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RESEARCH

Assessing portfolio vulnerability to systemic risk: a vine copula and APARCH-DCC approach

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Abstract

This study evaluates the sensitivity and robustness of the systemic risk measure, Conditional Value-at-Risk (CoVaR), estimated using the vine copula and APARCH-DCC models. We compute the CoVaR for the two portfolios across five allocation strategies. The novel vine copula captures the complex dependence patterns and tail dynamics. The APARCH DCC incorporates volatility clustering, skewness, and kurtosis. The results reveal that the CoVaR estimates vary based on portfolio strategy, with higher values for the cryptocurrency portfolio. However, CoVaR appears relatively robust across strategies compared to ΔCoVaR. The cryptocurrency portfolio has a greater overall vulnerability. The findings demonstrate the value of CoVaR estimated via the vine copula and APARCH-DCC in assessing portfolio systemic risk. This advanced approach provides nuanced insights into strengthening risk management practices. Future research could explore the sensitivity of the CoVaR to different weighting schemes, such as equal versus market-weighted portfolios. Incorporating the Gram–Charlier expansion of normal density into the APARCH specification enables a nonparametric, data-driven fitting of the residual distribution. Furthermore, comparing the CoVaR to another systemic risk measure could provide further insights into its reliability as a systemic risk measure.

Keywords: Cryptocurrency, Systemic risk, Vulnerability, CoVaR

JEL Classification: C20, G10, G15, G19

Introduction

Since the introduction of Bitcoin by Nakamoto (2019) in 2008, the cryptocurrency market has grown significantly, with approximately 19,800 cryptocurrencies in existence and a total of 930 billion USD in market capitalization at the time of this writing. More than 6000 cryptocurrencies are actively traded on 527 cryptocurrency exchanges, with daily volume transactions of 117 billion USD. These statistics indicate the importance of the cryptocurrency market size. Many studies have been conducted on these new types of assets in terms of their technology (Yermack 2015, 2017), volatility (Dwyer 2015; Katsiampa 2017; Scaillet et al. 2020), and price formation (Cheah and Fry 2015). Cryptocurrencies have become an important class of assets, and their speculative nature may result in large gains or losses. Therefore, it is important to adequately model the risk of the built portfolio and the marginal risk contribution of each asset in the portfolio. Value-at-Risk (VaR) is the most widely used risk measure owing to its simplicity. It has the advantage of being convex and easy to calculate and implement when solving portfolio optimization problems. It provides a single number that summarizes the risk exposure of a portfolio, can be used to set risk limits, and evaluates the effectiveness of risk management strategies (Jorion 2007; Embrechts et al. 2013; Embrechts et al. 2013). However, in 1999, this measure was criticized for the first time by Artzner et al. (1999) for lack of consistency, particularly for not being a sub-additive and therefore overestimating risk. A second criticism of VaR is its definition: a VaR of the 95% confidence level gives no idea of the magnitude of the loss if the potential loss exceeds the fixed 5% quantile. Before gradually shifting to coherent risk measures like the Conditional Value-at-Risk (CVaR), for a long time, regulators have used VaR to determine the amount of capital to be set aside by financial institutions against possible future market risk. The CVaR was introduced by Rockafellar and Uryasev (2002). This provides a measure of risk that captures the potential severity of losses beyond the VaR level by considering the shape of the loss distribution. However, it can be more computationally intensive than VaR and sensitive to the shape of the loss distribution and choice of confidence level. The CVaR has diverse applications, including long-term investment, portfolio optimization, and electricity price risk management. CVaR is a useful tool for capturing downside risks beyond VaR and can be used in various contexts to improve risk management and decision-making. However, VaR focuses on a single asset in isolation.

Since VaR fails to go beyond idiosyncratic risk to capture systemic risk by focusing on an asset in isolation, the scale of the risk facing a portfolio cannot be obtained from a portfolio's VaR. Systemic risk in the Benoit et al. (2017) sense can be defined as the risk that spillovers to the whole portfolio when many assets are simultaneously affected by severe losses. Adrian and Brunnermeier (2016) introduce the Conditional Value-at-Risk (CoVaR) systemic risk measure, which captures the Value-at-Risk (VaR) of an asset under normal market conditions against the VaR of the portfolio conditional on the fact that a given asset is in distress. Other systemic risk measures include the Systemic Expected Shortfall (SES) introduced by Acharya et al. (2010) in terms of the Marginal Expected Shortfall (MES), the systemic risk measure (SRISK) introduced by Acharya et al. (2012), which is a variant of the systemic expected shortfall (SES). The cryptocurrency market has been subject to multiple crises, in which the prices of most cryptocurrencies plunged to nearly 40% in just a few days. Therefore, it is important for investors to assess the risk of their portfolios conditional on an asset being in distress. This indicates the assets to be closely monitored when the cryptocurrency market is in distress.

In this study, we consider the CoVaR of Adrian and Brunnermeier (2016) to analyze the vulnerability of two different portfolios consisting of ten major cryptocurrencies, a mixture of three cryptocurrencies, and seven top world indices. To evaluate the robustness of these systemic measures, we used three portfolio strategies: the global minimum variance (GMV) portfolio, the most diversified portfolio (MDP), and the market-weighted portfolio (MWP). Within the cryptocurrency market, Ji et al. (2019) use return and volatility spillovers to examine connectedness among the six largest cryptocurrencies. Borri (2019) uses the CoVaR to estimate the conditional tail risk of Bitcoin, Ethereum, Litecoin, and Ripple. These findings indicate that cryptocurrencies are highly exposed to tail risk. The CoVaR estimation is based on quantile regressions. Mba (2022) uses CoVaR to systematically classify important cryptocurrencies into a pool of 10 major cryptocurrencies. To compute CoVaR, we use the Flexible Dynamic Conditional Correlation (FDCC) by Engle (2002b) combined with the APARCH model. The idea behind the FDCC was that high- and low-capitalization cryptocurrencies do not have the same dynamics. In the FDCC model, the dynamics are constrained to be equal only among the groups of random variables. In this study, our CoVaR computation was based on vine copula-APARCH models. Unlike the FDCC model, the vine copula uses a cascade of bivariate copulas to model the dynamics among random variables. In our model, we allow the auto-selection of a suitable copula pair for a given pair of random variables. What makes our model interesting is the following. On the one hand, besides the leptokurtic property, the APARCH model introduced by Ding et al. (1993) captures other stylized facts in financial time series, such as volatility clustering and the leverage effect, as well as the long memory property. The power of the APARCH model stems from the fact that it encompasses several other generalized GARCH heteroskedasticity models. However, beyond linear correlation, the copula proposed by Sklar (1959) has shown great capability in modeling the dependence structure among random variables using a joint distribution from marginal distributions of the given random variables. In addition to elliptical copulas with multidimensional capabilities, Archimedean copulas are limited to bivariate random variables. To benefit from this large family of copulas, Bedford and Cooke (2001) introduced a vine copula, a flexible graphical model for describing multivariate copulas constructed using a cascade of bivariate copulas or pair copulas. Popular vine copulas include the regular vine (R-vine) copula with its subclasses, the canonical vine (C-vine) copula, and the drawable vine (D-vine) copula. They differ in their tree constructions–in how they associate pairwise random variables to model the dependence structure.

This study aimed to assess a portfolio's vulnerability to its assets' tail risk and investigate the sensitivity of CoVaR to various portfolio allocation and optimization strategies, namely, market-weighted portfolio (MWP), global minimum variance (GMV), most diversified portfolio (MDP), mean-variance differential evolution (MVDE), and mean-variance particle swarm optimization (MVPSO). Our analysis represents a significant contribution to portfolio risk management as it offers a comprehensive evaluation of portfolio vulnerability that considers the interdependence of asset returns and the impact of extreme events on portfolio performance. In particular, we contribute to the literature as follows:

(1) Developing a novel approach for assessing portfolio vulnerability based on CoVaR that accounts for the tail risk of individual assets and their contribution to overall portfolio risk.
(2) We conduct an extensive empirical analysis to assess the sensitivity of the CoVaR to different portfolio allocation and optimization strategies, including market-weighted portfolios, global minimum variance portfolios, most diversified portfolios, mean–variance-based differential evolution, and particle swarm optimization portfolios.

(3) This study provides insights into the relative effectiveness of these strategies in mitigating tail risk and improving portfolio performance, as well as identifying potential trade-offs between risk reduction and return maximization.

Overall, our findings highlight the importance of incorporating CoVaR-based risk measures into portfolio management practices and suggest that optimal portfolio allocation and optimization strategies may vary depending on the level and nature of tail risk exposure. By providing a rigorous and systematic analysis of portfolio vulnerability, our study offers valuable insights for investors, asset managers, and risk professionals seeking to improve their understanding of portfolio risk and enhance their risk management strategies.

The remainder of this paper is organized as follows. Sect. "Literature review" presents a literature review, Sect. "Methodological framework" presents the methodological framework, and Sect. "Empirical results and discussion" presents the empirical results and discussion. Sect. "Sensitivity and robustness check" provides a Sensitivity and Robustness check, and Sect. "Conclusions" summarizes the findings and indicates future work directions.

Literature review

The rapid development and adoption of cryptocurrency and blockchain technologies have created new opportunities and challenges for investors and asset managers. Cryptocurrencies like Bitcoin, Ethereum, and Ripple are digital assets operating on decentralized networks. Blockchain is an underlying technology that enables secure and transparent cryptocurrency transactions. Blockchain has been hailed as a revolutionary innovation that can transform various industries and sectors, such as finance, supply chains, healthcare, and education (Xu et al. 2019). However, investing in cryptocurrencies also involves significant risks such as volatility, security breaches, regulatory uncertainty, and market manipulation. Cryptocurrencies exhibit complex and dynamic behaviors influenced by various factors such as supply and demand, network effects, technological innovations, media attention, and investor sentiment. These factors can lead to extreme price movements and spillover effects across cryptocurrencies and traditional assets (Fang et al. 2022). Therefore, investors and asset managers must understand and measure cryptocurrencies' risks and interactions with other assets. Traditional risk measures such as value-at-risk (VaR) or standard deviation may not adequately capture the tail risk of cryptocurrencies, which is the risk of extreme losses in the lower or upper tails of the return distribution. Tail risk can severely affect portfolio performance and financial stability.

One of the most popular tail risk measures is the Conditional Value-at-Risk (CoVaR), defined as the VaR of a system conditional on an institution being under distress (Adrian and Brunnermeier 2016). The CoVaR captures the systemic risk contribution of an individual asset or institution to the overall system. CoVaR can be extended to measure the bilateral tail risk between two assets or institutions, called ΔCoVaR. ΔCoVaR measures the change in CoVaR due to the distress of another asset or institution. However, CoVaR and ΔCoVaR are not easy to estimate, as they require modeling the joint distribution of returns and accounting for various stylized facts, such as fat-tails, volatility clustering, skewness, and long memory. Moreover, CoVaR and ΔCoVaR may be sensitive to the portfolio composition and optimization strategy investors and asset managers use. Different portfolio strategies may result in different weights, diversification benefits, and risk-return trade-offs. Finding the right algorithm that provides better diversification benefits and risk-return trade-offs can be considered a multiple-criteria decision-making (MCDM) problem, and MCDM techniques can be used to select the best method for a problem at hand (Kou et al. 2014). Sebastião and Godinho (2021) examined the predictability of three major cryptocurrencies (Bitcoin, Ethereum, and Litecoin) and the profitability of trading strategies devised using machine learning techniques. Kou et al. (2019) survey existing research and methodologies for assessing and measuring financial systemic risk combined with machine learning technologies, including big data, network, and sentiment analysis.

Allen et al. (2012) propose a measure of aggregate systemic risk called CATFIN, which complements microlevel systemic risk measures by focusing on direct interbank connections and can determine the macroeconomic implications of aggregate risk-taking in the financial system. This can provide bank regulators with early warning signals to calibrate a micro-level systemic risk premium (or tax) to macroeconomic conditions. Huang et al. (2011) present a systemic risk indicator measured as a financial firm's marginal contribution to the financial sector's distress insurance premium. This systemic risk approach indicates that a bank's contribution to systemic risk is roughly linear in its default probability but highly nonlinear concerning institution size and asset correlation. This risk is associated with the degree of interdependence among financial firms, as in Hartmann et al. (2006). In the same direction of research, we can mention the works of Billio et al. (2012), Ang and Longstaff (2013), Diebold and Yılmaz (2014), Hautsch et al. (2015), and Georgosouli and Goldby (2017). In a different direction, Brownlees and Engle (2017) introduce a systemic risk measure called SRISK, defined as the expected capital shortfall of a financial entity conditional on a prolonged market decline. It considers the size of the firm, its degree of leverage, and expected equity loss, conditional on market decline. This measure is related to Acharya et al. (2010) Systemic Expected Shortfall (SES), which measures a financial firm's conditional capital shortfall. In the SES framework, the financial system is vulnerable because financial institutions do not factor in the negative externality costs they generate during a crisis. To compute the SRISK, Brownlees and Engle (2017) use Engle's (2002a) GARCH-DCC model, which is widely used in financial time series analysis because of its ability to capture the stylized facts of financial data well. To measure systemic risk in the Chinese banking sector, Xu et al. (2018) used the CoVaR approach based on a DCC-MIDAS-t model. The performance of this model, which introduces the Student's t distribution into the standard DCC-MIDAS to adequately capture fat-tailed returns, was compared to those of DCC-MIDAS-N and DCC-GARCH in the CoVaR measurement. The findings show that the DCC-MIDAS-t model outperforms conventional models in volatility prediction and CoVaR measurements. Adrian and Brunnermeier's (2016) CoVaR links the systemic risk contribution of a financial institution in distress to an increase in the VaR of the entire financial system. The CoVaR has been widely used owing to its simplicity and effectiveness. Traditional CoVaR-based models include quantile regression, multivariate GARCH, and copulas, such as Girardi and Ergün (2013), Rösch and Scheule (2016), Karimalis and Nomikos

(2018), and Xu et al. (2018). Adrian and Brunnermeier (2016) employ linear quantile regressions to obtain CoVaR estimates. However, the estimates derived from this procedure do not include the time-varying exposure to the institution's VaR. Numerous studies indicate that the correlation between financial series is not constant over time (Engle, (2002a) Patton, (2006)). This tends to be more pronounced during downturns than during upturns, a stylized feature that should be incorporated in estimating systemic risk. Girardi and Ergün (2013) incorporate time-varying correlations into their CoVaR estimates through a three-step procedure based on a univariate GARCH-type model and the bivariate DCC model of Engle (2002a). However, their approach requires numerical integration, which can be computationally intensive and time-consuming. In addition, the specification of the marginal distribution in this procedure depends on the choice of bivariate distribution of assets R_1 andR_2. In practice, as Karimalis and Nomikos (2018) point out, the distributional characteristics of R_1 and R_2 can differ substantially; hence, by restricting the marginal specification, a misspecification bias can be introduced in the CoVaR estimation. Focusing on a portfolio of large European banks, Karimalis and Nomikos (2018) use copula functions to estimate the CoVaR and measure the contribution of each bank to systemic risk. They show that the ranking of systemically important institutions and the magnitude of their corresponding CoVaRs are affected by the choice of underlying distributions modeled by a broad range of copula families (Frank, Gumbel, Clayton, and BB7). However, instead of relying only on bivariate copulas, such as Frank, Gumbel, and Clayton, to model dependence, the vine copulas introduced by Bedford and Cooke (2001) appear more appropriate in a multivariate setting. These flexible graphical models describe the multivariate copulas constructed using a cascade of bivariate copulas. Copula has been used in the computation of CoVaR (Usman et al. 2019), Ji et al. (2018a, b), Sun et al. (2020), Ji et al. (2018a, b). Copula models perform better than other models and are more suitable for dealing with nonlinear, non-stationary, and tail dependence between random variables (Patton 2012; Kayalar et al. 2017).

This study proposes a novel approach to estimate CoVaR and ΔCoVaR using Vine copula-APARCH models. The vine copula is a flexible tool that can capture complex non-linear dependencies and tail dynamics across assets. APARCH is a generalized version of GARCH that incorporates volatility clustering, skewness, leptokurtosis, and long memory into conditional variance. We apply our approach to two portfolios comprising major cryptocurrencies and world indices. We also consider five portfolio allocation and optimization strategies: market-weighted portfolio (MWP), global minimum variance (GMV) portfolio, maximum diversification portfolio (MDP), mean–variance differential evolution (MVDE), and mean–variance particle swarm optimization (MVPSO). We conduct a sensitivity analysis to assess the impact of portfolio composition and optimization strategy on CoVaR and ΔCoVaR.

Our results demonstrate that the CoVaR values are highly sensitive to portfolio strategy. The tracking lines for each asset are not parallel to the horizontal lines, indicating that different portfolio strategies can significantly impact the tail risk of individual assets and the overall portfolio. Moreover, the high CoVaR values for cryptocurrencies compared with traditional indices suggest that investing in cryptocurrencies carries a higher level of risk. Furthermore, the results suggest that CoVaR is even more sensitive to portfolio strategies than CoVaR. The GMV strategy provides the most vulnerable portfolio in terms of ΔCoVaR, emphasizing the importance of carefully selecting a portfolio strategy to mitigate tail risk.

Our findings have important implications for investors and asset managers seeking to enhance portfolio resilience and mitigate tail risk. First, our results suggest that traditional portfolio optimization strategies may not adequately mitigate tail risk in today's rapidly changing and complex financial markets. Therefore, investors and asset managers may need to consider more sophisticated portfolio optimization techniques that explicitly account for tail risk, such as CoVaR and ΔCoVaR. Second, our findings suggest that investing in cryptocurrencies carries a higher level of risk than traditional indices do; therefore, investors should exercise caution when considering investing in cryptocurrencies. We recommend that investors conduct thorough due diligence on cryptocurrencies and consider their risk tolerance before investing. Overall, this study highlights the value of CoVaR estimated via the vine copula-APARCH in assessing portfolio systemic risk. This study gives investors and risk managers meaningful insights to enhance portfolio resilience. Our approach represents a significant methodological contribution to applying advanced econometric modeling to produce robust systemic risk metrics.

This study contributes to the literature by:

(1) Including the vine copula in the CoVaR estimation allows a collection of copulas to be used in modeling the dependence structure. The model autoselects a suitable copula for each pair of random variables based on their shared characteristics.
(2) Our study is also related to that of Brownlees and Engle (2017), who use the standard GARCHDCC model to capture the stylized facts of financial data. However, instead of the standard GARCH-DCC, we use the APARCH-DCC. What makes our model interesting is that, besides the leptokurtic property, volatility clustering, and leverage effect, the APARCH model introduced by Ding et al. (1993) captures other stylized facts in financial time series, such as the long memory property. The power of the APARCH model stems from the fact that it encompasses several other generalized GARCH heteroskedasticity models.
(3) Unlike Karimalis and Nomikos (2018), who consider one portfolio and check the robustness of their procedure across different systemic risk measures (CoVaR and CoES), we consider one systemic risk measure (CoVaR) and check its robustness across different portfolio strategies, namely, global minimum variance, maximum diversification, and market portfolios. We believe that for the CoVaR to be a reliable systemic risk measure, it should be less sensitive to the type of portfolio strategy chosen. We apply these three portfolio strategies to two portfolios consisting of cryptocurrencies and stock indices.
(4) We also aim to assess the impact of a shock to the cryptocurrency market on the traditional stock market and vice versa. Some previous systemic risk assessments of the cryptocurrency market include that of Ji et al. (2019), who use return and volatility spillovers to examine connectedness among the six largest cryptocurrencies. Borri (2019) uses CoVaR to estimate the conditional tail risk of Bitcoin, Ethereum, Litecoin, and Ripple and finds that these cryptocurrencies are highly exposed to tail risk.

Methodological framework

Portfolio theories

Modern portfolio theory and global minimum variance portfolio

Harry Markowitz was the pioneer of portfolio theory. In 1952, he published a seminal paper titled "Portfolio Selection" in the Journal of Finance. Markowitz (1959) introduced the concept of Modern Portfolio Theory (MPT). This theory is considered one of the foundations of modern portfolio construction and is widely used in investment management. MPT assumes that investors are risk averse and seek to maximize the expected return for a given level of risk or minimize risk for a given expected return. Despite many criticisms (Assumptions of rational investors (see, Shefrin 2002b; Shefrin 2002a; Prast 2003; Tversky and Kahneman 1981), Efficient Market Hypothesis (EMH) (see, Malkiel 2003; Sewell 2011), Single period optimization (see, Grossman 1995), it is still widely used in practice. In the Markowitz mean–variance portfolio theory, the goal is to optimally choose portfolio weighting factors, that is, one in which the portfolio achieves an acceptable baseline expected return with minimal volatility.

Consider a portfolio P consisting of n assets. Let r_i be the return series for asset i and $r=(r_1,r_2,\ldots,r_n)$. Set $\mu_i=E[r_i]$, $\mu=(\mu_1,\mu_2,\ldots,\mu_n)$ and $\Sigma=cov(r)$ the positive semi-definite variance–covariance matrix of the portfolio's assets. Given a set of weights $\omega=(\omega_1,\omega_2,\ldots,\omega_n)$ associated with the portfolio,

$r_p=\sum_{i=1}^{n} r_i\omega_i$, $\mu_p=\mu^T\omega$ and $\sigma_p^2=\omega^T\Sigma\omega$ are respectively the return, the mean, and the variance of the portfolio. If μ_p is the acceptable baseline expected return, then in the MPT framework, the optimization problem for the minimal variance portfolios can be stated as:

$$\begin{cases} \min\limits_{\omega} \frac{1}{2}\omega^T\Sigma\omega \\ Subject\ to\ \omega^T\mu \geq \mu_p, and \omega^T i = 1 \end{cases}$$

where $\boldsymbol{i}$ is a column vector of ones.

According to this function, given a target return μ_p, the weight vector for a minimal variance portfolio is given by

$$\omega^* = \mu_p\omega_1^* + \omega_2^* \tag{1}$$

where $\omega_1^*=\frac{1}{d}\left(c\Sigma^{-1}\mu-b\Sigma^{-1}\boldsymbol{i}\right)$, $\omega_2^*=-\frac{1}{d}\left(b\Sigma^{-1}\mu-a\Sigma^{-1}\boldsymbol{i}\right)$. The portfolio standard deviation is given by

$$\sqrt{\frac{1}{d}\left(c\mu_b^2-2b\mu_b+a\right)} \tag{2}$$

where $a=\mu^T\Sigma^{-1}\mu$, b$=\mu^T\Sigma^{-1}\boldsymbol{i}$, c$=\boldsymbol{i}^T\Sigma^{-1}\boldsymbol{i}$, and $d=ac-b^2$. Merton (1972) was consulted for a detailed explanation. Equation 2 describes a hyperbola for efficient mean–variance portfolios. The apex of this hyperbola is the Global Minimum Variance (GMV) portfolio with weights given by $\omega_{GMV}^*=-\frac{1}{c}\Sigma^{-1}\boldsymbol{i}$. The GMV portfolio is the point on the efficient frontier where the portfolio has the lowest possible risk for a given expected return or the highest expected return for a given level of risk. This is important because it provides a baseline portfolio that can be used as a starting point for constructing efficient portfolios or as a standalone portfolio for investors prioritizing risk minimization over return maximization. This motivates our choice in this study to include the GMV portfolio in evaluating the sensitivity of the CoVaR measure.

CAPM and APT

In practice, one would like to better understand the risk-return trade-off because we want to maximize returns while minimizing risk. One way to achieve this is to solve a quadratic programming

$$\begin{cases} \min\limits_{\omega} \frac{1}{2}\omega^T\Sigma\omega - \lambda\omega^T\mu \\ Subject\ to\ \omega^T i = 1 \end{cases}$$

where $\boldsymbol{i}$ is a column of ones.

The MPT analysis assumes that asset *f* is added with a risk-free return, r_f. Let ω_0 be the weight to be assigned to the asset *f*. The Markowitz quadratic program can be written as follows:

$$\begin{cases} \min\limits_{\omega} \frac{1}{2}\omega^T\Sigma\omega \\ Subject\ to\ r_f\omega_0\omega^T\mu \geq \mu_b, and \omega_0 + \omega^T i = 1 \end{cases}$$

Let $r_M = r_f\left(1 - \boldsymbol{i}^T\Sigma^{-1}\left(\mu - r_f\boldsymbol{i}\right)\right) + r\Sigma^{-1}\left(\mu - r_f\boldsymbol{i}\right)$ be the market portfolio and let $r = \omega_0 r_f + \mu^T\omega$ be any efficient portfolio with $\omega_0 = 1 - \alpha\boldsymbol{i}^T\Sigma^{-1}\left(\mu - r_f\boldsymbol{i}\right)$ and $\omega = \alpha\Sigma^{-1}\left(\mu - r_f\boldsymbol{i}\right)$ for some value α. Then the expected return μ_r of the portfolio *r* satisfies the equation

$$\mu_r = r_f + \frac{\mu_M - r_f}{\sigma_M}\sigma_r$$

where $\mu_r = E[r], \sigma_r^2 = variance(r), \mu_M = r_f + \left(\mu - r_f\boldsymbol{i}\right)^T\Sigma^{-1}\left(\mu - r_f\boldsymbol{i}\right)$ and $\sigma_M^2 = \left(\mu - r_f\boldsymbol{i}\right)^T$ $\Sigma^{-1}\left(\mu - r_f\boldsymbol{i}\right)$. This line describes the efficient frontier and is called the *capital market line*. The slope of this line $\frac{\mu_M - r_f}{\sigma_M}$ is called the *price of risk*. Consider a portfolio that combines r_M and asset *i*. The capital

market line (CML) is tangent to the mean-standard deviation curve for this portfolio at portfolio r_M, so that

$$\mu_i = r_f + \beta_i\left(\mu_M - r_f\right)$$

where μ_i is the expected return of asset *i*, $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$, and σ_{iM} is the covariance of the return on asset *i*

and market portfolio r_M. This is the Capital Asset Pricing Model (CAPM). Sharpe (1964) first introduced this concept. While MPT provides the rules for making investment decisions and a systematic approach to determining a set of efficient portfolios and selecting optimal portfolios to evaluate financial assets, the CAPM and the Arbitrage Pricing Theory (APT) models assume that all investors respect these rules for making investment decisions. The CAPM and APT contribute significantly to the understanding of the linear relationship between return, risk, and asset valuation in the capital market. APT, developed by Ross in the 1970s, provides a multifactor model for security pricing based on the concept of arbitrage. APT assumes that securities are priced based on multiple factors, not just a single market risk factor, and that these factors interact to determine the expected return on the security.

$$r_i = r_f + \lambda_1\beta_{i1} + \lambda_2\beta_{i2} + \cdots + \lambda_k\beta_{ik}$$

where r_i is the expected return of the asset i; r_f is the risk-free rate; $\lambda_j\left(j = 1, 2, \ldots, k\right)$ is the factor risk premium related with the j-th factor and β_{ij} is the return sensitivity of the asset i to the value of the factor $j, j = 1, 2, \ldots, k$. The key shortcoming of the APT model is that it does not specify the systemic risk factors. Many attempts have been made to evaluate these factors through factor analysis (Roll and Ross 1980; Dhrymes et al. 1984), the specification of macroeconomic factors (Faruque 2011; Zhu 2012; Jamaludin et al. 2017), and the specification of microeconomic factors (Tudor 2010; Uwubanmwenand and Obayagbona 2012; Idris and Bala 2015).

These portfolio theories suggest that diversification is a key element in portfolio construction. The MPT is built on the concept of diversification and argues that investors can maximize the expected returns for a given level of risk or minimize the risk for a given expected return by diversifying their investments across different asset classes. The CAPM is an extension of the MPT and incorporates diversification as a key factor in determining the expected returns for individual assets within a portfolio. Diversification comprises at least two dimensions. The first addresses the underlying common characteristics of diverse assets. The second one seeks to measure the degree of diversification regarding underlying characteristics. The variance–covariance matrix of returns is often used to assess the riskiness of assets and the dependencies between them. However, diversification is defined regarding the overall return dependencies.

Choueifaty and Coignard (2008) and Choueifaty et al. (2011) address portfolios' theoretical and empirical properties when diversification is used as a criterion. They establish a measure to assess the degree of diversification of a long-only portfolio.

Most diversified portfolio

The diversification ratio (DR) is given by Eq. (3)

$$DR_{\omega\epsilon\Omega} = \frac{1}{\sqrt{\rho + CR - \rho CR}} \tag{3}$$

where ρ and CR denote the volatility-weighted average correlation and the volatility-weighted concentration ratio, respectively.

The diversification ratio measures the degree of portfolio diversification. The higher the DR, the more diversified the portfolio. Portfolio solutions characterized by highly concentrated allocations or highly correlated asset returns qualify as poorly diversified. The most diversified portfolio (MDP) is obtained by maximizing DR:

$$P_{MDP} = \underset{\omega\epsilon\Omega}{\text{argmax}} DR \tag{4}$$

The diversification ratio is maximized by minimizing $\omega^T C\omega$, where C denotes the correlation matrix of the initial asset returns. Hence, the objective function coincides with that of a GMV portfolio. However, instead of using the variance–covariance matrix, a correlation matrix is employed. In the GMV approach, asset volatilities enter directly into the quadratic form of the objective function to be minimized, whereas the impact of asset volatilities is smaller for an MDP.

This study assesses the sensitivity and robustness of the CoVaR measure for portfolio strategies. In other words, is it a reliable measure independent of the portfolio strategy considered? To achieve this, we consider five portfolio strategies: global minimum variance (GMV) portfolio, most diversified portfolio (MDP), mean–variance differential evolution (MVDE), mean–variance particle swarm optimization (MVPSO), and market-weighted portfolio (MWP). In a *market-weighted portfolio* is the one in, weights are determined by the market share of each asset.

APARCH model

Consider a return series $r_t = \mu_t + a_t$, where μ_t is the conditional expected return and $a_t = \sigma z_t$ is a zero-mean white noise, where $z_t \sim D(0, 1)$, and D is the skew Student's t distribution. (Ding et al. 1993) found that $|a_t|^s$ often displays a strong and persistent autocorrelation for various values of s illustrating a long memory property. We say that $a_t \sim APARCH(p, q)$ if

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i (|a_{t-i}| - \gamma_i a_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta \tag{5}$$

where $\omega > 0, \alpha_i \geq 0, \beta_j \geq 0, \delta > 0$ and $-1 < \gamma_i < 1$. The parameter γ_i captures the leverage effect. In this study, we use $(p, q) = (1, 1)$ since it is usually the option that best fits the financial time series. Brooks (2002) proves that using a GARCH class model with one lag order is sufficient to describe volatility clustering in asset returns. The innovations in this model are assumed to follow a skewed Student's t-distribution with a density function:

$$d(x; \eta, \lambda) = \begin{cases} bc\left(1 + \frac{1}{\eta-2}\left(\frac{bx+a}{1-\lambda}\right)^2\right)^{-\frac{\eta+1}{2}}, ifx > -\frac{a}{b} \\ bc\left(1 + \frac{1}{\eta-2}\left(\frac{bx+a}{1+\lambda}\right)^2\right)^{-\frac{\eta+1}{2}}, ifx \leq -\frac{a}{b} \end{cases}$$

where $a = 4\lambda c \frac{\eta-2}{\eta+1}, b = 1 + 3\lambda^2 - a^2, c = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\Gamma\left(\frac{\eta}{2}\right)}$ and Γ is the gamma function.

Vine copula

Although the DCC-GARCH model allows for a time-varying conditional correlation, it fails to reproduce the non-linear dependence that may exist between variables and does not provide information about tail dependence. Tail dependence corresponds to the possibility of joint events, such as a low or high occurrence of extreme events. To achieve this, an alternative approach based on copula functions was adopted. The main advantage of copulas is that they separate the dependence structure from the marginals without making any assumptions about the distribution. Using several copula functions, Nguyen and Bhatti (2012) provide evidence of left-tail dependence in Vietnam but not China. In the case of six CEE countries (Bulgaria, Czech Republic, Hungary, Poland, Romania, and Slovenia), Aloui et al. (2013) also find left-tail dependence. Copula functions are statistical tools for modeling the dependence structure by coupling multiple marginal distributions to represent a joint distribution function. This was introduced by Sklar (1959) in the following theorem:

Theorem 3.1 *Assume $F = (F_1, \ldots, F_n)$ is an n-dimensional joint distribution function with the marginal distribution function $F_i (i = 1, \ldots, n)$. Then there exists a copula C such that for all $\boldsymbol{x} = (x_1, \ldots, x_n) \epsilon I^n$,*

$$F(x) = C(F_1(x_1), \ldots, F_n(x_n))$$

If $F_1, \ldots, F_n$ continue, then C is unique. Otherwise, C is not unique to I^n. In addition, if $F_1, \ldots, F_n$ are distribution functions on I and if C is a copula, then the function $F(x) = C(F_1(x_1), \ldots, F_n(x_n))$ is a joint distribution function on I^n. The canonical representation of the copula density function is

$$c(u_1, \ldots, u_n) = \frac{\partial^n C(u_1, \ldots, u_n)}{\partial u_1 \ldots \partial u_n} \tag{7}$$

To obtain the density of the n-dimensional distribution F, the following relationship is used:

$$f(x_1, \ldots, x_n) = c(F_1(x_1), \ldots, F_n(x_n)) \prod_{i=1}^{n} f_i(x_i) \tag{7}$$

where f_i is the density of the marginal distribution F_i.

In risk management, one is interested in capturing tail dependence and describing the behavior of random variables during extreme events. We distinguish between symmetric and asymmetric copula based on how they model tail dependence. The pairwise upper and lower tail coefficients, denoted respectively by λ_U and λ_L, are given by the following equations:

$$\lambda_U = \lim_{u \to 1} \frac{1 - 2u + C(u, u)}{1 - u} \tag{8}$$

$$\lambda_L = \lim_{u \to 1} \frac{C(u, u)}{u} \tag{9}$$

This copula is not limited to 2 dimensions. This approach could be extended to arbitrarily large dimensions. However, the disadvantage is its practicality. At higher dimensions, the copula becomes rigid and tends to lose several useful properties. Therefore, the vine copula described by Bedford and Cooke (2001, 2002) addressed this high-dimensional probabilistic modeling issue. Instead of using a multidimensional copula directly, it first decomposes the probability density into conditional probabilities and then decomposes the conditional probabilities into bivariate copulas.

For example, let Y_1, Y_2 and Y_3 be three random variables with distribution functions G_1, G_2 and G_3 respectively. The joint density can be decomposed as:

$$g(y_1, y_2, y_3) = g_{3|12}(y_3|y_1, y_2) g_{2|1}(y_2|y_1) f_1(y_1)$$

where $g_{2|1}(y_2|y_1) = c_{12}(G_1(y_1), G_2(y_2))g_2(y_2)$,

$$g_{3|12}(y_3|y_1, y_2) = c_{13,2}(G_{1|2}(y_1|y_2), G_{3|2}(y_3|y_2))g_{3|2}(y_3|y_2)$$

$g_{3|2}(y_3|y_2) = c_{23}(G_2(y_2), G_3(y_3))g_3(y_3)$,

with $G(y|\boldsymbol{\nu}) = \frac{\partial C_{y,\nu_j,\boldsymbol{\nu}_{-j}}(G(y|\boldsymbol{\nu}_{-j}),G(\nu_j|\boldsymbol{\nu}_{-j}))}{\partial G(\nu_j|\boldsymbol{\nu}_{-j})}$ for every ν_j of the vector $\boldsymbol{\nu}$ with $\boldsymbol{\nu}_{-j} = \boldsymbol{\nu} - \{\nu_j\}$ in the general case.

Aas et al. (2009) described the statistical inference techniques for two classes of vine copulas, namely, canonical vine (C-vine) and drawable vine (D-vine), which are the most commonly used in practice for dependence modeling. Owing to its star-like structure, the C-vine is useful when a key variable governs the interactions among random variables. This appears suitable in the cryptocurrency market context, where the top cryptocurrencies, Bitcoin and Ethereum, largely govern behavioral patterns in many other cryptocurrencies.

Co-risk measure: CoVaR

Let $r = (r_1, \ldots, r_d)$ be a vector of the asset returns in the portfolio. Let $r_p = \sum_{i=1}^{d} \omega_i r_i$ be the portfolio return, where ω_i is the weight of asset i. We define $CoVaR_q^{p|i}$ with confidence level q as the VaR of the portfolio conditional on asset i in a state of distress (i.e., at VaR_q^i). It is the q-quantile of the conditional probability distribution

$$P\left(r_p \leq CoVaR_q^{p|i} | r_i = VaR_q^i\right) = q \tag{10}$$

$CoVaR$ is calculated as a quantile-based measure of systemic risk. It estimates the potential losses of a portfolio, given the severe loss experienced by asset i which pushes asset i to the lower quantile of its distribution. The calculation of CoVaR involves two steps. First, the Value-at-Risk (VaR) of the at-risk asset is calculated at a specified confidence level (e.g., 95%). VaR measures the maximum loss an asset is expected to incur within a specified time horizon at a given confidence level. Second, the portfolio's CoVaR is calculated as its expected losses beyond its VaR conditional on asset i being in a low quantile (VaR). We measure the vulnerability of the portfolio to tail-risk in asset i with $\Delta CoVaR_q$ which is given by

$$\Delta CoVaR_q^{p|i} = \left(CoVaR_q^{p|i} | r_i = VaR_q^i\right) - \left(CoVaR_q^{p|i} | r_i = VaR_{0.5}^i\right) \tag{11}$$

This is the difference between the CoVaR of the portfolio conditional on the distress of a particular asset i and the median state (i.e., $q = 0.5$), that is, during normal market conditions. Therefore, the larger (in absolute value) the ΔCoVaR, the higher the vulnerability of the portfolio to contagion from tail-risk events of the asset i.

To compute the CoVaR, the below steps are followed:

(1) Compute daily returns data.
(2) Fit the multivariate DCC-APARCH to the data.
(3) Extract the standardized residuals.

(4) Estimate the coefficients of the marginal distribution (skewed student distribution) for the C-vine copula simulation.
(5) Simulate new data from C-vine copula.
(6) Apply the inverse transform of the skewed Student's t-distribution to the new data to recover the returns structure.
(7) Compute variance–covariance matrix.
(8) Compute the VaR_i of each asset i in the portfolio.
(9) Compute the $CoVaR^{p|i}$ of the portfolio given that asset i is at its VaR_i.

Data description

The data span from November 10, 2017, to April 07, 2022, from Yahoo Finance (https://finance.yahoo.com/), the weighted average prices from all the market exchanges. The rationale for using a weighted average is that markets with higher volumes generally have higher liquidity and are less prone to price fluctuations. The analysis is based on two portfolios, each with ten assets.

Portfolio 1 consists of ten major cryptocurrencies, namely, Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Cardano (ADA), Chainlink (LINK), Litecoin (LTC), Bitcoin Cash (BCH), Stellar (XLM), Binance Coin (BNB) and Dogecoin (DOGE). BTC was the first cryptocurrency created in 2009 and was still the largest by market capitalization. It was designed as a decentralized alternative to traditional currencies with a fixed supply of 21 million coins. ETH, created in 2015, is the second-largest cryptocurrency by market capitalization. It differs from Bitcoin in that it allows the creation of decentralized applications (dApps) on its blockchain using smart contracts. The XRP is a cryptocurrency designed for global payments and remittances. It is used by financial institutions and is known for its fast transaction speed and low fees. The ADA, created in 2017, is a third-generation blockchain platform that aims to be more secure, sustainable, and scalable than before its introduction. LINK is a decentralized oracle network that provides off-chain data to smart contracts in the blockchain. It was created in 2017 and aims to bridge the gap between blockchain technology and real-world applications. LTC is a cryptocurrency created in 2011 that is similar to Bitcoin but has faster transaction speeds and lower fees. It is designed as a more efficient alternative to Bitcoin for everyday transactions. BCH is a fork of Bitcoin created in 2017. It was designed to increase the block size limit of Bitcoin to allow for more transactions per block and lower fees. XLM is a cryptocurrency created in 2014 and has similar goals as XRP in revolutionizing cross-border payments; however, it has different target audiences, governance models, consensus mechanisms, transaction speeds and costs, and cryptocurrency use cases. The BNB, created in 2017, is a native token of the Binance Exchange, one of the world's largest cryptocurrency exchanges. It is used to pay trading fees in exchange. It can also be used for other services, including payment for goods and services, investment in other cryptocurrencies, and participation in the Binance Launchpad platform for token sales. DOGE, a cryptocurrency created in 2013, was originally designed as a lightweight and fun alternative to BTC. Its transactions are generally processed much faster and at lower fees than BTC and many other cryptocurrencies, making it a popular choice for micropayments and tipping on social media platforms. To create a cryptocurrency portfolio, one might consider including a mix of assets with different characteristics and risk profiles, such as large-cap cryptocurrencies (BTC, ETH, BNB, XRP, and ADA), mid-cap cryptocurrencies (DOGE and LTC), and smaller-cap cryptocurrencies with higher growth potentials (LINK, BCH, and XLM).

Portfolio 2 consists of seven world indices and three cryptocurrencies: the NYSE COMPOSITE (NYA), NASDAQ Composite (IXIC), S&P 500 (GSPC), Euronext 100 Index (N100), FTSE 100 (FTSE), CAC 40 (FCHI), Dow Jones Industrial Average (DJI), Bitcoin (BTC), Ethereum (ETH) and Litecoin (LTC). NYA is an index that tracks the performance of all stocks listed on the New York Stock Exchange. It includes more than 2000 companies and covers a wide range of industries. The IXIC tracks the performance of all stocks listed on the NASDAQ stock exchange. These include technology, biotech, and other growth-oriented companies. The GSPC tracks the performance of the 500 largest publicly traded companies in the United States. It includes companies from various industries, such as technology, healthcare, and finance. The N100 tracks the performance of the 100 largest companies listed on the Euronext stock exchange, which operates in Amsterdam, Brussels, Dublin, Lisbon, Oslo, and Paris. The FTSE tracks the performance of the 100 largest companies listed on the London Stock Exchange. It includes companies from various industries, such as oil and gas, mining, and financial services. The FCHI tracks the performance of the 40 largest companies listed on the Euronext Paris Stock Exchange. It includes companies from various industries, such as healthcare, technology, and consumer goods. The DJI tracks the performance of 30 large publicly traded companies in the United States. It includes companies from various industries, such as technology, finance, and consumer goods. These indices track the performance of large and influential companies in their respective regions and industries. They can provide investors with a way to gauge the overall health of the stock market and economic conditions. To form Portfolio 2, we added to these seven indices BTC and ETH indices, the two largest cryptocurrencies in terms of market capitalization, and LTC, which is similar to BTC but offers faster transaction speeds and lower fees. The aim is to assess the level of impact that cryptocurrencies might have on the traditional stock market when in distress. However, we assess which assets are more vulnerable in both portfolios. Because many of the selected cryptocurrencies were created only in 2017, we collected data on November 10, 2017.

Table 1 presents the descriptive statistics of the assets in Portfolio 1. Except for BCH, which had a negative mean return, the remaining nine cryptocurrencies had positive mean returns. This finding suggests that investment in these cryptos during the study period will likely generate an overall positive return. The lowest percentage change in investment value over the specified period was significantly high in absolute value, with LINK having the highest at 61.5%. This indicates the worst-case scenario and potential downside risk of these cryptocurrencies. The highest percentage changes over the specified period were for XRP, ADA, and DOGE at 61%, 86%, and 152%, respectively. This indicates the best-case scenario and upside potential of these cryptocurrencies. The two leading cryptocurrencies, BTC and ETH, have the lowest maximum returns. Should this be an indication that these two cryptocurrencies are slowly reaching maturity?

From minimum to maximum through the mean returns, we can see that the returns are widely dispersed with a standard deviation (SD) that is much larger than the mean,

Table 1 Descriptive statistics for assets in Portfolio 1

	BTC	ETH	XRP	ADA	LINK	LTC	BCH	XLM	BNB	DOGE
Mean	0.117	0.148	0.083	0.23	0.275	0.041	− 0.068	0.116	0.341	0.3
Min	− 46.473	− 55.073	− 55.05	− 50.364	− 61.458	− 44.906	− 56.135	− 40.995	− 54.308	− 51.512
Max	22.512	23.474	60.689	86.154	48.062	38.932	42.081	55.918	52.922	151.633
SD	4.128	5.201	6.628	6.983	7.318	5.666	6.596	6.388	6.135	8.104
Kurtosis	12.284	10.319	15.911	24.111	6.993	8.826	10.298	10.268	14.149	84.791
Skewness	− 0.818	− 0.984	0.853	2.029	− 0.055	− 0.114	− 0.112	0.839	0.406	4.948
ACF	0.079	0.084	0.145	0.189	0.11	0.184	0.121	0.17	0.267	0.081

ACF: Autocorrelation Function; SD: Standard Deviation

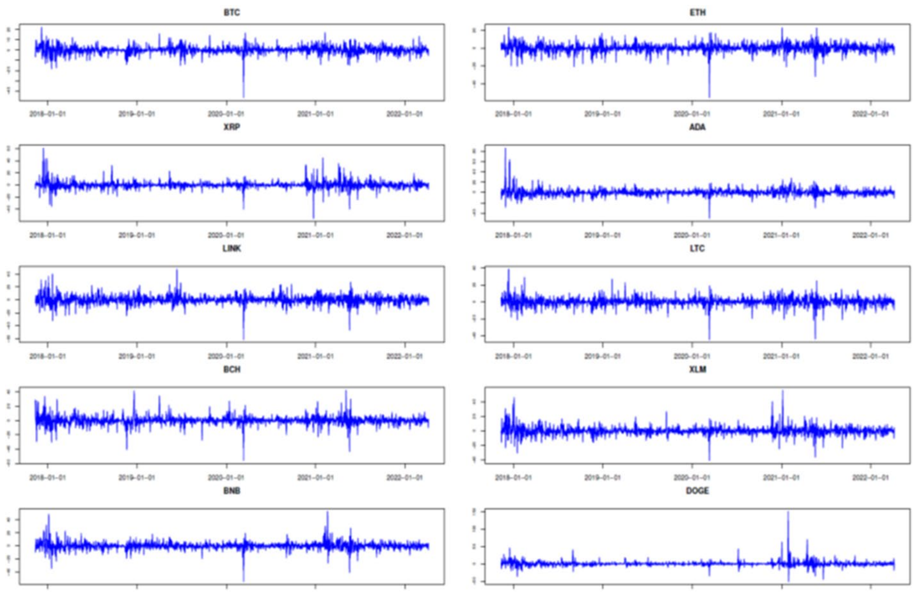

Fig. 1 Plot for returns of assets in Portfolio 1

Table 2 Correlation matrix for assets in Portfolio 1

	BTC	ETH	XRP	ADA	LINK	LTC	BCH	XLM	BNB	DOGE
BTC	1									
ETH	0.792	1								
XRP	0.703	0.801	1							
ADA	0.646	0.757	0.736	1						
LINK	0.424	0.555	0.485	0.536	1					
LTC	0.784	0.835	0.763	0.744	0.517	1				
BCH	0.727	0.783	0.739	0.708	0.495	0.830	1			
XLM	0.643	0.714	0.774	0.770	0.538	0.691	0.680	1		
BNB	0.666	0.711	0.647	0.665	0.482	0.698	0.643	0.645	1	
DOGE	0.697	0.684	0.693	0.612	0.405	0.678	0.644	0.626	0.593	1

indicating a high level of variability (See SD values in Table 1) exhibited by each cryptocurrency. They all have a higher kurtosis than the normal distribution. BTC, ETH, LINK, LTC, and BCH are skewed to the left, whereas XRP, ADA, XLM, BNB, and DOGE are skewed to the right. These stylized facts suggest a leptokurtic and skewed distribution for modeling crypto assets. Figure 1 illustrates the volatility clustering exhibited by all crypto assets; therefore, volatility is not constant over time. This finding suggests that a GARCH-type volatility model can be used to model these cryptographic assets. This study uses the APARCH model with a skewed student's t-distribution, which can capture stylized facts well.

The ACF values are all positive and close to zero, indicating little or no correlation between the values in the time series and their lagged values. This suggests the presence of random or unpredictable data patterns. Table 2 illustrates the high correlation among all selected crypto assets with BTC and LINK, with the lowest correlation of

Table 3 Descriptive statistics for assets in Portfolio 2

	BTC	ETH	LTC	NYA	IXIC	GSPC	N100	FTSE	FCHI	DJI
Mean	0.173	0.218	0.059	0.027	0.067	0.051	0.016	0.002	0.017	0.036
Min	− 46.473	− 55.073	− 44.906	− 12.595	− 13.149	− 12.765	− 12.752	− 11.512	− 13.098	− 13.842
Max	22.512	34.352	53.984	9.564	8.935	8.968	7.859	8.666	8.056	10.764
SD	4.93	6.344	6.683	1.293	1.534	1.338	1.241	1.157	1.329	1.385
Kurtosis	9.37	7.694	9.394	20.263	9.599	17.083	15.786	15.35	14.551	20.623
Skewness	− 0.778	− 0.667	0.188	− 1.333	− 0.793	− 0.971	− 1.371	− 1.197	− 1.125	− 1.019
ACF	0.069	0.081	0.274	0.428	0.476	0.501	0.089	0.149	0.095	0.435

ACF: Autocorrelation Function; SD: Standard Deviation

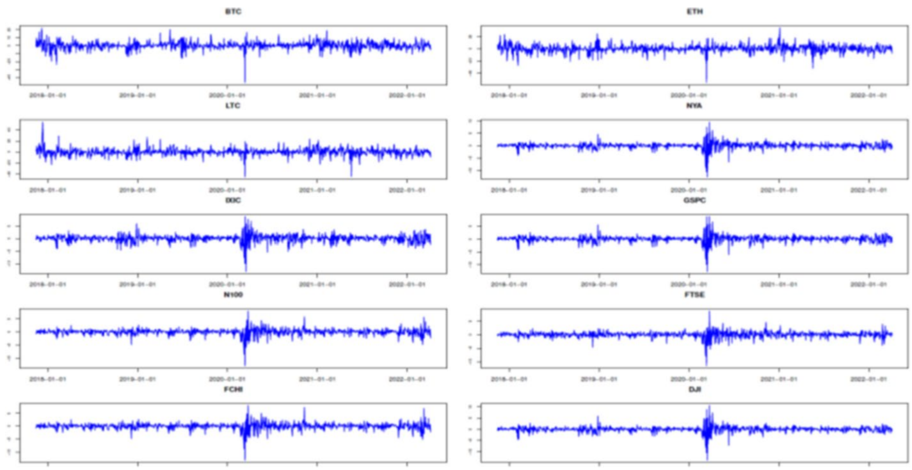

Fig. 2 Plot for returns of assets in Portfolio 2

Table 4 Correlation matrix for assets in Portfolio 2

	BTC	ETH	LTC	NYA	IXIC	GSPC	N100	FTSE	FCHI	DJI
BTC	1									
ETH	0.769	1								
LTC	0.784	0.835	1							
NYA	0.144	0.178	0.176	1						
IXIC	0.181	0.198	0.183	0.786	1					
GSPC	0.156	0.186	0.174	0.924	0.922	1				
N100	0.124	0.133	0.133	0.615	0.478	0.550	1			
FTSE	0.084	0.096	0.110	0.521	0.352	0.435	0.820	1		
FCHI	0.122	0.125	0.127	0.599	0.438	0.524	0.972	0.804	1	
DJI	0.129	0.170	0.156	0.932	0.772	0.927	0.560	0.464	0.554	1

0.424, whereas LTC and BCH display the highest correlation of 0.830. This indicates high systemic risk in Portfolio 1.

Table 3 presents the descriptive statistics of the assets in Portfolio 2. Compared with cryptocurrencies, the indices have lower standard deviations and are less extreme in terms of max and min. They had high kurtosis and were skewed to the left. NYA, IXIC, GSPC, and DJI display strong positive autocorrelations, which means that the values in the time series are highly correlated with their lagged values. This could indicate a pattern of persistence or a trend in the data, indicating long-memory dependence.

Additionally, Fig. 2 shows the volatility of these indices. In addition to stylized facts such as skewness, high kurtosis (meaning a high probability mass in the tail), and volatility clustering, the APARCH model can capture long memory dependence. Therefore, for Portfolio 1, to model Portfolio 2, we also use APARCH with a skewed Student's t-distribution. Table 4 shows that the indices are weakly correlated with BTC, ETH, and LTC. The GSPC was strongly positively correlated with NYA, IXIC, and DJA, with a correlation coefficient of approximately 0.9. The N100 was strongly and positively correlated with the FTSE and FCHI. Thus, there seem to be two clusters of strongly correlated assets in Portfolio 2: (1) GSPC, NYA, IXIC, and DJA, and (2) N100, FTSE, and FCHI. Because they represent approximately 80% of the global traditional stock market, any systemic risk inference for Portfolio 2 can be generalized to this market.

The five portfolio strategies are applied to each portfolio, and the corresponding ΔCoVaR is computed to assess the portfolio's vulnerability concerning each asset and the robustness of this measure.

Recall that CoVaR is the Value-at-Risk (VaR) of the portfolio conditional on an individual asset being in distress. A higher CoVaR value indicates higher systemic risk. ΔCoVaR measures the systemic impact of an individual asset on the portfolio, and a higher value indicates that the asset has a larger impact on systemic risk. Generally, higher CoVaR and CoVaR values indicate a higher level of systemic risk and a larger impact of an individual asset on the overall portfolio.

Empirical results and discussion

Portfolio 1 & 2 under MWP, GMV, MDP, MVDE and MVPSO

From Table 5, with a 95% confidence level, we realize that when Bitcoin is in distress (i.e., at its VaR = 13%), Portfolio 1 will suffer a loss of at least 15% of its value. In contrast, any of the ADA, BCH, XLM, or BNB in distress will lead Portfolio 1 to incur a loss of at least 18%. While Xu, Zhang, and Zhang (2021) observed ETH to be the largest risk transmitter, we found, based on $\boldsymbol{aL_i}$ values that LTC was the largest systemic risk transmitter, followed by ADA, XML, and BNB. This is not contradictory, given that the degree of total connectedness increases steadily over time. Although BTC appears to be the lowest-risk transmitter in distress, it has the highest impact on the portfolio. This follows from its ΔCoVaR value. Portfolio 1 appears more vulnerable to distress in BTC and ETH and less vulnerable to distress in DOGE. Knowing this may assist crypto enthusiasts/investors in identifying which assets to watch out for closely during market turmoil. Moreover, LTC distress not only significantly affects Portfolio 1 (11%) but also induces the largest additional loss (11%) on the other assets in Portfolio 1 (see the last column of Table 5). These findings highlight the importance of going beyond VaR to assess systemic risk. Limiting VaR alone would place BCH, ETH, ADA, LINK, and LTC at equal risk, ignoring the risk of spillover and its impact on individual assets. Furthermore, DOGE and XRP appear to be the riskiest in terms of VaR but appear to be among the last three lowest systemic transmitters. A similar study by Xu et al. (2021) ranked LTC, XLM, and XRP among the top 5 largest systemic risks. So, in terms of risk, while constructing their portfolios, investors should look at minimizing the VaR and the CoVaR, ΔCoVaR, and $\boldsymbol{aL_i}$ associated with each asset. Can this be feasible simultaneously? Further analysis is required to address this question. LTC, as the largest systemic transmitter, together with the two top cryptocurrencies in terms of market capitalization, is included in Portfolio 2, which consists mostly of world indices for assessing the impact of cryptocurrencies on the global market. Our study is similar to that of Li and Huang (2020), who use BTC, LTC, XRP, and major financial assets to unravel how cryptocurrencies could influence global financial systemic risk. They find that XRP is more connected to traditional assets than BTC and LTC. Overall, our findings show that LTC seems to be the largest systemic

Table 5 Estimated CoVaR and ΔCoVaR measures for Portfolio 1 under Market-weighted portfolio (MWP) strategy

Asset	ω_i	$VaR_q(L_i)$	$CoVaR_q^{p\|i}$	$CoVaR_{0.5}^{p\|i}$	$\Delta CoVaR_q^{p\|i}$	aL_i
BTC	0.59	0.13	0.15	0.03	0.12	0.04
ETH	0.28	0.14	0.17	0.05	0.12	0.08
XRP	0.03	0.18	0.17	0.11	0.07	0.06
ADA	0.03	0.14	0.18	0.09	0.09	0.09
LINK	0.01	0.14	0.17	0.11	0.07	0.07
LTC	0.01	0.14	0.17	0.07	0.11	0.11
BCH	0	0.14	0.18	0.08	0.1	0.1
XLM	0	0.13	0.18	0.09	0.09	0.09
BNB	0.05	0.15	0.18	0.08	0.09	0.09
DOGE	0.01	0.18	0.17	0.11	0.06	0.06

$q = 0.05$; ω_i are the weights corresponding to market capitalization; L_i is the vector of profit/loss; aL_i is the additional loss on the other components of the portfolio induced by the loss incurred by asset *i*. It is given by $aL_i = \Delta CoVaR_q^{p|i} - \omega_i VaR_q(L_i)$; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset *i* being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. This measures the vulnerability of the portfolio to contagion from the tail risk events of asset *i*

Table 6 Estimated CoVaR and ΔCoVaR measures for Portfolio 2 under Market-weighted portfolio (MWP) strategy

Asset	ω_i	$VaR_q(L_i)$	$CoVaR_q^{p\|i}$	$CoVaR_{0.5}^{p\|i}$	$\Delta CoVaR_q^{p\|i}$	aL_i
BTC	0.01	0.08	0.06	0.06	0	0
ETH	0	0.07	0.06	0.06	0	0
LTC	0	0.07	0.07	0.06	0.01	0.01
NYA	0.28	0.07	0.07	0.01	0.06	0.04
IXIC	0.13	0.07	0.09	0.03	0.05	0.04
GSPC	0.39	0.07	0.07	0.01	0.06	0.04
N100	0.04	0.07	0.09	0.03	0.05	0.05
FTSE	0.02	0.08	0.09	0.04	0.05	0.05
FCHI	0.03	0.08	0.09	0.04	0.05	0.04
DJI	0.11	0.07	0.08	0.02	0.06	0.05

NYSE COMPOSITE (NYA), NASDAQ Composite (IXIC), S&P 500 (GSPC), Euronext 100 Index (N100), FTSE 100 (FTSE), CAC 40 (FCHI) and Dow Jones Industrial Average (DJI), Bitcoin (BTC), Ethereum (ETH) and Litecoin (LTC). $q = 0.05$; ω_i are the weights corresponding to market capitalization; L_i is the vector of profit/loss; aL_i is the additional loss on the other components of the portfolio induced by the loss incurred by asset *i*. It is given by $aL_i = \Delta CoVaR_q^{p|i} - \omega_i VaR_q(L_i)$; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset *i* being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset *i*

risk transmitter in the cryptocurrency market and that distress in each cryptocurrency is likely to affect the remaining market participants.

Portfolio 2 is well-diversified. From Table 6, when BTC and ETH are at their VaRs, the VaR of Portfolio 2 is 0.06, the same as that of Portfolio 2 under normal market conditions. This illustrates that distress in BTC and ETH has no or negligible impact on Portfolio 2. A similar observation holds for LTC. This may be due to the small size of the cryptocurrency market compared with that of the equity market. It may also be because cryptocurrencies are weakly correlated to equities, see Sajeev and Afjal (2022). This confirms the diversification benefits of cryptocurrencies (Anyfantaki and Topaloglou 2018). Similar observations are made by Li and Huang (2020), who find that cryptocurrencies function as a separate source of risk from traditional assets. Among BTC, ETH, and LTC, the largest systemic risk transmitter appears to be LTC, although it is the lowest compared with traditional stocks.

Similar to the crypto-only portfolio (Portfolio 1), LTC is likely to produce a larger systemic risk impact than the other cryptocurrencies. Therefore, LTC may have unique characteristics that require further investigation. Gemici and Polat (2021) examined the volatility spillovers between BTC, LTC, and ETH related to structural breaks. Their findings indicate that there is a one-way causality-in-mean from BTC to LTC and ETH, but there is no causality-in-mean from LTC and ETH to BTC. Considering structural breaks, they found short-term causality-in-variance from LTC to BTC and long-term causality-in-variance from BTC to LTC. Jana et al. (2019) analyze the informational efficiency of LTC using computationally efficient and robust estimators of long-range dependence and find evidence of market inefficiency and the multifractality of LTC returns. Using Vulnerability CoVaR (*V*CoVaR) to assess the systemic risk among BTC, ETH, LTC, XMR, and XRP, Waltz et al. (2022) found that LTC had the largest impact on BTC. Are these LTC properties able to explain the high systemic risk transmission? However, further studies are needed to confirm this hypothesis.

For traditional stocks, while Portfolio 2 is more vulnerable to NYA, GSPC, and DJI, which share the same VaR percentage (7%), the N100, FTSE, and DJI appear to be the largest systemic risk transmitters.

In the previous section, portfolio allocation in portfolios 1 and 2 was based on market capitalization; that is, the weight of each asset was proportional to its market capitalization. This section's weights are derived from the global minimum variance and most diversified portfolio optimization approaches. Generally, a portfolio with weight allocation based on market capitalization tends to be more heavily weighted towards larger companies, which may lead to a higher concentration risk in a particular sector or industry. As shown in Table 5, BTC and ETH account for approximately 87% of the shares, and in Table 6, GSPC and NYA account for nearly 67%. Such allocations appear to be similar to the Global Minimum Variance (GMV), except that the GMV portfolio tends to have a more concentrated allocation in a few assets with the lowest possible level of risk. This is because the GMV portfolio seeks to minimize risk by reducing the exposure to assets with high levels of volatility or correlation.

This justifies the concentration weights of BTC and XRP being greater than 95%, as shown in Table 15. The Most Diversified Portfolio seeks to minimize concentration risk by allocating weights to create a less sensitive portfolio to individual assets or market movements. Unlike Karimalis and Nomikos (2018), who consider one portfolio and check the robustness of their procedure across different systemic risk measures (CoVaR and CoES), we assess the sensitivity and robustness of the CoVaR systemic risk measures using these five portfolio allocation and optimization techniques.

To compute the CoVaR, ΔCoVaR, and the $\boldsymbol{aL_i}$ from the GMV, MDP, MVDE, and MVPSO, we need to first find the portfolio weights allocation from each of the portfolio optimization techniques. Table 15 lists the obtained weights. The two portfolio strategies, GMV and MWP, are much more concentrated, with BTC having the largest weight. As its name indicates, the MDP appears to be well-diversified. In addition, MVDE and

Table 7 CoVaR and ΔCoVaR computed from GMV for Portfolio 1

Asset	ω_i	$VaR_q(L_i)$	$CoVaR_q^{p\|i}$	$CoVaR_{0.5}^{p\|i}$	$\Delta CoVaR_q^{p\|i}$	aL_i
BTC	0.9	0.13	0.14	0.01	0.13	0.01
ETH	0.03	0.14	0.18	0.07	0.11	0.1
XRP	0.06	0.18	0.18	0.11	0.07	0.06
ADA	0	0.14	0.18	0.1	0.08	0.08
LINK	0	0.14	0.18	0.11	0.06	0.06
LTC	0	0.14	0.18	0.08	0.1	0.1
BCH	0	0.14	0.18	0.09	0.1	0.1
XLM	0	0.13	0.18	0.1	0.09	0.09
BNB	0.01	0.15	0.18	0.09	0.09	0.09
DOGE	0	0.18	0.17	0.12	0.06	0.06

$q = 0.05$; ω_i are the weights corresponding to market capitalization; L_i is the vector of profit/loss; aL_i is the additional loss on the other components of the portfolio induced by the loss incurred by asset *i*. It is given by $aL_i = \Delta CoVaR_q^{p|i} - \omega_i VaR_q(L_i)$; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset *i* being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset *i*

Table 8 CoVaR and ΔCoVaR computed from MDP for Portfolio 1

Asset	ω_i	$VaR_q(L_i)$	$CoVaR_q^{p\|i}$	$CoVaR_{0.5}^{p\|i}$	$\Delta CoVaR_q^{p\|i}$	aL_i
BTC	0	0.13	0.17	0.08	0.08	0.08
ETH	0	0.14	0.17	0.08	0.09	0.09
XRP	0.17	0.18	0.17	0.08	0.09	0.06
ADA	0.12	0.14	0.17	0.07	0.1	0.08
LINK	0.18	0.14	0.17	0.08	0.09	0.06
LTC	0	0.14	0.17	0.07	0.09	0.09
BCH	0.1	0.14	0.17	0.08	0.09	0.08
XLM	0	0.13	0.17	0.07	0.1	0.1
BNB	0.22	0.15	0.17	0.07	0.09	0.06
DOGE	0.21	0.18	0.17	0.08	0.09	0.05

$q = 0.05$; ω_i are the weights corresponding to market capitalization; L_i is the vector of profit/loss; aL_i is the additional loss on the other components of the portfolio induced by the loss incurred by asset *i*. It is given by $aL_i = \Delta CoVaR_q^{p|i} - \omega_i VaR_q(L_i)$; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset *i* being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset *i*

MVPSO appear quite diverse, although they allocate weights differently. We assessed the implications of co-risks.

Under the GMV portfolio strategy in Table 7, the VaR of Portfolio 1, conditional on BTC being in distress, is the lowest compared with the remaining assets. This may explain why the GMV allocates the highest weight to BTC. BTC appears to be the transmitter with the lowest systemic risk. As with the market weights (Table 5), LTC and BCH remain the top systemic risk transmitters with GMV allocations (Table 7). The CoVaR estimations of the MWP and GMV remain very close. Therefore, the CoVaR estimation for Portfolio 1 seems less sensitive to the MWP and GMV portfolio allocation and optimization. Because the GMV tends to result in a more concentrated portfolio with high exposure to a few assets, the CoVaR computed using the GMV portfolio seems biased towards these few assets' tail risk (see Table 7).

Table 9 CoVaR and ΔCoVaR computed from GMV for Portfolio 2

Asset	ω_i	$VaR_q(L_i)$	$CoVaR_q^{p\|i}$	$CoVaR_{0.5}^{p\|i}$	$\Delta CoVaR_q^{p\|i}$	aL_i
BTC	0	0.08	0.07	0.07	0	0
ETH	0	0.07	0.07	0.07	0	0
LTC	0	0.07	0.07	0.07	0	0
NYA	0	0.07	0.09	0.03	0.06	0.06
IXIC	0	0.07	0.1	0.05	0.04	0.04
GSPC	0.33	0.07	0.09	0.04	0.06	0.04
N100	0	0.07	0.09	0.02	0.06	0.06
FTSE	0.66	0.08	0.08	0.02	0.07	0.02
FCHI	0	0.08	0.09	0.04	0.06	0.06
DJI	0	0.07	0.09	0.03	0.06	0.06

NYSE COMPOSITE (NYA), NASDAQ Composite (IXIC), S&P 500 (GSPC), Euronext 100 Index (N100), FTSE 100 (FTSE), CAC 40 (FCHI) and Dow Jones Industrial Average (DJI), Bitcoin (BTC), Ethereum (ETH) and Litecoin (LTC). $q = 0.05$; ω_i are the weights corresponding to market capitalization; L_i is the vector of profit/loss; aL_i is the additional loss on the other components of the portfolio induced by the loss incurred by asset *i*. It is given by $aL_i = \Delta CoVaR_q^{p|i} - \omega_i VaR_q(L_i)$; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset *i* being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset *i*

However, because MDP results in a more evenly distributed portfolio with lower exposure to any one asset, the CoVaR computed using MDP seems to be more representative of the tail risk of the overall portfolio (see Table 8). With this approach, BTC appears to be among the six largest systemic risk transmitters based on $\boldsymbol{aL_i}$ values, as shown in Table 8. Therefore, it is essential to consider a portfolio optimization strategy when computing the CoVaR and interpreting the results. The strengths and weaknesses of the GMV and MDP approaches depend on individual investor circumstances and investment objectives.

With the MDP strategy, the CoVaR of Portfolio 1 (Table 8) conditional on an asset in distress is the same irrespective of the asset. Though XLM is assigned a weight of zero, any distress experienced by XLM will likely affect Portfolio 2 more than distress in any remaining assets based on its ΔCoVaR value. This may be due to the strong correlation between the portfolio's XLM shares and other assets. This is attested to by the largest additional loss incurred by other assets when XLM is in distress. We realize that even though the weight of an asset may be zero, as long as it is correlated with other assets, being in a state of distress will still impact the portfolio (see BTC, ETH, LTC, and XLM).

A significant difference can be observed between the values of $\boldsymbol{aL_i}$ for BTC given by the GMV and MDP, although their respective CoVaR values remain similar. This can be attributed to differences in the representation of the tail risk. Overall, all the assets in Portfolio 1 have an equal impact on the portfolio when in distress.

Moreover, each cryptocurrency is significantly affected if a joint distress event occurs among the remaining portfolio assets. To mitigate the risks and benefits of diversification, cryptocurrencies should be combined with other traditional classes of assets, such as Portfolio 2.

From Table 16, similar to Table 15, the GMV strategy concentrates the weights on two assets: GSPC and FTSE. The MDP, although quite concentrated on the IXIC (30.30) and FTSE (37.90), involves all assets in Portfolio 2 in the allocation. MVDE and MVPSO are

Table 10 CoVaR and ΔCoVaR computed from MDP for Portfolio 2

Asset	ω_i	$VaR_q(L_i)$	$CoVaR_q^{p\|i}$	$CoVaR_{0.5}^{p\|i}$	$\Delta CoVaR_q^{p\|i}$	aL_i
BTC	0.07	0.08	0.07	0.05	0.01	0
ETH	0.03	0.07	0.06	0.06	0.01	0.01
LTC	0.06	0.07	0.07	0.05	0.01	0.01
NYA	0	0.07	0.08	0.03	0.05	0.05
IXIC	0.3	0.07	0.08	0.03	0.04	0.02
GSPC	0	0.07	0.08	0.03	0.05	0.05
N100	0	0.07	0.07	0.02	0.05	0.05
FTSE	0.38	0.08	0.07	0.02	0.05	0.02
FCHI	0.12	0.08	0.08	0.03	0.05	0.04
DJI	0.04	0.07	0.08	0.03	0.05	0.04

NYSE COMPOSITE (NYA), NASDAQ Composite (IXIC), S&P 500 (GSPC), Euronext 100 Index (N100), FTSE 100 (FTSE), CAC 40 (FCHI) and Dow Jones Industrial Average (DJI), Bitcoin (BTC), Ethereum (ETH) and Litecoin (LTC). $q = 0.05$; $\boldsymbol{\omega_i}$ are the weights corresponding to market capitalization; L_i is the vector of profit/loss; aL_i is the additional loss on the other components of the portfolio induced by the loss incurred by asset *i*. It is given by $aL_i = \Delta CoVaR_q^{p|i} - \omega_i VaR_q(L_i)$; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset *i* being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset *i*

quite diversified, although they differ in their allocation, except for asset IXIC, to which their weight allocations are very close: 18 and 17, respectively.

As shown in Table 6, under the market-weighted portfolio strategy, we observe from Table 9 that the vulnerability of Portfolio 2 to distress in crypto-assets (BTC, ETH, and LTC) is negligible, and they induce no additional loss on the other assets, indicating negligible risk spillover from the cryptocurrency market to the traditional stock market. While Portfolio 2 seems more vulnerable to distress in the FTSE, among the other indices in Portfolio 2, distress in the FTSE induces the least additional loss in the remaining assets. Does this relate to the weak correlation between the FTSE and other indices? This aspect should be further investigated at later stages of development. Let's notice that NYA, N100, FCHI, and DJI are the top largest systemic risk transmitters in this portfolio and have a high impact on the portfolio when in distress, based on $\boldsymbol{aL_i}$ and ΔCoVaR values respectively in Table 9.

We observe from Table 10 that crypto assets still have a negligible impact on Portfolio 2. However, the VaR of Portfolio 2, when any of the crypto assets (BTC, ETH, or LTC) is in distress, remains similar to that of the equities in the portfolio. Under the MDP portfolio strategy, Portfolio 2's vulnerability to the distress of any of its equities is harmonized and has almost the same impact. The IXIC and FTSE seem to induce fewer additional losses on the remaining assets than any other equity in Portfolio 2. Portfolio 2 is less risky than Portfolio 1 under MWP, GMV, and MDP portfolio allocation and optimization. In recent years, cryptocurrencies have experienced significant price volatility. Although they can generate high returns, they are subject to significant price fluctuations and regulatory uncertainties. In contrast, traditional stocks and indices are generally less volatile and are backed by established companies with a track record of revenue and earnings. They are also subject to regulatory oversight and have established markets with a history of trading volume. Combining cryptocurrencies with traditional stocks and indices in one portfolio can provide additional diversification benefits by spreading risk across different asset classes and geographies. They may also provide opportunities for higher returns through exposure to emerging technologies and market inefficiencies.

The remaining computations of CoVaR and ΔCoVaR from the portfolio strategy MVDE for Portfolio 1 and Portfolio 2 are presented in Table 11 and Table 12, respectively, while the computations from MVPSO for Portfolio 1 and Portfolio 2 are in Table 13 and Table 14, respectively. The summary of these computations is used in the next section to assess the sensitivity and robustness of CoVaR and ΔCoVaR as co-risk measures. Tables 15 and 16 display the weights allocation from the five portfolio strategies and use Tables S4 to S14 to compute the CoVaR and ΔCoVaR values.

Sensitivity and robustness check

Comparing CoVaR and ΔCoVaR from Portfolio 1

Table 17 displays the values of CoVaR and ΔCoVaR from the five portfolio strategies: MWP, GMV, MDP, MVDE, and MVPSO for Portfolio 1. From the values in Table 17, we plotted Figs. 3 and 4.

In Fig. 3, each line tracks the CoVaR value of each asset for each portfolio allocation strategy. BTC had the lowest CoVaR for the MWP and GMV. This finding supports that the GMV allocates the largest weights to less-risky assets (see Table 15). Firstly, the

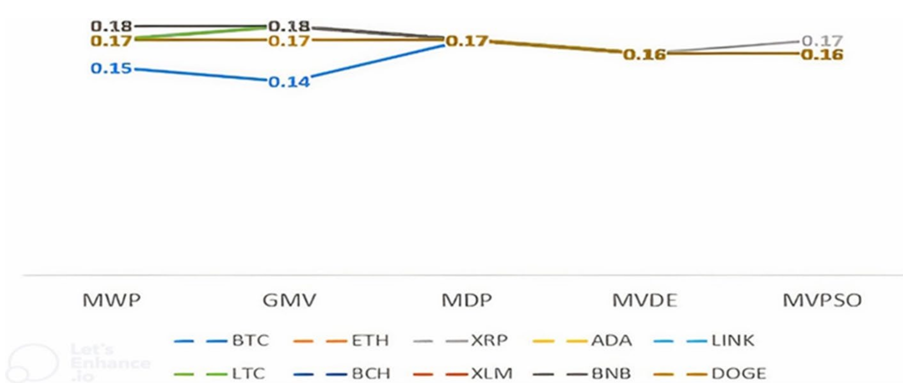

Fig. 3 Robustness and sensibility of CoVaR for Portfolio 1

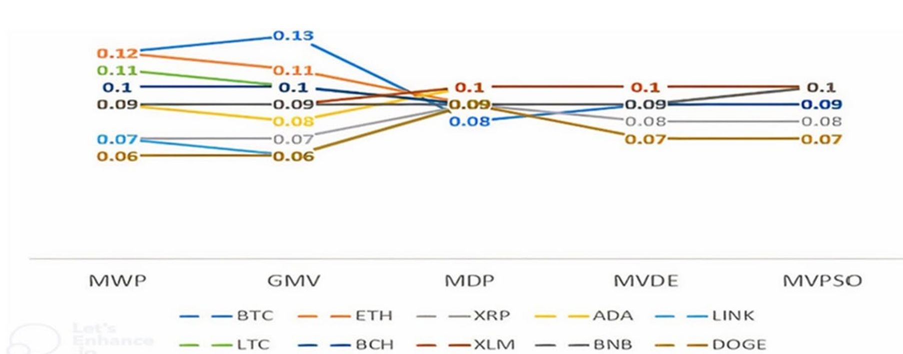

Fig. 4 Vulnerability of Portfolio 1 to its assets tail risk

CoVaR tracking lines of the remaining assets are close to one another, indicating that the distress experienced by any of these assets is likely to have an equal impact on the portfolio. Second, except for BTC, the CoVaR lines for other assets are not parallel to the horizontal line but remain quite close to it. This result indicates that the sensitivity of the CoVaR measure is less pronounced regarding portfolio allocation and optimization. Therefore, it can be used as a reliable measure of systemic risk.

Figure 4 displays the ΔCoVaR lines for assets in Portfolio 1. It can be observed that the level of vulnerability of a portfolio to an asset tail risk depends on the portfolio allocation and optimization used. For example, Portfolio 1 is more vulnerable to distress in BTC under GMV than the contribution of other assets but less vulnerable to the same BTC under MDP. Under the MDP, MVDE, and MVPSO. XLM has the highest impact on Portfolio 1; that is, under these portfolio allocation strategies, Portfolio 1 is more vulnerable to distress in XLM than in any of the remaining assets. By contrast, when in distress, DOGE has the lowest impact on Portfolio 1 under MWP, GMV, MVDE, and MVPSO. This suggests that investors should investigate the systemic risk profile of their portfolios using each portfolio allocation optimization approach they may choose.

Several studies expand the CoVaR measure to include multiple cases by incorporating more than one variable into a conditional event. Cao (2013) introduces the multi-CoVaR (MCoVaR) with the condition of several assets being simultaneously in distress. Bernardi et al. (2021) proposed the System-CoVaR (SCoVaR), in which the conditional variables are aggregated via their sum. Waltz et al. (2022) introduced Vulnerability-CoVaR (*V*CoVaR), which is defined as the Value-at-Risk (VaR) of a financial system or institution, given that at least one other institution is equal to or below its VaR. *V*CoVaR relaxes the normality assumptions and is estimated using a copula. Although the *CoVaR* measure in this study is a condition in which one asset is in distress, it is estimated using a vine copula to carefully capture tail dependence. Our findings reveal that our CoVaR estimation approach is less sensitive to the portfolio allocation strategy. This is because of the ability of the C-vine copula to model tail dependence.

Comparing CoVaR and ΔCoVaR from Portfolio 2

Table 18 displays the values of CoVaR and ΔCoVaR from the five portfolio strategies: MWP, GMV, MDP, MVDE, and MVPSO for Portfolio 2. From the values in Table 18, we plotted Figs. 5 and 6.

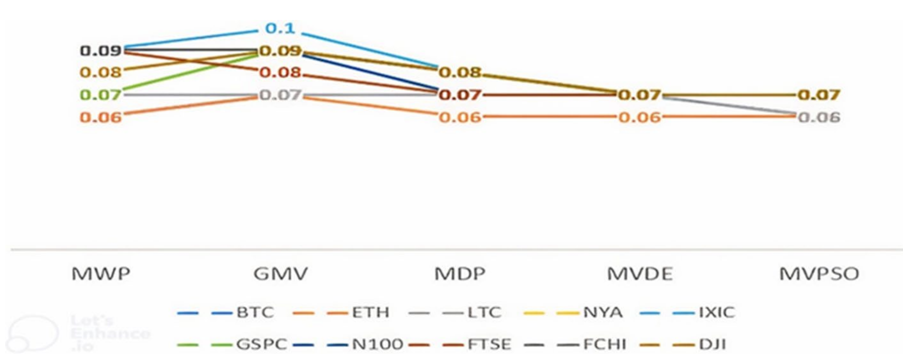

Fig. 5 Robustness and sensitivity of CoVaR for Portfolio 2

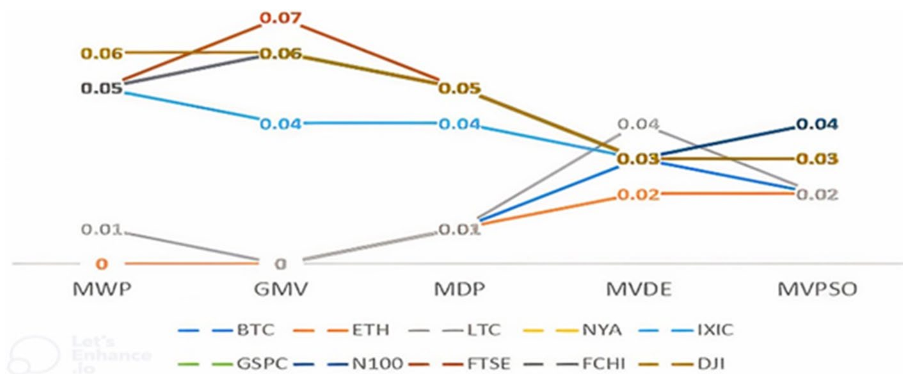

Fig. 6 Vulnerability of Portfolio 2 to its assets tail risk

Figure 5 shows the CoVaR tracking lines for each asset in Portfolio 2 for each portfolio allocation and optimization technique used. Regarding systemic risk, Portfolio 2 is less risky than Portfolio 1: Its highest CoVaR value is 0.1 for IXIC under GMV compared to Portfolio 1, with the highest value of 0.18 corresponding to XLM under GMV and MWP. It can be observed that these lines are not parallel to the horizontal line, except their last segments corresponding to MDP, MVDE, and MVPSO for some assets. Therefore, the CoVaR measure depends on the allocation and optimization techniques used. Figure 3 suggests that MVDE and MVPSO are preferred to GMV and MDP regarding contagion and risk spillover effects during market downtime. Although combining cryptocurrencies with traditional stocks also involves a higher level of risk owing to the volatile nature of cryptocurrencies, it provides additional diversification benefits by spreading risk across different asset classes and geographies. Thus, Portfolio 1 is significantly riskier than Portfolio 2 across the three portfolio allocations.

Figure 6 indicates that Portfolio 2 is more vulnerable to each of the seven indices than BTC, ETH, and LTC under MWP, GMV, MDP, and MVPSO. MVDE presents a scenario in which Portfolio 2 is as vulnerable to BTC as any of the indices. Moreover, under MVDE, LTC has the highest ΔCoVaR value, indicating that Portfolio 2 is more vulnerable to LTC than the remaining assets. Furthermore, MVDE and MVPSO appear to provide an overall equal level of vulnerability to Portfolio 2, which is lower than that of MDP. GMV and MWP. The optimizers DE and PSO in MVDE and MVPSO are population-based.

In summary, the ΔCoVaR of an asset quantifies the potential impact of an asset's distress on a portfolio's vulnerability to systemic risk. The higher the ΔCoVaR of an asset, the more significant its contribution to the overall systemic risk of the portfolio, and the more vulnerable the portfolio is to the asset's distress. While investors can use ΔCoVaR to evaluate their portfolio's exposure to systemic risk and identify the assets that pose the greatest risk to their portfolios during financial stress or crisis, they should also adequately select their allocation and optimization approach. By understanding the ΔCoVaR of each asset, investors can make informed decisions regarding portfolio construction and risk management strategies.

Conclusions

This study aimed to assess the sensitivity and robustness of the Conditional Value-at-Risk (CoVaR) as a reliable co-risk measure for tail risk spillover. Robustness was evaluated regarding the different portfolio allocations and optimization strategies that investors may utilize. To achieve this, we consider two portfolios comprising major cryptocurrencies and global indices. Portfolio 1 consists of ten major cryptocurrencies and Portfolio 2 consists of seven stock indices and three cryptocurrencies. These two portfolios reflect two markets: the cryptocurrency and traditional global markets. The rationale behind this choice is to incorporate all the differences between the two markets. The cryptocurrency and traditional global markets are fundamentally different, with the former being decentralized, highly volatile, and largely unregulated. In contrast, the latter is heavily regulated, less volatile, and offers a wider range of investment options. In addition to the two portfolios, we consider five portfolio allocation and optimization strategies: the MWP, in which weights are computed from the respective market capitalization, GMV portfolio, MDP, MVDE, and MVPSO. The rationale behind this two-stage approach is to assess the sensitivity of CoVaR and the vulnerability of each portfolio concerning the portfolio composition and allocation strategy. To compute CoVaR, we use the C-vine copula APARCH model to capture the dependence structure and various stylized facts of the return data, including fat tail, volatility clustering, and skewness.

Our approach's key novelty lies in using vine copula-APARCH models to estimate the CoVaR. The Vine copula captures the complex non-linear dependence and tail dynamics across assets. The APARCH model incorporates volatility clustering, skewness, leptokurtosis, and long memory. This advanced approach provides more accurate CoVaR estimates than traditional methods. The results of our analysis demonstrate that CoVaR values are highly sensitive to the portfolio strategy used.

The tracking lines for each asset are not parallel to the horizontal lines, indicating that different portfolio strategies can significantly impact the tail risk of individual assets and the overall portfolio. Moreover, the high CoVaR values for cryptocurrencies compared with traditional indices suggest that investing in cryptocurrencies carries a higher level of risk. Furthermore, the results suggest that CoVaR is even more sensitive to portfolio strategies than CoVaR. The GMV strategy provides the most vulnerable portfolio in terms of ΔCoVaR, emphasizing the importance of carefully selecting a portfolio strategy to mitigate tail risk.

These findings have important implications for investors and asset managers. First, our results suggest that traditional portfolio optimization strategies may not adequately mitigate tail risk in today's rapidly changing and complex financial markets. Therefore, investors and asset managers may need to consider more sophisticated portfolio optimization techniques that explicitly account for tail risk, such as CoVaR and ΔCoVaR. Second, our findings suggest that investing in cryptocurrencies carries a higher level of risk than traditional indices do; therefore, investors should exercise caution when considering investing in cryptocurrencies. We recommend that investors conduct thorough due diligence on cryptocurrencies and consider their risk tolerance before investing. Overall, this study highlights the value of CoVaR estimated via the vine copula-APARCH in assessing portfolio systemic risk. This study provides investors and risk managers with meaningful insights to enhance portfolio resilience. Our approach represents a significant methodological contribution to applying advanced econometric modeling to produce robust systemic risk metrics (Additional file 1).

In summary, the sensitivity analysis provides evidence that CoVaR is a relatively reliable co-risk measure, while portfolio composition and optimization strategy significantly impact the overall tail risk. The novel modeling approach underpins the robustness of the CoVaR estimates. Based on the results of this study, there are several areas for future research. Firstly, it would be interesting to explore the sensitivity of CoVaR and ΔCoVaR to different weighting schemes, such as equal-weighted versus market weighted portfolios, as this could significantly impact the results. Second, the study focused on a specific set of assets and periods. It would be valuable to extend the analysis to include additional assets and periods to assess the generalizability of our findings. Thirdly, while our study focused on CoVaR and ΔCoVaR as risk measures, other risk measures could be explored, such as Conditional Drawdown at Risk (CDaR) or Expected Shortfall (ES), which could provide additional insights into the tail risk of a portfolio. Finally, the study assumed a static portfolio allocation, and it would be interesting to investigate the impact of dynamic portfolio allocation strategies on CoVaR and ΔCoVaR. This could involve exploring machine learning techniques to determine the optimal portfolio allocation strategies that account for tail risk. These potential research areas could provide further insight into portfolio tail risk and help improve risk management practices for investors and asset managers.

Appendix 1

This section contained some tables displaying the results obtained and cited in the analysis (see Tables 11, 12, 13, 14, 15, 16, 17 and 18).

Table 11 Estimated CoVaR q and ΔCoVaR measures for Portfolio 1 under MV-DE portfolio (MVDE) Strategy

Asset	ω_i	$VaR_q(L_i)$	$CoVaR_q^{p\|i}$	$CoVaR_{0.5}^{p\|i}$	$\Delta CoVaR_q^{p\|i}$	aL_i
BTC	0.09	0.13	0.16	0.08	0.09	0.07
ETH	0.03	0.14	0.16	0.07	0.09	0.09
XRP	0.11	0.18	0.16	0.08	0.08	0.06
ADA	0.14	0.14	0.16	0.06	0.1	0.08
LINK	0.3	0.14	0.16	0.07	0.09	0.05
LTC	0	0.14	0.16	0.07	0.09	0.09
BCH	0.01	0.14	0.16	0.07	0.09	0.09
XLM	0.07	0.13	0.16	0.06	0.1	0.09
BNB	0.16	0.15	0.16	0.07	0.09	0.06
DOGE	0.09	0.18	0.16	0.09	0.07	0.06

$q = 0.05$; ω_i are the weights corresponding to market capitalization; L_i is the vector of profit/loss; aL_i is the additional loss on the other components of the portfolio induced by the loss incurred by asset i. It is given by $aL_i = \Delta CoVaR_q^{p|i} - \omega_i VaR_q(L_i)$; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset i being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset i

Table 12 Estimated CoVaR q and ΔCoVaR measures for Portfolio 2 under MV-DE portfolio (MVDE) Strategy

Asset	ω_i	$VaR_q(L_i)$	$CoVaR_q^{p\|i}$	$CoVaR_{0.5}^{p\|i}$	$\Delta CoVaR_q^{p\|i}$	aL_i
BTC	0.12	0.08	0.07	0.04	0.03	0.02
ETH	0.02	0.07	0.06	0.04	0.02	0.02
LTC	0.35	0.07	0.07	0.03	0.04	0.01
NYA	0.01	0.07	0.07	0.04	0.03	0.03
IXIC	0.18	0.07	0.07	0.04	0.03	0.02
GSPC	0.15	0.07	0.07	0.04	0.03	0.02
N100	0.13	0.07	0.07	0.04	0.03	0.02
FTSE	0.04	0.08	0.07	0.04	0.03	0.02
FCHI	0	0.08	0.07	0.04	0.03	0.03
DJI	0	0.07	0.07	0.04	0.03	0.03

$q = 0.05$; ω_i are the weights corresponding to market capitalization; L_i is the vector of profit/loss; aL_i is the additional loss on the other components of the portfolio induced by the loss incurred by asset i. It is given by $aL_i = \Delta CoVaR_q^{p|i} - \omega_i VaR_q(L_i)$; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset i being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset i

Table 13 Estimated CoVaR q and ΔCoVaR measures for Portfolio 1 under MV-PSO portfolio (MVPSO) Strategy

Asset	ω_i	$VaR_q(L_i)$	$CoVaR_q^{p\|i}$	$CoVaR_{0.5}^{p\|i}$	$\Delta CoVaR_q^{p\|i}$	aL_i
BTC	0.07	0.13	0.16	0.07	0.09	0.08
ETH	0.16	0.14	0.16	0.06	0.1	0.08
XRP	0.08	0.18	0.17	0.09	0.08	0.06
ADA	0.22	0.14	0.16	0.06	0.1	0.07
LINK	0.17	0.14	0.16	0.08	0.09	0.06
LTC	0.05	0.14	0.16	0.06	0.1	0.09
BCH	0	0.14	0.16	0.07	0.09	0.09
XLM	0	0.13	0.16	0.07	0.1	0.1
BNB	0.19	0.15	0.16	0.07	0.1	0.07
DOGE	0.07	0.18	0.16	0.1	0.07	0.06

$q = 0.05$; ω_i are the weights corresponding to market capitalization; L_i is the vector of profit/loss; aL_i is the additional loss on the other components of the portfolio induced by the loss incurred by asset i. It is given by $aL_i = \Delta CoVaR_q^{p|i} - \omega_i VaR_q(L_i)$; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset i being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset i

Table 14 Estimated CoVaR *q* and ΔCoVaR measures for Portfolio 2 under MV-PSO portfolio (MVPSO) Strategy

Asset	ω_i	$VaR_q(L_i)$	$CoVaR_q^{p\mid i}$	$CoVaR_{0.5}^{p\mid i}$	$\Delta CoVaR_q^{p\mid i}$	aL_i
BTC	0.19	0.08	0.06	0.04	0.02	0.01
ETH	0.07	0.07	0.06	0.04	0.02	0.01
LTC	0.12	0.07	0.06	0.04	0.02	0.01
NYA	0.16	0.07	0.07	0.03	0.04	0.03
IXIC	0.07	0.07	0.07	0.03	0.03	0.03
GSPC	0.12	0.07	0.07	0.03	0.04	0.03
N100	0.01	0.07	0.07	0.03	0.04	0.03
FTSE	0.11	0.08	0.07	0.03	0.03	0.02
FCHI	0	0.08	0.07	0.04	0.03	0.03
DJI	0.16	0.07	0.07	0.03	0.04	0.03

$q = 0.05$; ω_i are the weights corresponding to market capitalization; L_i is the vector of profit/loss; aL_i is the additional loss on the other components of the portfolio induced by the loss incurred by asset i. It is given by $aL_i = \Delta CoVaR_q^{p|i} - \omega_i VaR_q(L_i)$; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset i being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset i

Table 15 Weights allocation from GMV, MDP, MWP, MVDE and MVPSO portfolio strategies for assets in Portfolio 1

	GMVw	MDPw	MWPw	MVDEw	MVPSOw
BTC	89.80	0.03	58.53	9	7
ETH	2.62	0.00	27.96	3	16
XRP	6.17	16.80	2.65	11	8
ADA	0.00	12.30	2.54	14	22
LINK	0.37	18.00	0.52	30	17
LTC	0.00	0.00	0.56	0	5
BCH	0.00	9.76	0.45	1	0
XLM	0.00	0.00	0.36	7	0
BNB	1.00	22.40	5.07	16	19
DOGE	0.00	20.70	1.37	9	7

GMVw represents the weights from the Global minimum variance (GMV); MDPw represents the weights from the Maximum diversification portfolio (MDP); MWPw represents the weights corresponding to market capitalization

Table 16 Weights allocation from GMV, MDP, MWP, MVDE and MVPSO for assets in Portfolio 2

	GMVw	MDPw	MWPw	MVDEw	MVPSOw
BTC	0.29	7.44	1.00	12	7
ETH	0.00	2.84	0.00	2	16
LTC	0.00	5.67	0.00	35	8
NYA	0.00	0.00	28.00	1	22
IXIC	0.01	30.30	13.00	18	17
GSPC	33.40	0.00	39.00	15	5
N100	0.00	0.00	4.00	13	0
FTSE	66.30	37.90	2.00	4	0
FCHI	0.00	12.10	3.00	0	19
DJI	0.00	3.76	11.00	0	7

NYSE COMPOSITE (NYA), NASDAQ Composite (IXIC), S&P 500 (GSPC), Euronext 100 Index (N100), FTSE 100 (FTSE), CAC 40 (FCHI) and Dow Jones Industrial Average (DJI), Bitcoin (BTC), Ethereum (ETH) and Litecoin (LTC). GMVw represents the weights from the Global minimum variance (GMV); MDPw represents the weights from the Maximum diversification portfolio (MDP); MWPw represents the weights corresponding to market capitalization MVDEw represents the weights from the Differential Evolution (DE) Optimization; MVPSOw represents the weights from the Particle Swarm Optimization (PSO)

Table 17 ΔCoVaR and CoVaR under MWP, GMV, MDP, MVDE and MVPSO in Portfolio 1

$\Delta CoVaR_q^{p\|i}$						$CoVaR_q^{p\|i}$				
Asset	**MWP**	**GMV**	**MDP**	**MVDE**	**MVPSO**	**MWP**	**GMV**	**MDP**	**MVDE**	**MVPSO**
BTC	0.12	0.13	0.08	0.09	0.09	0.15	0.14	0.17	0.16	0.16
ETH	0.12	0.11	0.09	0.09	0.1	0.17	0.18	0.17	0.16	0.16
XRP	0.07	0.07	0.09	0.08	0.08	0.17	0.18	0.17	0.16	0.17
ADA	0.09	0.08	0.1	0.1	0.1	0.18	0.18	0.17	0.16	0.16
LINK	0.07	0.06	0.09	0.09	0.09	0.17	0.18	0.17	0.16	0.16
LTC	0.11	0.1	0.09	0.09	0.1	0.17	0.18	0.17	0.16	0.16
BCH	0.1	0.1	0.09	0.09	0.09	0.18	0.18	0.17	0.16	0.16
XLM	0.09	0.09	0.1	0.1	0.1	0.18	0.18	0.17	0.16	0.16
BNB	0.09	0.09	0.09	0.09	0.1	0.18	0.18	0.17	0.16	0.16
DOGE	0.06	0.06	0.09	0.07	0.07	0.17	0.17	0.17	0.16	0.16

Market weighted portfolio (MWP); Global minimum variance (GMV) portfolio; Maximum diversification portfolio (MDP), Mean–variance Differential Evolution (MVDE) and Mean–variance Particle Swarm Optimization (MVPSO).; $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset *i* being in a state of distress;$\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset *i*; q = 0.05

Table 18 ΔCoVaR and CoVaR under MWP, GMV and MDP in Portfolio 2

$\Delta CoVaR_q^{p\|i}$						$CoVaR_q^{p\|i}$				
Asset	**MWP**	**GMV**	**MDP**	**MVDE**	**MVPSO**	**MWP**	**GMV**	**MDP**	**MVDE**	**MVPSO**
BTC	0	0	0.01	0.03	0.02	0.06	0.07	0.07	0.07	0.06
ETH	0	0	0.01	0.02	0.02	0.06	0.07	0.06	0.06	0.06
LTC	0.01	0	0.01	0.04	0.02	0.07	0.07	0.07	0.07	0.06
NYA	0.06	0.06	0.05	0.03	0.04	0.07	0.09	0.08	0.07	0.07
IXIC	0.05	0.04	0.04	0.03	0.03	0.09	0.1	0.08	0.07	0.07
GSPC	0.06	0.06	0.05	0.03	0.04	0.07	0.09	0.08	0.07	0.07
N100	0.05	0.06	0.05	0.03	0.04	0.09	0.09	0.07	0.07	0.07
FTSE	0.05	0.07	0.05	0.03	0.03	0.09	0.08	0.07	0.07	0.07
FCHI	0.05	0.06	0.05	0.03	0.03	0.09	0.09	0.08	0.07	0.07
DJI	0.06	0.06	0.05	0.03	0.03	0.08	0.09	0.08	0.07	0.07

NYSE COMPOSITE (NYA), NASDAQ Composite (IXIC), S&P 500 (GSPC), Euronext 100 Index (N100), FTSE 100 (FTSE), CAC 40 (FCHI) and Dow Jones Industrial Average (DJI), Bitcoin (BTC), Ethereum (ETH) and Litecoin (LTC). Market weighted portfolio strategy (MWP); Global minimum variance (GMV) portfolio strategy; Maximum diversification portfolio (MDP) strategy; Mean–Variance Differential Evolution (MVDE); Mean–Variance Particle Swarm Optimization (MVPSO). $CoVaR_q^{p|i}$ is the VaR of the portfolio conditional upon asset *i* being in a state of distress; $\Delta CoVaR_q^{p|i} = CoVaR_q^{p|i} - CoVaR_{0.5}^{p|i}$. It measures the vulnerability of the portfolio to the contagion from tail-risk events of the asset *i; q = 0.05*

Abbreviations

VaR	Value-at-risk
CVaR	Conditional value-at-risk
CoVaR	Conditional value-at-risk
NYA	NYSE composite
IXIC	NASDAQ composite
GSPC	S&P 500
N100	Euronext 100 index
FTSE	FTSE 100
FCHI	CAC 40
DJI	Dow jones industrial average
BTC	Bitcoin
ETH	Ethereum
XRP	Ripple

ADA	Cardano
LINK	Chainlink
LTC	Litecoin
BCH	Bitcoin cash
XLM	Stellar
BNB	Binance coin
DOGE	Dogecoin
GMV	Global minimum variance
MDP	Maximum diversification portfolio
MWP	Market weighted portfolio
MVDE	Mean–variance differential evolution
MVPSO	Mean–variance particle swarm optimization

Supplementary Information

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Additional file 1: Data file.

Author Contribution

JC designed the research, conducted the research and analysed the data, and wrote the article.

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Competing Interests

The author declare that he/she has no conflict of interest.

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