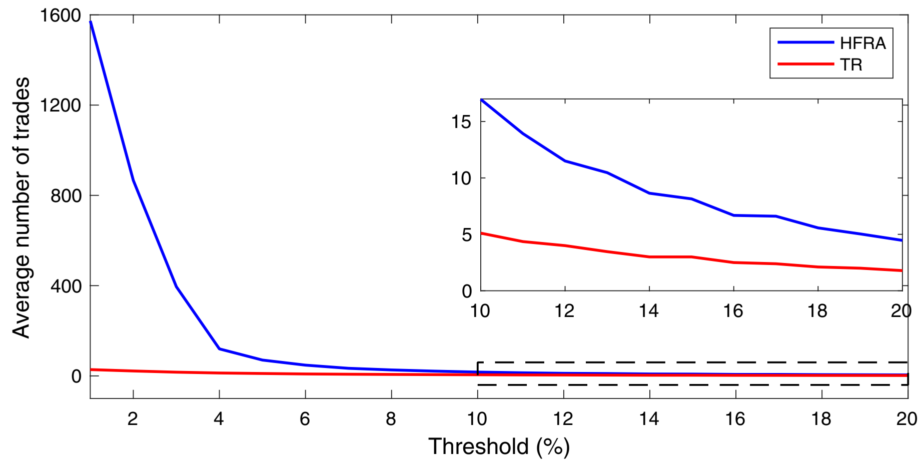

Fig. 12 Average number of trades per month for each threshold between January 1, 2021 and April 30, 2023

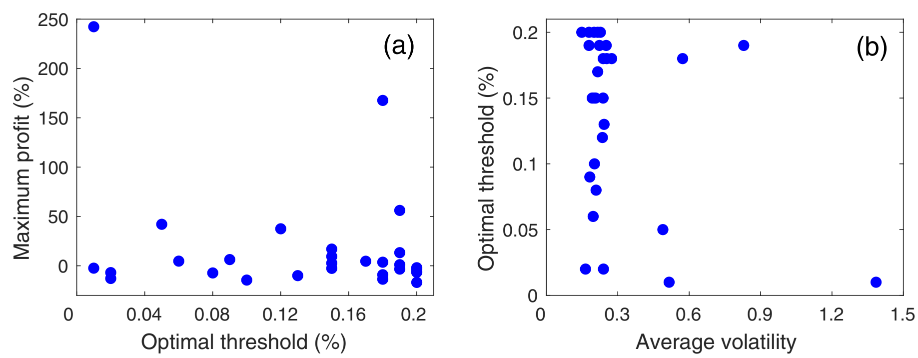

Fig. 13 Optimal threshold (profit margin) versus **a** maximum profit and **b** average volatility of 5 crypto-assets for the 28 months between January 1, 2021 and April 30, 2023

the HFRA is calculated as the average profit of the algorithm when the threshold is iterated from 1 to the 20%.

Conclusion

In this study, the HFRA, which rebalances asset allocation in a portfolio by pairs trading, is proposed and applied to a real dataset to select the optimal portfolio size according to the trend and volatility information of cointegrated price data. This dataset includes the per-minute closing price ratio of 40 crypto-assets from the Binance Exchange for 2021. The numerical results of the application are systematically presented for six different trends and volatility regimes. These regimes are up-trend with high volatility, up-trend with low volatility, down-trend with high volatility, down-trend with low volatility, no-trend with high volatility, and no-trend with low volatility.

Using a real data set, it is observed that increasing the number of assets in a portfolio assists the profitability of the HFRA for all trend regimes with high volatility. For low-volatility regimes, although increasing portfolio size marginally enhances the HFRA's profitability, the profits of portfolios of varied sizes do not significantly differ. Additionally, when volatility is relatively high and the trend is upward, HFRA can provide a substantial return via large portfolios. In addition, the outcomes of real price series applications reveal that increasing the number of assets in a portfolio lessens the potential loss of the HFRA for down-trends with a high volatility regime.

To demonstrate the profitability of a long-term implementation of the HFRA and compare its performance with that of the TR and PR strategies, which are common in practice for investment portfolios (DeMiguel et al. 2013; Das et al. 2014; Das and Goyal 2015; Zilinskij 2015; Costabile and Gaudenzi 2017; Symitsi and Chalvatzis 2019), we implement it to a special portfolio of five crypto-assets that are of the largest market cap (ADA/BTC, BNB/BTC, DOGE/BTC, ETH/BTC and XRP/BTC) for the last 28 months (between January 1, 2021 and April 30, 2023). The comparisons show that HFRA outperforms the TR and PR strategies over long-term implementation. This result is supported by visual analysis and validated statistically using Fisher–Pitman permutation test. These comparisons indicate that the use of high-frequency data in the HFRA creates a remarkable difference in the profitability of the rebalancing approach.

The final part of the study showed the lack of a meaningful relationship between the optimal threshold (or profit margin) value and the volatility of the crypto-assets thus, it is not possible to predetermine (or predict) an appropriate threshold for a portfolio with existing parameters.

In light of the analysis in this study, it can be concluded that extending the portfolio size improves the profitability (or reduces the risk of loss) of the proposed algorithm for cointegrated price data, regardless of the trend and volatility regimes. These findings are in accordance with those of (Liang et al. 2020; Lin et al. 2021; Figá-Talamanca et al. 2021; Tadi and Kortchemski 2021), who demonstrated that the diversification of portfolios with multi-asset pairs enhances the profitability of pairs trading strategies.

Importantly, as other prior studies already reported (Tadi and Kortchemski 2021; Bouri et al. 2019; Kaya Soylu et al. 2020; Figá-Talamanca et al. 2021), crypto-assets are highly volatile, which indicates that the suggested HFRA strategy can yield reasonable profits via the practical implementation of cryptocurrency exchange markets. Furthermore, both the HFRA and the optimal portfolio selection procedure are independent of the exchange market; thus, they can be applied to any stock exchange market under appropriate trends and volatility regimes.

The main limitation of this study is that portfolios of various sizes are constructed by including price series with similar trends and volatility characteristics, such as upward trends with high volatility. Another limitation is the determination of the capital size for each asset in the portfolio. If capital size exceeds the trading volume capacity of a market, there will be a price impact that can cause liquidation, and consequently, trading losses. This issue can be circumvented by determining capital according to the daily trading volume of the assets in a portfolio. In addition, selecting a low threshold and/or considering high-volume markets reduce the risk of trading losses emerging from price impacts.

Future research using a different approach can make the portfolio selection process independent of the trend and/or volatility dynamics of the associated price series. Moreover, the profitability of the HFRA can be improved by determining the optimal threshold (or profit margin) using volatility and/or trend information and new methodologies in future studies.

Appendix

See the Matlab script file *Rebalancing_Algorithm.m.*

```
%Matlab code of the rebalancing algorithm.
%Calculate average profits of portfolios that include 2, 3 and 5 assets.
clc; clear all; close all;
load('Sample_series_1.mat'); price1=price;   %load sample dataset
load('Sample_series_2.mat'); price2=price;
load('Sample_series_3.mat'); price3=price;
load('Sample_series_4.mat'); price4=price;
load('Sample_series_5.mat'); price5=price;
trd_fee=1/1000;   %trading fee 0.1%
sizes=[2, 3, 5];   %number of price series (assets) in the portfolio
for cases=1:3   %three cases n=2, n=3, n=5
    n=sizes(cases);
    if n==2
        prices=[price1, price2];
    end
    if n==3
        prices=[price1, price2, price3];
    end
    if n==5
        prices=[price1, price2, price3, price4, price5];
    end
    for T=1:20   %iterate threshold (profit margin)
        quantity=ones(1,n)*1;   %initial quantity is 1
        margin=0.01*T;
        for j=1:length(price)
            for i=1:n
                P(i)=prices(j,i)*quantity(i);   %total value of each asset
            end
            if(max(P)>min(P)*(1+margin))
                for m=1:n
                    if P(m)==max(P)
                        s=m; %index of sell pair
                        P_sell=P(s);
                    end
                    if P(m)==min(P)
                        b=m; %index of buy pair
                        P_buy=P(b);
                    end
                end
                sell=0.5*margin*P_sell*(1-trd_fee);   %sell amount
                buy=(sell/prices(j,b))*(1-trd_fee);   %buy amount
                quantity(s)=quantity(s)*(1-(0.5*margin)); %last quantity of sold asset
                quantity(b)=quantity(b)+buy;   %last quantity of bought asset
                P(s)=quantity(s)*prices(j,s);
                P(b)=quantity(b)*prices(j,b);
            end
        end
        last_quantity=sum(quantity);
        if last_quantity==n   %if there is no trade
            continue;
        end
        total_value(T)=sum(P);   %total value of the portfolio for each threshold
    end
    sprintf('Profit of %d assets = %.2f%s', n, ((mean(total_value)/n)-1)*100, '%')
end
for i=1:5   %plot price series
    plot(prices(:,i),'Linewidth',0.7)
    hold on;
end
axis([0 length(prices) min(min(prices)) max(max(prices))])
xlabel('Time [min]')
ylabel('Price ratio')
legend('P_1','P_2','P_3','P_4','P_5','Location','north','Orientation','horizontal')
```

Abbreviations

HFRA	High-frequency rebalancing algorithm
PR	Periodic rebalancing
TR	Threshold rebalancing algorithm
AT	Algorithm trading
HFT	High-frequency trading
ML	Machine learning
SVM	Support vector machine
VECM	Vector error-correction model
VAR	Vector autoregressive

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MB carried out the methodology and analysis of the research, and drafted the manuscript. PKS prepared the literature review and contributed to the writing and organization of the manuscript. All the authors read and approved the final version of the manuscript.

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Availability of data and materials
The real dataset used in this study is publicly available on Binance cryptocurrency exchange market (https://www.binance.com), and a sample experimental dataset is available on Google Drive [here]. The datasets used and/or analyzed in the current study are available from the corresponding author upon reasonable request.

Declarations

Competing interests
The authors declare that they have no competing interests.

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