Financial Innovation

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Intelligent option portfolio model with perspective of shadow price and risk-free profit

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Abstract

Since Markowitz proposed modern portfolio theory, portfolio optimization has been being a classic topic in financial engineering. Although it is generally accepted that options help to improve the market, there is still an improvement for the portrayal of their unique properties in portfolio problems. In this paper, an intelligent option portfolio model is developed that allows selling options contracts to earn option fees and considers the high leverage of options in the market. Deep learning methods are used to predict the forward price of the underlying asset, making the model smarter. It can find an optimal option portfolio that maximizes the final wealth among the call and put options with multiple strike prices. We use the duality theory to analyze the marginal contribution of initial assets, risk tolerance limit, and portfolio leverage limit for the final wealth. The leverage limit of the option portfolio has a significant impact on the return. To satisfy the investors with different risk preferences, we also give the conditions for the option portfolio to gain a risk-free return and replace the Conditional Value-at-Risk. Numerical experiments demonstrate that the intelligent option portfolio model obtains a satisfactory out-of-sample return, which is significantly positively correlated with the volatility of the underlying asset and negatively correlated with the forecast error of the forward price. The risk- free option model is effective in achieving the goal of no drawdown and gaining satisfactory returns. Investors can adjust the balance point between returns and risks according to their risk preference.

Keywords: Option portfolio, Linear programming, Deep learning, Risk appetite

Introduction

As of November 2021, China's index options market has maintained a substantial growth this year. The total trading volume of the domestic stock index options market has reached more than 13 billion pieces in 2020 with an annual increase of about 153% (see Fig. 1). Furthermore, the CSRC[1] has put forward six major work priorities for the construction of the financial market, including "improving the field and path of futures and options". The application of index options in the financial market has attracted more

[1] CHINA SECURITIES REGULATORY COMMISSION: supervise the national securities and futures market, maintain the order of the securities and futures market and ensure its legal operation. The abbreviations mentioned in the article are summarized in Appendix 5.

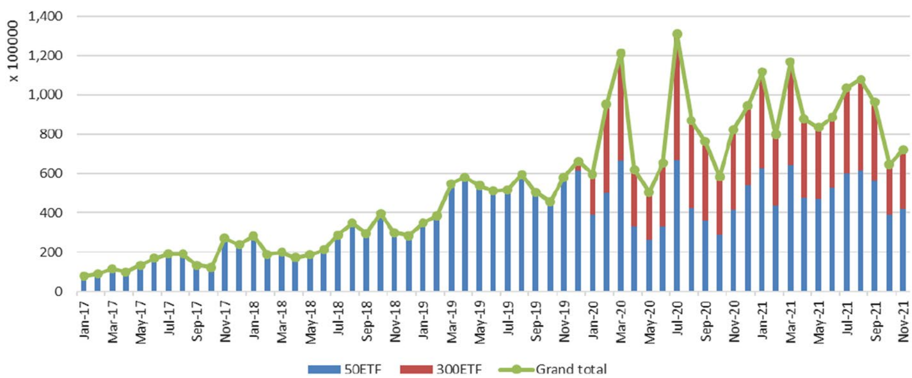


Fig. 1 Monthly trading volume of index options in China

and more attention for financial institutions. How to use this financial tool still has broad research prospects.

Portfolio optimization has been being a classic topic in financial engineering since Markowitz (Markowitz 1952) proposed the modern portfolio theory. Under different objectives and constraints, the optimal capital allocation of financial assets (such as stocks and bonds) has been discussed in the portfolio among a large amount of literature. While it is generally accepted that options are helpful to improve the vitality of financial market, research on how to highlight the characteristics of options and how to use them effectively in the market remains promising. Due to the highly leveraged nature of the options contracts, option portfolios may generate unexpected returns that may outperform the benchmark index. It is difficult to find a stochastic process that accurately reflects the behavior of option return. As a result, the option portfolio has received less attention than the portfolio of stocks or bonds.

The problem of option portfolio originated at the end of the last century. Several scholars studied how to build an effective option portfolio from different perspectives (Korn and Trautmann 1999). Liu and Pan (2003) modelled stochastic volatility and jump processes and derived the optimal portfolio policy of a CRRA investor. Constantinides et al. (2012) studied portfolios made up of either call or put options with a targeted moneyness. In consideration of the non-normal distribution of returns and the short sample of option returns available, Faias and Santa-Clara (2017) proposed a method to optimize a portfolio of European options, held to maturity, with a myopic objective function that overcomes these limitations. Guasoni and Mayerhofer (2020) identified the combination of asset specific option payoffs that maximized the Sharpe ratio of the overall portfolio, which mainly analyzed the optimal solution of the portfolio from the perspective of optimization. Some scholars considered the hedging attributes of options and regarded the options with small or zero risk premium as an important hedging tool to reduce the total portfolio risk (e.g. Han et al. 2015; Wang and Huang 2019; Khodamoradi et al. 2020). Most of the recent studies were considered from the perspective of mispricing to obtain arbitrage (Constantinides et al. 2017) or investment diversity (Yuan and Rieger 2020). In light of the current research progress, the existing studies still lack the description of leverage in the option portfolio. On the other hand, the strategy about the combination of options contracts is not rich enough. For example, some institutions often earn option fees from selling options contracts, which is also a common and simple strategy. Research on the strategy of selling options contracts still needs to be deepened in the model analysis. With the development of options, both call options and put options are applicable to almost all asset classes, including stocks, bonds, commodities, currencies, and relative indexes (e.g.Topaloglou et al. 2011; Yuan and Rieger 2020). For a specific call or put options contract, different execution prices in at-the-money (ATM), in-the-money and out-the-money (OTM) options. Several studies claimed on overpricing in both OTM puts and ATM straddles (Jackwerth 2000). Based on mispricing, Constantinides et al. (2012) built a portfolio that mainly includes short call options, which was particularly profitable in the case of short-term and high volatility. With so many investment opportunities available, capturing the balance between the option income and option fees through a broad and diversified portfolio might bring the possibility of low risk and high return. Therefore, it is necessary to study how to fully utilize options through further research (Merton 1977).

This paper develops a model to find option portfolio that maximizes the terminal wealth in a market where call and put options are available with many strike prices while selling option contract to obtain the option fee. The model is a single-period problem, which reflects the monthly update of a portfolio among the most-active contracts. At the beginning of the period, the data-driven method is used to forecast forward price of the underlying asset, which rely on the historical data of underlying assets. Meanwhile, investors construct the portfolio according to the value of each contract (the balance between the expected return and the option fee). At the end of the contract, the investor collects the option payoffs, then, makes the next round of investment decisions. In addition, this paper proposes a new option portfolio model, which can be adjusted to pursue higher expected returns or arbitrage through different risk constraints.

The main performance of the model is to maximize the final wealth of the portfolio, and the return depends greatly on the judgment of the price of an underlying asset. Since Fama put forward the efficient market hypothesis in 1970, many scholars have extensively demonstrated the efficient market hypothesis from theoretical and empirical perspectives. Its birth and development have been greatly supported in the market, including weak efficient market, semi-strong efficient market, and strong efficient markets. Three types of efficient markets have been verified by different scholars (Fama 1970; Jensen and Benington 1970). However, as the statistical models of empirical studies have gradually matured, some anomalies have emerged in the capital market that are inconsistent with the theoretical studies of the efficient market hypothesis and even difficult to be explained by the classical financial theory. If the anomalies behind abnormal returns can be well explained, the market will become effective or appear arbitrage and speculation. For fore- casting the price of financial assets (especially for stocks and stock market indexes), a large number of scholars have proved the effectiveness of machine learning and applied it to the portfolio (e.g., Wang et al. 2020; Yu and Chang 2020). Therefore, this paper uses the deep learning approach to forecast the forward price of the underlying asset, which is applied as a key parameter to the portfolio model. It is worth mentioning that this paper also provides a risk-free option portfolio model for risk-averse investors to reduce the loss caused by parameter sensitivity. In general, there are three main contributions of our option portfolio model as follows

Table 1 Characteristics of index options daily return

Conf Int		99%	95%		90%		
Option	**Mean**	**LB**	**UB**	**LB**	**UB**	**LB**	**UB**
50ETF option	0.0022	− 0.0034	0.0079	− 0.0021	0.0065	− 0.0014	0.0058
300ETF option	− 0.0017	− 0.0069	0.0036	− 0.0057	0.0024	− 0.0050	0.0017

This table displays confidence intervals for the time series of realized option returns with the confidence interval in 99%, 95% and 90%

- This paper analyzes the statistical characteristics of index options contracts in China, and finds that there is a peak and tail phenomenon in option return. It is not only a prerequisite for an option portfolio to be effectively profitable but also makes the portfolio potentially implicitly risky. In analyzing the return of index options with one-month maturity, it is found that there is a leftward bias in that return and a significant overpricing of options.
- The model is more realistic, incorporating both call and put options at multiple strike prices, and the model allows for the sale of signed contracts for the option fee. The final return of the option portfolio is the balance between the expected return of options and the option fee. Therefore, the model is a general high-dimensional portfolio problem. We use the deep learning method to determine the future price of the underlying index to obtain the expected return of the options contract, and demonstrate that it is a guarantee to obtain the return of our intelligent option portfolio.
- L_1 norm is introduced to control the leverage of the portfolio, which also ensures that the model is bounded. The treatment transforms the L_1 norm into a continuous linear form to analyze the shadow prices of the initial assets, the upper limit of risk tolerance, and the upper limit of portfolio leverage. It is found that the leverage ratio plays an important role in improving the return of the portfolio.

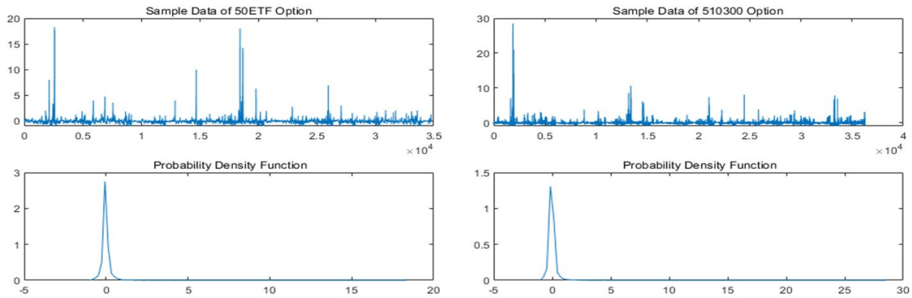
The contents of this paper are structured as follows. Section Model construction introduces the project motivation, supported by a statistical analysis of the option market. In the traditional CVaR model, we put forward an intelligent option portfolio model. Section Model analysis analyzes the shadow prices of the input factors in the model and gives a risk-free condition to satisfy investors with different risk preferences. Section Empirical results conducts an experimental study on the proposed models. Some conclusions are drawn in Section Conclusions.

Model construction

Characteristics of options in China

To start with, we make a statistical analysis of the returns of index options in China to explore some possible profit motives of the market. Table 1 displays the confidence intervals for return of two types options in the time series of 2020, each of which contains nearly thirty-six thousand data points. The selected data are options contracts expiring within one month, which are also the contracts we will use in the model.

From Fig. 2, it can be seen there are obvious fat tail characteristics in the distribution of China's index options yields, which is both an opportunity and a challenge for



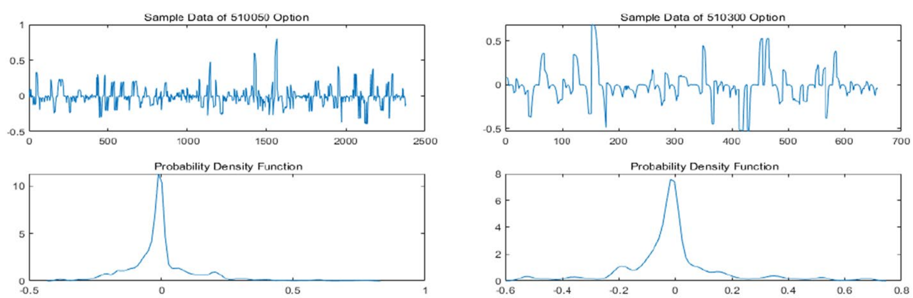
(a) 50ETF option (b) 510300

Fig. 2 The probability density distribution of daily return of 50ETF and 300ETF option

Table 2 Return characteristics of option with one-month hold to maturity

Conf Int		99%	95%		90%		
Option	**Mean**	**LB**	**UB**	**LB**	**UB**	**LB**	**UB**
50ETF option	− 0.0021	− 0.0090	0.0048	− 0.0073	0.0032	− 0.0065	0.0023
300ETF option	− 0.0100	− 0.0283	0.0083	− 0.0239	0.0039	− 0.0217	0.0016

This table displays confidence intervals for the time series of realized option returns with the confidence interval in 99%, 95%, and 90%



(a) 50ETF option (b) 300ETF option

Fig. 3 The probability density distribution of 50ETF and 300ETF option with one month hold to maturity

building an effective option portfolio. It also reflects the failure effectiveness of China option market.

On the other hand, we select one-month maturity index options as the analysis sample (50ETF options issued from 2015 and 300ETF options issued from 2020). The hold-to-maturity yields of options contracts are shown in Table 2 and Fig. 3. The yield to maturity is the difference between the contract income and the option fee. A positive value indicates that the buyer of the contract will make a profit, and conversely the seller of the contract will make a gain with a negative value. It can be seen that the average return of the contract during in one month is negative and the distribution of the maturity yields clearly presents a left-skewed pattern. This also amply demonstrates the existence of mispricing of index options and that it is profitable to sell short options contracts.

Since options are one of the few financial assets that can be shorted in China, the model constructs a portfolio in the risk-free asset and options contracts according to the returns of the Chinese market (but the portfolio assumes that borrowing is not allowed). A major contribution of the proposed model is that the model includes both call options and put options, and options can be bought or sold. These settings make it possible for the model to have extremely high tail returns, but also potentially significant risk. Therefore, based on the return distribution of China's options market, we adopt CVaR to measure the portfolio risk, which is more appropriate for our requirements to prevent extreme risks.

Option portfolio based on CVaR constraint

A single-period portfolio strategy is implemented for a risk-free asset with fixed return r_f, and the number of call options and put options of the index respectively is C and P at different exercise prices. To represent the real market in the largest extent, we allow options to be bought or sold to hedge market risk. A key point of an option portfolio is how to determine the future price of the underlying asset S_{t+1}, as we will discuss in detail in the next subsection.

In addition, for buying options, the exposure to loss is limited, but the risk is infinite for selling options. Some scholars (e.g.Alexander et al. 2006; Xue et al. 2015) have utilized $CVaR\beta(x)$ constraint to minimize the expectation of tail losses in top $100(1-\beta)\%$, which control the loss of uncertainty risk on sold contracts.

Suppose that r is a discrete random variable with G possible scenarios $\{r_g\}_{g=1}^{G}$, where the probability of each scenario is q_g satisfying $\sum_{g=1}^{G} q_g = 1$. Thus, we can calculate $CVaR_{\beta}(x)$ by minimizing an auxiliary function $F_{\beta}(x,\alpha)$. (Rockafellar and Uryasev, 2002), where

$$F_{\beta}(x,\alpha) = \alpha + \frac{1}{1-\beta}\sum_{g=1}^{G}\left\{q_g(-r^T x-\alpha)\mathbb{I}_{[-r^T x\geq\alpha]}\right\}, \tag{1}$$

here, $I_{[\cdot]}$ denotes the indicator function of an event. By introducing slack variables $d_g\left(g=1,2,\ldots,G\right)$, the formula (1) is equivalent to

$$F_{\beta}(x,\alpha) = \alpha + \tfrac{1}{1-\beta}\sum_{g=1}^{G} q_g d_g, x, d_g, \alpha \in \mathcal{X}, \tag{2}$$

where $\mathcal{X} := \left(x, d_g, \alpha\right) \in \mathbb{R}^N \times \mathbb{R}^G \times \mathbb{R} : d_g \geq -r_g^T x - \alpha, d_g \geq 0, g = 1,2,\ldots,G$

In sum, $x_f \in R^+$ is the share invested in risk-free assets. $x_c, x_p \in R^{N+}$ represent the amount of buying or selling call or put options, which are calculated through maximization of the investor's expected utility at each period as follows:

$$\begin{aligned}
&max\ E[U(W_1-OF)]\\
&s.t.\ W_1 = x_f r_f + \sum_{c=1}^{C} x_c C_1 + \sum_{p=1}^{P} x_p P_1\\
&\quad OF = \sum_{c=1}^{C} x_c OF_c + \sum_{p=1}^{P} x_p OF_p,\\
&\qquad x_f + \sum_{c=1}^{C} x_c OF_c + \sum_{p=1}^{P} x_p OF_p \leq W_0,\\
&\qquad F_{\beta}([x_c,x_p],\alpha) \leq \rho,\\
&\qquad [x_c,x_p], d_g, \alpha \in \mathcal{X}
\end{aligned} \tag{3}$$

where W_1 is the total wealth next time, which consists of risk-free return and option returns. OF represents the option fee, greater than 0 meaning the net expenditure of the option fee of the portfolio, and less than 0 meaning the earned option fee. OF_c and OF_p are the premium of call option and put option, respectively. Therefore, the meaning of objective function is to maximize the utility of terminal wealth. $C_1 = max\,(S_1 - K_c, 0)$, $P_1 = max\,(K_p - S_1, 0)$ represent the benefits of different options contracts with striking price K separately. ρ is the upper limit of risk tolerance.

Intelligent option portfolio with short position and leverage constraint

Compared with stock and futures, the option is a more complex but flexible quantitative investment tool. In China's stock market, investors can only gain income in the upward market by their market timing ability. However, whether it is a trending market or a fluctuating market, there are corresponding strategies for the investor to use options contracts wisely to capture profits and control risk in almost all market expectations. For institutional investors, a common trading method is used to earn option fees by selling options contracts. In this paper, we consider the trading behavior of selling options, i.e., we emphasize that the weight vector of the portfolio invested in options is the set of real numbers. This makes our model more flexible and changeable. In such a high vector space composed of multiple options and different execution prices, we will find the balance point between the expected return of options and the cost of options.

On the other hand, since the expected return of selling options contracts may be higher than the return of risk-free assets, this will lead to selling infinite number of contracts to obtain the option fee in exchange for a risk-free return. The solution of the model (3) will also tend to infinity. If the forward price forecast is completely accurate, such a portfolio will certainly be welcomed by investors. But in a more effective financial market, 100% accurate forecast is a near-impossible task. If there is no restriction on the number of purchase and sale contracts, investors will face unlimited risk exposure. Based on this consideration, another contribution of the proposed model is to control the leverage of the option portfolio. Introducing L_1-norm to meet the requirements of investors with different types of risk preference for portfolio leverage, we convert model (3) into vector form as follows

$$
\begin{aligned}
&\max_{X,d,\alpha} R^T X \\
&s.t.\ C^T X \le W_0, \\
&\quad \beta q^T d + \alpha \le \rho, \\
&\quad -AX - d - \alpha \mathbf{1} \le 0, \\
&\quad \| \Theta \odot X \|_1 \le W_0 L, \\
&\quad d \ge 0,
\end{aligned}
\tag{4}
$$

Here, substitute constraint 1 and constraint 2 of the model (3) into the objective function. The utility function in the objective is in the form of linear function, which is expressed as the multiplication of yield and weight. $R = \left[r_f, C_1 - OF_c, P_1 - OF_p\right]'$ is the expected yield vector of options contract. $X = \left[x_f, x_c, x_p\right]'$ represents the investment weight of the portfolio on risk-free assets and options contracts. $C = \left[1, OF_c, OF_p\right]'$ is the investment cost vector. $\Theta = (1, \theta_1, \theta_2, \ldots, \theta_N)$, $\theta_i = S_0 * unit_i$, $unit_i$ denotes the trading unit of the ith options contract. Constraints 3, 4, and 5 are linear forms of CVaR measure.

$|\Theta \odot X|_1 / W_0$ can be regarded as the ratio of the total value of the underlying assets agreed by the portfolio to the initial wealth, i.e., the leverage ratio of the spot-right portfolio. L constrains the upper limit of option portfolio leverage.

The key aspect of the model (4) performance lies in the accurate prediction of the medium and forward price S_1 of the vector $R = g(S_1)$. Because we assume that the options contracts deliver at the realized price until the expiration date, whether option pricing OF has a bias toward the realized price has little impact on the model. The underlying asset of an options contract is an index, which has many convenient features in forward price prediction and portfolio construction. Compared with index options, the underlying asset prices of commodity options are often affected by factors such as season, supply, and demand, which may create new challenges to forecast. There are a variety of methods for predicting index prices, and a large number of scholars have demonstrated the effectiveness of machine learning for such problems and applied it to portfolio problems. Machine learning is often regarded as a black box problem in which the network logic is difficult to explain the economic significance. But it cannot be denied that it gives a new basis for analysis in the time series prediction. In view of the continuous improvement of data availability and computing power in recent years, deep learning has become a fundamental component of the new generation of time series prediction models and achieved excellent results (Gu et al. 2020; Avramov et al. 2021). We use the deep machine learning method to analyze the expected price of the underlying asset, which increases the decision-making ability of the model. Because it is difficult to calculate the yield of each options contract in an options portfolio without the forward price of the underlying asset. The model (4) with the deep machine learning method is described as an intelligent option portfolio. And it can also tell us which risk-free condition should be adopted to build a "gilt-edged, loss-free" portfolio (the specific strategy is reflected in Section Risk-free profit conditions). The motivation of this paper is to construct a class of option portfolio models with realistic constraints, which adapt to multiple methods for analyzing the underlying asset price. Therefore, we did not focus on comparing which type of method is more accurate, and give a commonly used deep learning method. Following the study of Cho et al. (2014), the content of the prediction methods of deep learning is explained in detail in Appendix 1.

After introducing the leverage constraint of intelligent option portfolio in model (4), investors can adjust the upper limit of portfolio leverage according to their personal requirements or their level of certainty about the current market. Leverage is one of the key factors that distinguishes the option portfolio from the general portfolio such as stocks. Reasonable adjustment of portfolio leverage can serve the purpose of amplifying returns or controlling risk for investors. Therefore, exploring how this parameter, the upper leverage limit L, affects the final wealth of a portfolio can help us better understand options investment. To facilitate the analysis and reduce the difficulty of the model, we transformed the L_1 norm by linear equivalence. Problem (4) is reformulated into the following equivalent form

$$
\begin{aligned}
&\max_{X,d,\alpha} R^T X \\
&s.t.\ C^T X \le W_0, \\
&\quad \beta q^T d + \alpha \le \rho, \\
&\quad -AX - d - \alpha \mathbf{1} \le 0, \\
&\quad e^T(2z - \Theta \odot X) \le W_0 L, \\
&\quad \Theta \odot X \le z, \\
&\quad z, d \ge 0,
\end{aligned}
\tag{5}
$$

The reformulation reduces half of the number of inequalities added to the constraints compared with model (4). It is worth mentioning that model (4) and (5) are both linear programming problems, and their dual forms can be derived according to the dual theory. But the dual form of L_1 norm is L_∞ norm, which will increase the analysis difficulty of dual conclusions. We reduce the complexity of the model after dual transformation by performing a linear equivalence transformation of the L_1 norm. This simplifies the original model and provides convenient support for our subsequent analysis of the shadow price of the model.

Model analysis

This chapter mainly discusses some extended properties of the model (5). On the one hand, the model's main factor inputs (capital, risk tolerance, leverage ratio) are analyzed from the perspective of shadow prices and the degree of their marginal contribution, which has implications on how investors allocate their assets and set their preferences. On the other hand, we propose a risk-free option portfolio condition, which guarantees that the expected portfolio return is non-negative. This condition is introduced into the intelligent option portfolio model as a risk constraint.

Shadow price

Option investment is an economic behavior with a high yield and high risk. Investors are exposed to certain risks when obtaining high rates of return. In addition, this risk is magnified by the characteristics of options trading, as one of the main features of options investment is the high leverage. In the case of China 50 ETF options, for example, the leverage of one options contract is about 10 times of total contract value. Therefore, how to measure the value of initial capital, risk tolerance, and preset leverage, or how the distribution of input factors affects the expected return of its portfolio is a matter of great interest and urgency for investors to explore. This section analyzes the shadow price of initial capital, risk tolerance and preset leverage from the perspective of duality theory. Shadow price reflects the scarcity of resources and the marginal contribution of resources to the objective function. It is an important basis for investors to realize a rational allocation of resources.

As input elements, the variables in the model (5) include initial capital W_0, the upper limit of risk tolerance ρ, and the upper limit of portfolio leverage L. According to duality theory, the Dual form of model (5) can be represented as follows

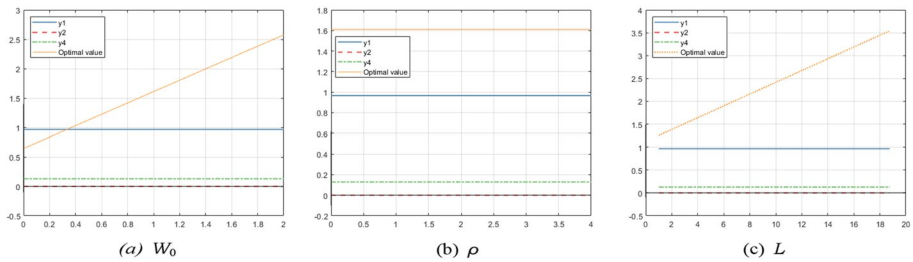


Fig. 4 Shadow Price of W_0, ρ, and L. Panel (**a**) shows how the objective function and three dual variables change when the capital W_0 varies from 0 to 2 at the beginning of the current period. The x-axis is the variation range of input factors, and the y-axis is the value of objective function value and the marginal contribution of y_1, y_2, and y_3. In the same way, Panel (**b**) and Panel (**c**) show the results of ρ and L respectively. When a parameter is changed, the initial value of other parameters is $W_0 = 1$, $\rho = 0.5$ and $L = 5$

$$
\begin{aligned}
& mix W_0 y_1 + \rho y_2 + L y_4 \\
& s.t.\ C y_1 - A^T y_3 - \mathbf{1} y_4 + y_5 = R, \\
& \quad \beta q y_2 - y_3 - y_7 = 0, \\
& \quad y_2 - \mathbf{1}^T y_3 = 0, \\
& \quad \mathbf{1} * 2 y_4 - y5 - y6 = 0, \\
& \quad y_1, y_2, y_3, y_4, y_5, y_6, y_7 \geq 0.
\end{aligned} \tag{6}
$$

where y_1, y_2, ...y_7 denote dual variables, $1 \in R^N$ is an n-dimensional full 1 vector, and the definitions of other variables are the same with model (5). y_1, y_2 and y_4 reflect the marginal contribution of their input factors to the objective function, respectively. In other word, they are the marginal value of total wealth under the optimal utilization of resources Because the optimal value of the objective function of the original problem is equivalent to that of the dual problem, which is the total wealth in the terminal period. y_1, $y_2$and y_4 also represent the marginal contribution of their corresponding elements to the final wealth in the economic sense. Detailed proofs can be viewed in Appendix 2. In this subsection, we consider the case of a single asset, in which, Θ is a single constant value and therefore is not reflected in the model (6).

Figure 4 illustrates how the marginal contribution varies with the objective function when each of the three input factors is changed. Combining Panel (a) and (b), it can be seen that the shadow prices (marginal contributions) of the three factors are approximately constant when the initial capital W_0 and risk tolerance ρ are increased, respectively. The value of y_1 is close to 1, which is the largest marginal contribution to the objective function among the three factors. It indicates that increasing 1 unit of initial investment will increase the same value of final wealth. This is also consistent with the economic explanation because the objective function represents the final wealth, and the initial wealth input most directly affects the final wealth. The second-largest marginal contribution is the upper limit of portfolio leverage L. The factor with the smallest marginal contribution is the risk tolerance of CVaR, which is close to 0. This result exceeds our expectations, but it also reflects the large number of lower-risk profit opportunities that exist in the Chinese index options market. Under the assumption, investors will not receive additional final wealth as a result of the increased risk tolerance.

The model (5) is a high-dimensional linear model. The obtained portfolio is the optimal solution in the current market environment, which also means to find a portfolio that maximizes the final wealth in the sample. The model allows for the purchase of options contracts to obtain exercise rights and also the sale of options contracts to earn option fees. The portfolio may be trapped in an infinite cycle of buying options contracts at the initial moment with option fees from selling. If the market liquidity is sufficient and the predicted forward price value is accurate, the return exposure of the method is the result of convergence to positive infinity. Therefore, the ability to magnify this return and reasonably control risk depends on another key element, the upper limit of portfolio leverage. In contrast, the upper limit of portfolio leverage ratio is more likely to generate wealth for investors, as reflected in Fig. 4. The leverage ratio of L starts at 1, indicating that the initial wealth is equal to the value of holding risk-free assets and options contracts. The situation is the same as that shown in panel (a) and (b), and the initial wealth is the main influencing factor. The leverage ratio remains a key parameter affecting the final wealth, maintaining a marginal contribution of about 0.2. This means that raising the leverage limit by 1 unit increases the final wealth by 0.2 units of value.

Risk-free profit conditions

Section Characteristics of options in China statistically analyzes the distribution of return of major financial options in options market of China. It is found that anomalies occur frequently and the distribution of returns is very asymmetric, which greatly improves the possibility of the deviation between the price and value of options contracts. Is there a condition that makes the expected return of the option portfolio model non-negative, and is there such an opportunity of" making a steady profit without loss" in the options market of China?

Dert and Oldenkamp (1997) proposed optimal guaranteed portfolios, which are composed of a single stock index and several European options that can control the maximum loss. The nature of this risk-free model is that the expected loss is almost 0, but the return may be very attractive. In model (5), there is a risk-free asset in the assets of the intelligent option portfolio. It ensures that the expected return is not lower than the risk-free return from the perspective of the model objective function. Proposition 1 reveals the inequality relationship that option portfolio weights should satisfy different forward prices S_1 in relation to the strike price K.

Proposition 1: Risk-free Profit Conditions

If the expected return on the option portfolio is guaranteed to be non-negative, the call option weight x_c and the put option weight x_p should satisfy

$$|S_1 - K|^T(x_c + x_p) \geq (K - S_1)^T(x_c - x_p) + 2(OF_c + OF_p)^T(x_c + x_p). \tag{7}$$

The inequality is equivalent to: if $S_1 > K$, then

$$x_c \geq \left(\frac{OF_c + OF_p}{S_1 - K - OF_c - OF_p}\right)^T x_p, \tag{8}$$

if $S_1 < K$, then

$$x_p \geq \left(\frac{OF_c + OF_p}{K - S_1 - OF_c - OF_p}\right)^T x_c. \tag{9}$$

From the proposition above, we determine the conditions under which the expected return on an options portfolio is non-negative for a different relationship between forward price and strike price. This condition can also be regarded as a way to control portfolio risk, as it controls the minimum return of the portfolio. Therefore, the risk measure of CVaR is replaced by the risk-free condition in Proposition 1. The original model (4) is transformed into the model as follows called the risk-free option model

$$\begin{aligned}
&\max_{X,d,\alpha} r_f x_f + (C - OF_c)^T X_c + (P - OF_p)^T X_p \\
&s.t.\ OF_c^T X_c + OF_p^T X_p + x_f \le W_0, \\
&\quad x_{c,i} \ge \frac{FC + FP}{S_1 - K - FC - FP} x_{p,i} \quad if\, S_1 \ge k_i, \\
&\quad x_{p,i} \ge \frac{FC + FP}{K - S_1 - FC - FP} x_{c,i} \quad if\, S_1 < k_i, \\
&\quad \| \theta_c \odot X_c \| + \| \theta_p \odot X_p \| \le L,
\end{aligned} \tag{10}$$

where $X_c = \left(x_{c,1}, x_{c,2}, \ldots, x_{c,N}\right)$ presents the weight vector of N call options contracts, $X_p = \left(x_{p,1}, x_{p,2}, \ldots, x_{p,N}\right)$ is the weight vector of N put options contracts.

The constraint formed by Proposition 1 controls the maximum loss of the model (10). Compared with model (4), the risk control based on CVaR is replaced by the constraint conditions. Of course, S_1 remains a crucial parameter in the model (10). When the forecast deviates, the option portfolio still fails to obtain a non-negative return. However, based on the judgment of history, it can be concluded that the monthly price fluctuation of the underlying asset will be within a range of [L, U]. With the monthly volatility of the underlying asset σ, we set the forward price range of the underlying asset as $[S_0(1 - \varphi * \sigma), S_0(1 + \varphi * \sigma)]$. ϕ reflects investors' risk preference. The larger the value, the less confidence the investors have in the future direction of prices. The range is partitioned to obtain a finite M number of possible forward prices. According to Proposition 1, M forward prices are converted into M constraints, which ensures that the return of the model (10) is non-negative in the range.

Empirical results

In this section, the out-of-sample performance of the models (5) and (10) are evaluated based on the data of 50ETF options and 300ETF options in the option market of China. In the case of single option asset, investors only allocate assets on risk-free assets and 50ETF options contracts. 50 ETF options have been around for a long time and there is more data available for analysis. While comparing the out-of-sample performance, the return distribution and regression analysis of the option portfolios are used to explain the model's superior performance. To further illustrate the characteristics of the model and reduce the instability in parameter estimation, 300 ETF options are introduced to investment on a single-asset basis.

Data

The return of the 50ETF index from February 2005 to October 2021 is used to simulate the underlying asset process, with data from the Wind database. 50ETF options contracts expire on the fourth Wednesday of each month. Options are settled in cash on

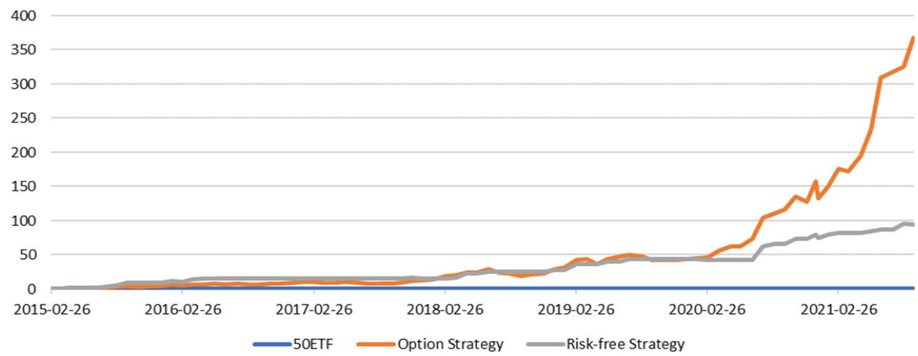


Fig. 5 Cumulative wealth of 50ETF, intelligent option model, and risk-free model. *Note*: This figure shows the change of accumulated wealth from 2015–02-26 to 2021–09-23, and adjusts each portfolio 80 times. The initial wealth is 1. Since the fluctuation of 50ETF is relatively small compared with the other two strategies, the change of cumulative income is not obvious. The maximum value during this period is 1.4640 and the minimum value is 0.7519

the business day after expiration. Our asset allocation uses risk-free assets and index options with a variety of different strike prices. The majority of trading activity in 50 ETF options are concentrated in the closest contracts less than 30 days from expiration, so the options contracts have high liquidity. Holding options until maturity will only incur transaction costs at the beginning of the transaction. We select options that expire in one month and establish the portfolio 80 times on the entire dataset with monthly investment frequency.

In the numerical experiment, we assume the transaction cost of 0.5%, the initial capital $W_0 = 1$, the upper limit of risk aversion $\rho = 0.5$, the upper limit of leverage $L = 5$. Regarding the setting of L, two main facets are considered. One aspect is the real situation of the options market. Usually, the leverage of financial option contracts is 10 times, but considering the margin of about 15%, the leverage of individual option contracts is about 5 times. In addition, a parametric sensitivity analysis was also performed for L. The detailed results are presented in Appendix 4. Note that we assume sufficient margin in our numerical experiments and do not take margin into account in the cost versus return.

Performance of intelligent options model under single asset

In this section, the performance of the index option portfolio is compared under a single underlying asset. Here we take the 50ETF option as an example (because 50 ETF options were issued earlier, there is a longer data set), which provides a sufficient sample size to perform an imputation analysis of the intelligent option portfolio returns.

The sample during the investment period is from 2015 to 2021, and the cumulative yield is shown in the Fig. 5. Table 3 describes the performance of the intelligent option model, risk-free model (the risk preference parameter $\phi = 1.5$ for no loss), the underlying assets and the classical option portfolio model. Overall, the intelligent option model has a higher average return, but it is also very volatile. The maximum drawdown ratio reaches 0.2901. The reason for this loss stems mainly from the deviation of the predicted price from the realized price. The model performs very well within the sample and even shows the possibility of an exponential increase in total assets in one month.

Table 3 Performance of 50 ETFs and different option models

	Mean	Std	Sharpe Ratio	MDD	Max	Min
50ETF	0.0064	**0.0733**	0.0653	0.1252	0.2446	− 0.2890
Intelligent option model	**0.0838**	0.1525	**0.5385**	0.2742	0.7634	− 0.1647
Risk-free model	0.0685	0.1734	0.3856	**0.0368**	**1.2326**	**0.0000**
CRRA option model	0.0731	0.1505	0.4746	0.2611	0.6763	− 0.2039
CVaR option model	0.0599	0.1423	0.4095	0.2178	0.7424	− 0.1430

This table displays the out-of-sample performance of 50ETF index and different option strategies with one month adjustment frequency. The performance of different strategies is judged from six dimensions (mean, variance, Sharpe ratio, maximum drawdown, maximum and minimum monthly return), and the bold content indicates the best numerical result

Although the maximum drawdown of the intelligent option strategy is approximately 1.2 times that of the 50 ETF. The average monthly return of the intelligent option portfolio is approximately 13.73 times that of the 50 ETF, and the Sharpe ratio is 8.10 times that of the 50 ETF. Of course, this does not completely eliminate the fears of the risk averse. Therefore, the paper also proposes a risk-free option strategy, which is also designed to balance the high-return, high-risk nature of the intelligent option strategy. Both the intelligent option model and risk-free model have higher Sharpe ratios than the underlying asset, the 50 ETF. Although the volatility of 50ETF exhibits minimal volatility, it is worth mentioning that the fluctuation of the risk-free model is entirely upward, which is pleasing to the investors. This is also reflected in the maximum drawdown of 0.0368 (a value of 0 when transaction costs are not taken into account).

We further illustrate the advantages of intelligent portfolio models from both classical option portfolio models and forward price analysis. In the model perspective, we compare with the CRRA utility function for the option portfolio strategy5 (Faias and Santa-Clara 2017) and the CVaR option portfolio strategy5 (Alexander et al. 2006). The predicted values of the forward prices of the above strategies are derived from the GRU method, which is the same as the intelligent portfolio model. In the Chinese financial derivatives market, the classical CRRA utility function model and the CVaR model still achieve good portfolio performance. In comparison, t the intelligent portfolio significantly outperforms the two classical portfolios in terms of mean return and Sharpe ratio. Specifically, the returns of the intelligent portfolio are 13.26% and 28.87% higher, and the Sharpe ratios are 11.92% and 23.99% higher, respectively. It is worth noting that we don't attempting to compare the risk-free option model with the classical option portfolio strategy. The two types of models have different motivations and are suitable for different types of investors.

One reason for the superior performance of the intelligent portfolio model is its leverage constraint, which is the L_1-norm constraint. This constraint makes the model sparse and concentrates portfolio weights. This may increase the volatility of the model but generally improves the Sharpe ratio. This response to the portfolio results may increase the volatility of the model, but generally the Sharpe ratio is also improved. Moreover, our proposed option portfolio model permits the selling of option contracts. By selling option contracts, the cost of constructing an options portfolio can be used to expand its size. This approach increases the return and risk

Table 4 Comparative of results under different price analysis methods

	Mean	Std	Sharpe Ratio	MDD	Max	Min
50ETF	0.0064	**0.0733**	0.0653	**0.1252**	0.2446	− 0.2890
Intelligent option model	**0.0838**	0.1525	**0.5385**	0.2742	**0.7634**	− 0.1647
Monte Carlo	0.0411	0.1423	0.2771	0.4506	0.3434	− 0.3883
Volatility correction	0.0429	0.1496	0.2755	0.4286	0.5523	− 0.4083

This table shows the results of the option portfolio model under different forward price predictive methods. The portfolio is held to maturity and the adjustment frequency is monthly

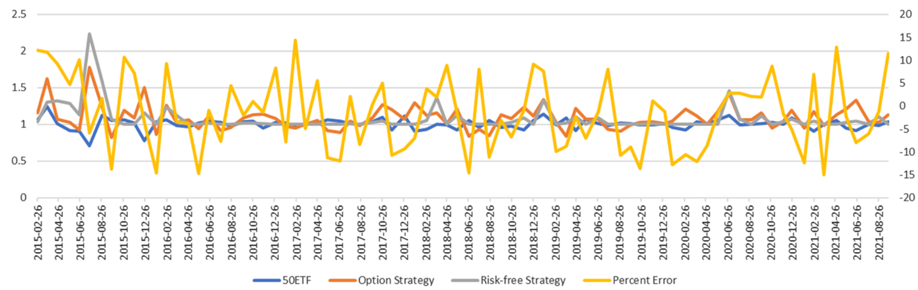


Fig. 6 Out-of-sample monthly return of different model and prediction error. *Note*: This figure shows the monthly yield of the underlying asset 50ETF, intelligent option model and risk-free model from 2015 to 2021. The yellow line indicates the prediction error. The ordinate on the left is the average monthly rate of return, and the ordinate on the right is the prediction error

exposure of the portfolio under the constraint of the target leverage ratio. Selling option contracts is an effective method for achieving higher returns if there is little deviation in the judgment of the future forward price of the underlying asset. To demonstrate the robustness of the model, we next compare the impact of different methods of forward asset price analysis on the model.

We consider the classical model Carlo simulation approach and the volatility correction approach5 (Faias and Santa-Clara 2017). The two traditional analytical methods are brought into the portfolio model proposed in this paper and compared with the intelligent portfolio model. The empirical results in Table 4 continue to compare the mean, standard deviation, maximum and minimum values of the portfolio return, the Sharpe ratio and the maximum drawdown ratio of the portfolio (the bolded font indicates the best results).

When the traditional forward price analysis method is substituted for the machine learning method, the return and Sharpe ratio metrics of the option portfolio model decrease to an extent. The option portfolios under the two traditional methods performed similarly, with the average value of return above 4%. Overall, the results of the Intelligent Portfolio outperformed both in terms of return and Sharpe ratio. The main reason for this result is the forecast errors of the different methods. The substitution of forecasting methods also exposes the parameter sensitivity of the model with respect to forward prices. In terms of root mean square error, mean absolute error, and mean absolute percentage error, the GRU method exhibits relative advantages over the traditional method for forward price prediction. The results are supplemented in Appendix A.

Figure 6 displays the returns and the forecast errors per period for both models. We can get two preliminary inferences. First, when the 50ETF option is just launched, the underlying index's returns are more volatile, while the two option portfolios possess

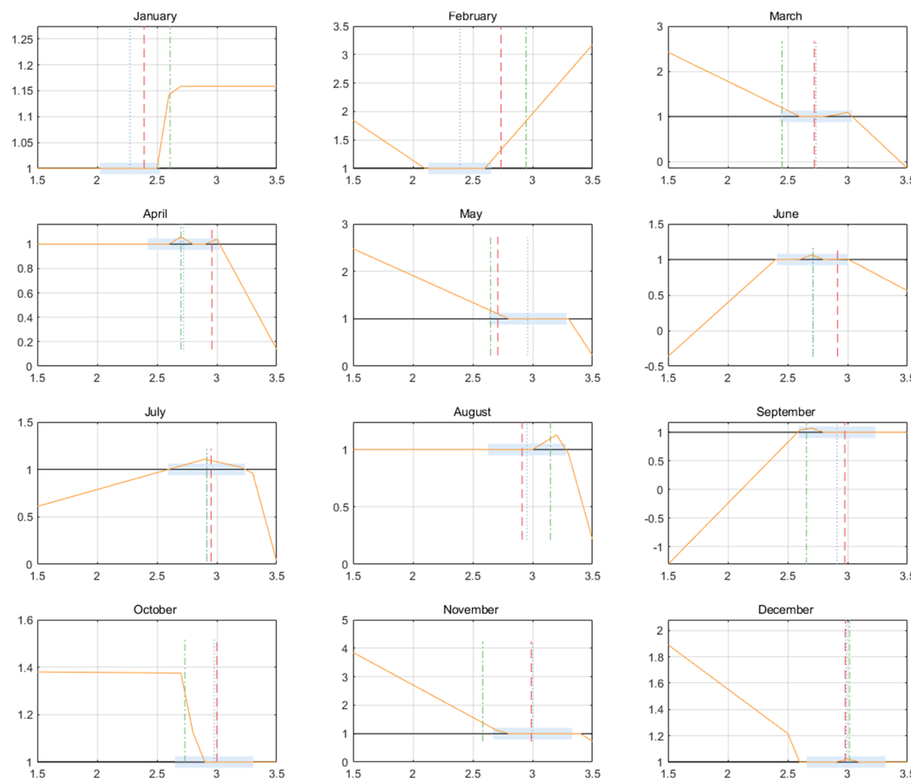


Fig. 7 12-month risk-free model terminal wealth. Note: The X-axis of charts is the forward price of the underlying assets, and the Y-axis is the expected wealth in the next period. The red line is the real forward price of the underlying asset. The green line is the predicted forward asset price. The x-axis covered by the blue block means the risk-free range

the higher return. In addition, forecast errors overall also have a significant impact on the returns of the model. This conclusion is also consistent with the nature of the model, because the predicted forward price is an important parameter affecting the weight of the option portfolio. We give a further explanation in the next subsection.

To visually explain how the risk-free model achieves the continuous rising trend, Fig. 7 presents the expected return of the portfolio in 12 portfolios throughout 2018 with forward prices within the range [L, U].

The numerical experiment assumes that the initial wealth of each period is 1. It can be seen that the expected wealth of the risk-free model stays above the initial wealth regardless of the price movement of the underlying asset. In other words, no matter how the underlying assets change, the return of a risk-free portfolio is always non-negative. The amount of yield in each period mainly depends on two aspects. One is the accuracy of the prediction method. In the 12 periods shown in Fig. 7, the return from executing the portfolio's contracts at the forecast price is higher than the return from executing at the realized return. On the other hand, the existence of contracts with price deviations allows the model (10) to construct a portfolio with an expected high return. It is also a key factor to obtain returns. For example, although the forecast deviation in February is relatively large, with a relative deviation of 7.65%.

Table 5 Summary statistics of option variables

	Vol	Err	IV	EL	Delta	Gamma	Vega	Theta	Rho
Mean	0.0691	0.0697	0.2567	− 0.1386	− 0.1502	− 0.6070	− 0.0784	0.1337	− 0.0441
Median	0.0561	0.0747	0.2625	− 0.6038	− 0.7318	− 0.4884	− 0.0618	0.1044	− 0.1532
Std	0.0420	0.0434	0.2188	0.8345	0.9062	1.3628	0.1844	0.3355	0.2111
No. Obs	80	80	80	80	80	80	80	80	80

This table provides summary statistics for the sample of option returns and factors used in the analysis. The sample period is 2015 to 2021. For option variables the table shows the yield and volatility of the underlying asset, the prediction error of the forward price, embedded leverage, Black–Scholes implied volatility (IV), option Delta, option Gamma, option Vega, option Theta and option Rho

However, because the portfolio in February may obtain huge expected returns, the real risk-free model finally produced a yield of 32.83% in February.

Regression analysis

The intelligent option model has a surprising performance, which makes us very curious about the potential driving factors. We explore the source of the return of the intelligence option model through the method of factor analysis. The explained variable is the return of the intelligent option model. The core explanatory variables include the volatility of the underlying asset and the forecast error. For other explanatory variables, we refer to some option factors mentioned by Buechner and Kelly (2022) and Kang and Kwon (2020). The time series regression is as follows

$$r_t = \alpha + \beta X_t + e_t, \tag{11}$$

where X indicates the different explanatory variables, including realized volatility, absolute value of predictive error, implied volatility (IV), embedded leverage (EL), and five Greek value of options.

A total of 80 observations are made for each variable, shown in the Table 5. The data in Table 4 are consistent with the out-of-sample time of the portfolio and cover monthly data from 2015 to 2021. *Vol* is the realized volatility of the futures market and is calculated from the weighted average of the square of the difference between the possible value and the expected value during time interval t. *Err* represents the absolute value of predictive error based on deep learning in time t. IV and EL are implied volatility and embedded leverage respectively, and refer to (Buechner and Kelly (forthcoming); Kang and Kwon 2020) for calculation.

Table 6 reports corresponding results for options markets from 2015 to 2021. Considering the relation with the intelligent option portfolio return, the observation conclusion in the previous subsection is confirmed.

Regression (2) shows that the volatility of the underlying asset is significantly and positively related to the return on our intelligent option portfolio. The more volatile the market, the higher the model's return. This also reflects the relationship between the options contract and the underlying asset. The greater the price fluctuation of the underlying asset, the more difficult it is to maintain the relationship between the option price and its expected rate of return under the no arbitrage pricing theory. This provides more profit for our intelligent option model. Regression (3) gives a significant negative correlation

Table 6 Regression of option returns on option characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
R	− 0.4520										− 0.1475
	(− 1.80)										(− 0.62)
Vol		1.4318									1.4281
		(3.45)									(3.37)
Err			− 1.1525								− 1.9285
			(− 2.80)								(− 5.00)
IV				0.0005							0.0617
				(0.01)							(0.88)
EL					0.0591						0.1208
					(2.75)						(1.98)
Delta						0.0370					0.2420
						(1.83)					(3.25)
Gamma							0.0223				0.0285
							(1.65)				(0.99)
Vega								0.1942			− 0.1582
								(1.96)			(− 0.50)
Theta									− 0.0980		− 0.1412
									(− 1.79)		(− 1.44)
Rho										0.0949	− 0.8711
										(1.08)	(− 2.60)
Constant	0.0958	− 0.0060	0.1732	0.0928	0.1011	0.0984	0.1064	0.1081	0.1060	0.0971	0.0271
	(5.23)	(− 0.18)	(5.57)	(5.13)	(5.61)	(5.32)	(5.30)	(5.47)	(5.39)	(5.14)	(0.17)
R^2	4.00%	13.22%	9.13%	0.01%	8.86%	4.10%	3.36%	4.68%	3.94%	1.46%	50.70%

The dependent variables are the monthly return of intelligent option portfolio. Regression (1) to (10) are univariate regression with intercept term. Regression (11) is the regression result for all independent variables. The sample of regressions is from 2015 to 2021

Table 7 The relationship between risk preference and out-of-sample performance of risk- free model

	Mean	Std	SpR	MDD	Max	Min
$\phi=0.25$	0.2671	0.5184	0.5121	0.8761	0.3614	− 0.8711
$\phi=0.5$	0.1909	0.4056	0.4665	0.7776	0.3029	− 0.7533
$\phi=1$	0.1148	0.2984	0.3791	0.4986	0.2742	− 0.4910
$\phi=1.5$	0.0685	0.1734	0.3856	0.0368	0.2232	0.0000
$\phi=2$	0.0462	0.1356	0.3285	0.0408	0.1953	0.0000
$\phi=2.5$	0.0381	0.1081	0.3371	0.0646	0.1621	0.0000
$\phi=5$	0.0250	0.0690	0.3374	0.0646	0.1328	0.0000

This table displays how the forward price range $[S_0(1-\varphi*\sigma), S_0(1+\varphi*\sigma)]$ of the underlying asset affects the risk-free model under different investor preferences ϕ. The out-of-sample performance is mainly compared from the mean, standard deviation, Sharpe ratio, maximum drawdown rate, maximum and minimum return

between prediction error and intelligent option portfolio return. This conclusion is consistent with our realistic cognition that forecast accuracy is critical to the return of the intelligent option portfolio. The exposure of expected return is also enlarged for the investors or forecast methods that can better capture the market trend and the price of underlying assets This is why we introduce deep learning to predict the underlying asset price and thus improve the model return. But R^2 of the two regressions are not ideal enough, indicating that some of the key explanatory variables are missing.

Finally, the R^2 of regression (11) reaches 50.70% after the inclusion of all control variables. The results show that volatility of the underlying asset also has a positively significant coefficient of 1.4281 with a *t*-statistic of 3.37, which means that a one-unit increase in volatility of underlying assets raises expected returns by 1.4281 per month after excluding transaction cost. Similarly, the coefficient for prediction error is statistically negatively significant at –1.9285 (*t*-statistic = –5.00). The embedded leverage also has a significant positive effect on the return, with a coefficient of 0.1208 that approximates the results we obtained at shadow prices. In addition, the *Delta* and *Rho* values of the portfolio also have a significant impact on the yield. High-yielding option strategies prefer high *Delta* and low *Rho* values.

The impact of forward price range on risk-free models

In this subsection, we compare the impact of the size of the risk-free range on returns. The risk-free range implies that when the forward price of the underlying asset is within this range it will not bring losses to the investors. If the range is too large, the constraints of the model may be too strict reducing the possibility of return. While a small range may make the forward price of the underlying asset jump out of the range, thus bringing losses. Therefore, how to set an appropriate range still needs to be selected according to investors' confidence in the market and their risk preferences.

Table 7 describes the underlying assets and the out-of-sample performance of risk-free option portfolios with different risk preferences ϕ from 2015 to 2021. Risk preference controls the risk-free range $[S_0(1-\phi^*\sigma), S_0(1+\phi^*\sigma)]$ of the option portfolio. It is observed that the larger the value of ϕ, the larger the risk-free range, corresponding to the stricter constraints of the model (10). ϕ has a clear effect on the performance of the portfolio over this 7-year sample horizon. An obvious positive correlation between the

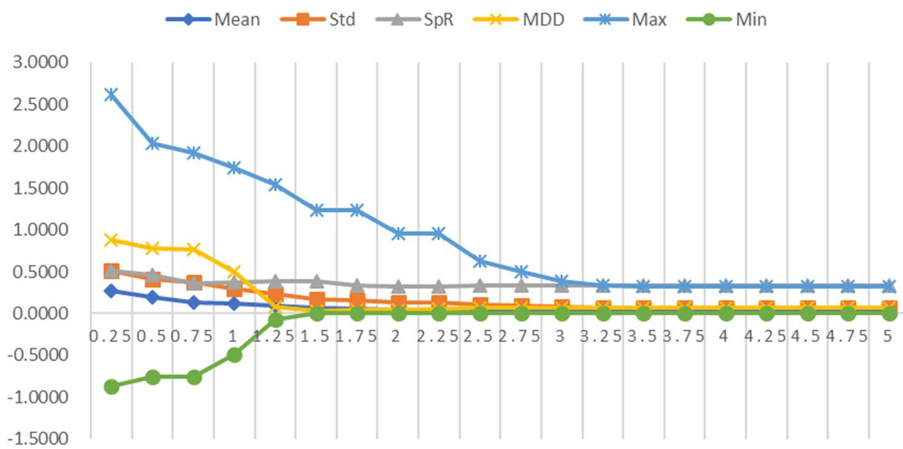


Fig. 8 The relation between the risk preference ϕ and out-of-sample performance. Note: This figure plots the impact of the change of investor's risk preference ϕ on the model (10). The x-axis is the parameter $\phi \in [0.25, 5]$, and the value interval is 0.25. There are 20 groups of data in total. The y-axis is the value corresponding to different indicators

average monthly return and the maximum monthly return in the sample period. For the standard deviation and minimum return within the full sample, they decrease with the increase of ϕ. This result is consistent with our theoretical expectation, since the expansion of the risk-free range necessarily leads to an increase in the constraints of the model. In terms of the in- sample performance of the model, an increase of ϕ necessarily results in a smaller feasible domain for the model (10), which in turn leads to a suboptimal solution.

Figure 8 further depicts the trend of risk preference on model performance. The performance of the model stabilizes after a single ϕ greater than 3. However, for the actual investment situation, a smaller ϕ value doesn't guarantee that the model can achieve the risk-free purpose. The maximum drawdown rate presents a decreasing and then increasing trend for ϕ. After the ϕ value is greater than 1.5, the minimum return of the portfolio is 0. The maximum drawdown rate at this point is 3.68%. Because we assume that the model has a transaction cost of 0.5% per portfolio construction, maintaining a return of 0 in the long run also results in a small drawdown ratio. This value is about 6.46% after model stabilization. Although the Sharpe ratio gradually decreases as ϕ increases in the results, the portfolio with the maximum drawdown of 0 may generate more temptation for risk-averse investors.

Performance of options portfolio model under multiple asset

We now compare the impact of multiple underlying assets versus a single underlying asset on the intelligent option model in the Chinese market. 300ETF options contracts are included in our portfolio. Because 300ETF options have been available for less than two years, a total of 20 option portfolios have been constructed in the next experiment of the dataset since 2020. The model (5) needs to forecast the forward price of the underlying assets. Increasing the number of underlying assets can theoretically increase the stability of the model. An increase in the number of forecast

Table 8 Performance of 50ETF, intelligent option model, and risk-free model under multi-asset

Index and model		Mean	Std	SpR	MDD	Max	Min
Underlying asset	50ETF	0.0058	0.0525	0.0782	0.1703	0.1271	− 0.0882
	300ETF	0.0101	0.0513	0.1651	0.1042	0.1369	− 0.0731
Intelligent option model	Multi-asset option model	0.1267	0.1468	**0.8519**	0.1497	0.4252	− 0.1447
	50ETF option model	0.1066	**0.1258**	0.8339	**0.0522**	0.4252	− 0.0472
	300ETF option model	**0.1889**	0.3065	0.6109	0.4135	0.9986	− 0.2909
Risk-free model	Multi-asset risk-free model	0.0557	0.1358	**0.3980**	**0.0149**	0.5798	0.0000
	50ETF risk-free model	0.0397	**0.0987**	0.3856	0.0248	0.4563	0.0000
	300ETF risk-free model	**0.0602**	0.1750	0.3343	0.0199	0.8021	0.0000

This table displays the out-of-sample performance of benchmark index, multi-asset option model, 50ETF option model, and 300ETF option model with one month adjustment frequency. For risk-free model setting, the experimental design is the same as subsection 4.2. The performance of different models is judged from six dimensions (mean, variance, Sharpe ratio, maximum drawdown, maximum return and minimum return), and the bold content indicates the best numerical result

forward prices of the underlying assets leads to a convergence of the average forecast error of the forecast methods, reducing the impact of extreme forecast errors.

Table 8 describes the out-of-sample performance of the intelligent option model and risk-free model under multiple assets from 2020 to 2021 and provides an experimental control for the single-asset case. Although the average monthly return of the 300ETF option model is higher, it also more volatile. Its maximum monthly loss reaches 29.09%, and the maximum drawdown rate is 41.35%. The multi-asset model enables the model (5) to have return and volatility between the two single underlying asset models and obtain a higher Sharpe ratio. For the intelligent option model, the result is in line with our expectation, achieving the purpose of increasing the underlying assets and making the intelligent option model more robust. The multi-asset model reduces the impact of extreme deviation in the predicted value of a single asset or sudden black swan event on the return of the portfolio.

The multi-asset risk-free model can attain the requirements of no loss and a higher Sharpe ratio. The maximum rate of return has also improved, indicating that there are fewer investment periods with a return of 0. The greater number of options contracts will increase the investment opportunities of the risk-free model, which in turn increases the return of the strategy. We try to explain the results of the multi-asset model through Fig. 9.

On the one hand, multi-asset may increase the maximum expected return of the model because of the access to increased investment opportunities. On the other hand, since multi-asset risk-free model needs to keep the final wealth larger than the initial wealth in the ranges of $[S_{50}(1-\varphi*\sigma_{50}), S_{50}(1+\varphi*\sigma_{50})]$ and $[S_{300}(1-\varphi*\sigma_{300}), S_{300}(1+\varphi*\sigma_{300})]$. This causes the M constraints for risk-free in the model (10) to become M^2. For all models, the option portfolio weight is based on the results in-sample and greatly affected by the forecast forward price. This also makes the final wealth of the risk-free model and the distance between the realized price and the forecast price negatively correlated. The single asset risk-free model only needs to ensure that a forecast forward price is as close as possible to the realized maturity price. The multi-asset risk-free model needs to guarantee that the error between multiple forecast prices and realized values is as small as possible. This requires more time to solve the multi-asset model and more accuracy in the forecast method. In general, both the

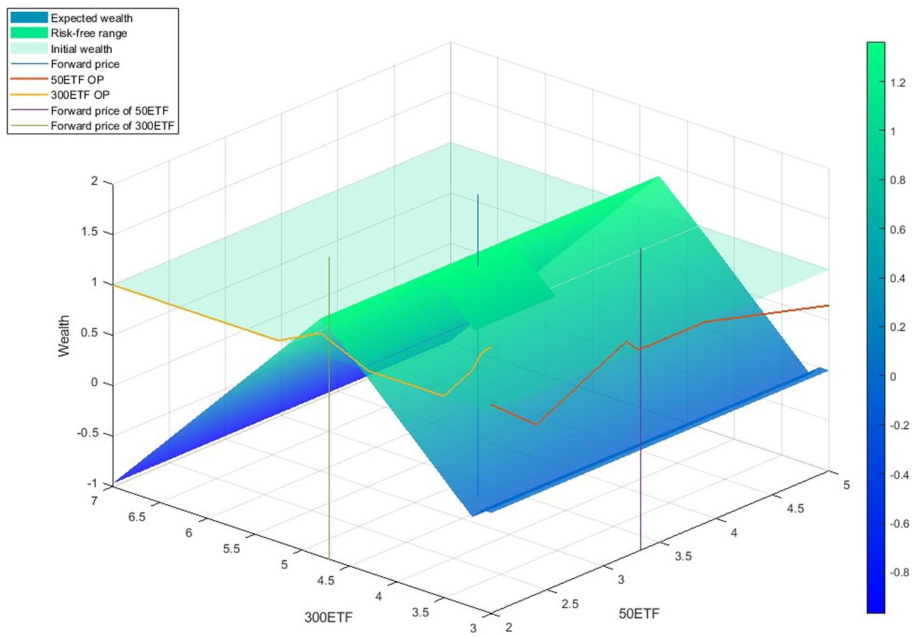


Fig. 9 The relationship between underlying asset prices and expected wealth of the risk-free model. Note: This graph depicts the effect of forward price changes on the expected wealth of the risk-free model. The curved surface is the expected wealth value of the multi-asset risk-free model. The dark green surface is the risk-free range of the model. The light green color indicates the initial wealth for comparison purposes. The orange and yellow lines are the expected wealth of the 50 ETF and 300 ETF option portfolios, respectively. The vertical line indicates the price of the underlying asset at the final exercise. The blue line then shows the position of the final wealth under the multi-asset model

intelligent option model and risk-free model bring their volatility between the volatility of the two single asset strategies. The number of times that the multi-asset risk-free model gain returns also increases significantly. Therefore, multi-asset option model can successfully build a stable portfolio.

Conclusions

This paper proposes a class of intelligent option portfolio models in a market where multiple call and put options with multiple strike prices are available. While allowing the option to be sold for a premium, we consider the highly leveraged nature of options and find option portfolios that maximize terminal wealth. The deep learning approach is applied to forecast the key parameter in the model—the forward price of the underlying asset. The dual theory is used to analyze the shadow price of the initial asset, the upper risk tolerance limit, and the upper portfolio leverage to the final wealth. A risk-free condition is given for the option portfolio to obtain returns without drawdown and combines it with our intelligent option portfolio model in place of the CVaR risk constraint. This risk- free strategy is a good application of the hedging features of options and offers investors with different preferences more choices. Strictly speaking, the two constraints are not substitutes, although to some extent they both express a risk constraint. CVaR is more focused on controlling risk when extreme risks occur and is a formalized approach to risk control. Risk-free strategies provide a more intuitive and simple approach to risk control for investors with different risk appetites. Numerical experiments demonstrate

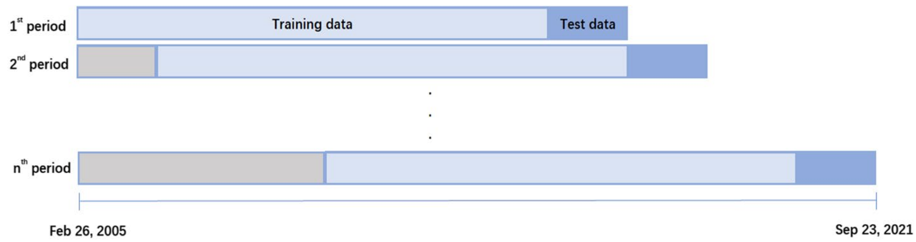


Fig. 10 Sliding window method for predicting the forward price

that the proposed intelligent option portfolio model can effectively obtain returns based on the mispriced options contracts and the peak and fat tail of options contract returns in the market of China. This out-of-sample return is positively correlated with the volatility of the underlying asset and negatively correlated with the forecast error of the forward price of the underlying asset. The risk-free model realizes the goal of no drawdown without considering transaction costs. Expanding the range of the underlying assets of the intelligent option portfolio model increases the Sharpe ratio and robustness of the intelligent option portfolio model.

Appendix

Appendix 1: Prediction method of forward price of underlying assets

The Gated Recurrent Unit (GRU) is a generation of Recurrent Neural Networks (RNN) and is very similar to an LSTM. To solve the vanishing gradient problem of a standard RNN, GRU uses the update gate and reset gate. These are two gates that decide what information should be passed to the output. These two gates can be trained to keep information from many time steps before the actual time step, without washing it through time, or to remove information that is irrelevant for the prediction (Cho et al. 2014). If carefully trained, GRU can perform extremely well even in complex scenarios (Dautel et al. 2020). In our paper, GRU is utilized to predict the forward price of options.

The sliding window method (see Fig. 10) is used in the simulation to reflect the actual price of the index. The sliding window method is used in the simulation to reflect the actual price of the index. The deep learning model is trained with the most up- to-date training data and gives the forward price of the index to build the intelligent option portfolio. Such an approach ensures that the machine learning model under each time window uses only historical data, making the results for the options portfolio out- of-sample. We select the monthly closing price of 50ETF from 2005 to 2021 as the full sample, and the time node is selected as the monthly delivery date of the options contract. After standardizing the data of full sample, it is divided into two subperiods: training data and test data. During each study period, the model is estimated on the training data and generate predictions for the test data, which contributes to maintain temporal consistency within and outside the sample.

The loss function minimizes the cross-entropy between predictions and actual target values. We stop the training if the loss on the validation set maintains non-decreasing for 20 epochs. The activation function uses the sigmoid function given by $\sigma(x) = (1 + e^{-x})^{-1}$ to enhance computational efficiency. Following the model architecture of Dautel et al. (2020) and Gupta et al. (2022), we set the GRU model with 3 hidden layers, 50

Table 9 The out-of-sample performance of the different methods

	RMSE	MAE	MAPE (%)
GRU model	2.0560	0.1948	7.001
Monte Carlo	2.3303	0.2016	7.673
Volatility correction	2.4363	0.2246	8.168

The table reflects the out-of-sample prediction results of the machine learning model and classic methods

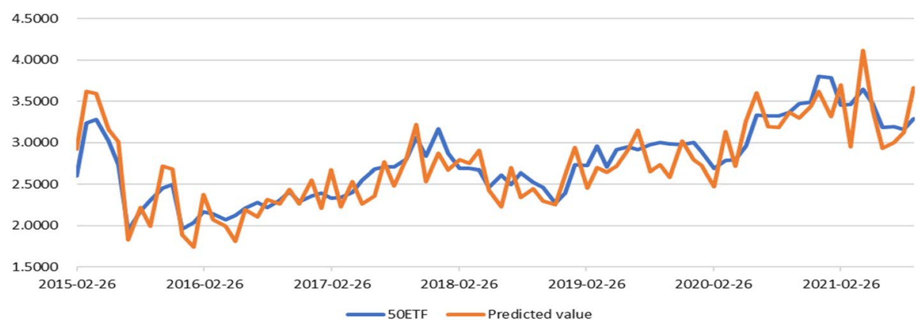


Fig. 11 Predictions and actual price of the 50ETF over time. Note: This graph depicts the time series of the predicted and realized values from Feb 2011 to October 2021. The orange line is the predicted value of the GRU model for a total of 80 time points

neurons per hidden layer. The dropout layers with dropout rate of 25 percent after each hidden layer. The proposed model is trained using Adam optimizer with learning rate 0.0005. The training batch size is 32.

To evaluate the performance of the GRU, three evaluation criteria are used in the study: (a) the root mean square error (RMSE), (b) the mean absolute error (MAE), and (c) the mean absolute percentage error (MAPE)

$$
\begin{aligned}
RMSE &= \sqrt{\frac{1}{n}\sum_{t=1}^{n}(A_t - P_t)^2}, \\
MAE &= \frac{1}{n}\sum_{t=1}^{n}|A_t - P_t|, \\
MAPE &= \frac{1}{n}\sum_{t=1}^{n}|\frac{A_t - P_t}{A_t}|,
\end{aligned}
\tag{12}
$$

where A_t is the realized closed price of index for the tth observation, P_t means the predicted value for the corresponding index, and n is the time of sliding window.

The Table 9 demonstrates the out-of-sample performance of the GRU model's and classic method's prediction results. The accuracy of the prediction results is in line with the general machine learning model requirements, and this result is also consistent with the review in Weng et al. (2018). Compared to traditional methods, GRU models can effectively improve forecasting performance to help option portfolio models achieve higher out-of-sample returns. A more intuitive relationship between the predicted and realized values is shown in the Fig. 11.

Cho et al. (2014).
Dautel et al. (2020)
Gupta et al. (2022)
He et al. (2015)
Weng et al. (2018)

Appendix 2: Transformation of dual form

Proof. The original problem (5) can be abbreviated as:

$$max\ Z = R^T X s.t.HX \leq b, \tag{13}$$

where $H = (C, 0, 0, 0; 0, \beta q, 1, 0; -A, -1, -1, 0; -e, 0, 0, 2e; 1, 0, 0, -1; 0, 0, 0, -1; 0, -1, 0, 0)$, $b = (W_0, \rho, 0, L, 0, 0, 0)^T$

The dual problem of this problem is:

$$min\ G = Yb\ s.t.YH = R, Y \geq 0 \tag{14}$$

where $y = (y_1, y_2, ..., y_7)$.

According to the dual theory of linear programming problem, the necessary condition for the existence of optimal solution $X^* = \left(x_1^*, x_2^*, \ldots, x_2N^*\right)^T$ is that the dual problem also has optimal solution $Y* = \left(y_1^*, y_2^*, \ldots, y_7^*\right)$, and $max\, Z = R^T X^* = min\, G = Y^* b$. The original problem pursues the maximization of the objective function. In the dual problem, Y reflects the reasonable valuation of the final income of each input element. If B is the optimal basis of the original problem, because

$$max\ Z = R^T X^* = min\ G = Y^* b = C_B B^{-z} b = b_1 y_1^* + b_2 y_2^* + \cdots + b_7 y_7^* \tag{15}$$

In the original problem $Z = R^t$, $max\ Z$ is a linear function of the decision variable X. Since the original problem is to solve the maximum value of the objective function under the constraint condition of $HX \leq b$, $max\ Z$ is constrained by the input element b. It can be expressed as

$$max\ Z = F(b) = b_1 y_1^* + b_2 y_2^* + ... + b_7 y_7^* \tag{16}$$

Based on $\frac{\partial Z}{\partial b_i} = y_i^*$, we know y_i^* represent the marginal benefit of the i element under the optimal option portfolio, which reflects the impact of the change of the i th element on the value of the objective function, and becomes the shadow price.

Since shadow price reflects the marginal value of resources under the optimal decision, there is no shadow price without the optimal decision. Under different economic problems and different optimal decisions (the corresponding linear programming problem may have more than one optimal solution), the shadow price of the same resource may be different. Therefore, the shadow price is restricted by the objective conditions of the economic structure itself. The relationship between the optimal solution of the primal problem and its dual problem shows that the shadow price of resources quantitatively reflects the value that resources should provide for total income under the optimal decision. In our problem, because the solution of the model (5) is unique, the shadow price of each element is also determined. The shadow price of the i th resource represents the estimation of the contribution of the i th resource to the final value of wealth under the condition of reasonable allocation of resources.

Appendix 3: Risk-free profit conditions

Proof. When considering the option fee, the expected return of the option portfolio can be expressed as:

$$([S_1 - K]_+ - OF_c)^T x_c + ([K - S_1]_+ - OF_P)^T x_c \tag{17}$$

where $[\cdot]_+$ represents a non-negative element of a vector, $[\cdot]_-$ has the opposite meaning. Then we make the expected return of the option portfolio greater than or equal to 0,

$$\begin{aligned} &([S_1 - K]_+ - OF_c)^T x_c + ([K - S_1]_+ - OF_p)^T x_c \geq 0 \\ &\Leftrightarrow (\frac{1}{2}(|S_1 - K| + S_1 - K) - OF_c))^T x_c + (\frac{1}{2}(|S_1 - K| - S_1 + K) - OF_p))^T x_p \geq 0 \\ &\Leftrightarrow \frac{1}{2}|S_1 - K|^T (x_c + x_p) \geq (\frac{1}{2}(K - S_1) - OF_c)^T x_c + (\frac{1}{2}(S_1 - K) - OF_p))^T x_p \\ &= (OF_c + OF_p)^T (x_c + x_p) + \frac{1}{2}(K - S_1)^T (x_c - x_p), \end{aligned} \tag{18}$$

This yields: if $S_1 > K$, then

$$\begin{aligned} &\tfrac{1}{2}(S_1 - K)^T (x_c + x_p) - \tfrac{1}{2}(K - S_1)^T (x_c - x_p) \geq (OF_c + OF_p)^T (x_c + x_p) \\ &\Leftrightarrow x_c \geq \left(\tfrac{OF_c + OF_p}{S_1 - K - OF_c - OF_p}\right)^T x_p, \end{aligned} \tag{19}$$

if $S_1 < K$, then

$$\begin{aligned} &\tfrac{1}{2}(K - S_1)^T (x_c + x_p) - \tfrac{1}{2}(K - S_1)^T (x_c - x_p) \geq (OF_c + OF_p)^T (x_c + x_p) \\ &\Leftrightarrow x_p \geq (\tfrac{OF_c + OF_p}{K - S_1 - OF_c - OF_p})^T x_c. \end{aligned} \tag{20}$$

For now, we obtain the result from Proposition 1. For option investors, after judging the relationship between forward price S_1 and strike price K, the portfolio weight can meet the conditions (19) or (20) to achieve the goal of non-negative expected return.

Appendix 4: Parameter sensitivity analysis of *L*

In the section on shadow price, the paper illustrates the positive effect of the upper limit of leverage L on the final wealth of the portfolio. For the realized data, increasing L does increase in-sample returns, but equally amplifies out-of-sample returns and risks. Therefore, the article also discusses about the setting and impact of L. Usually financial option contracts are leveraged by a factor of 10, but considering the margin of around 15%, the leverage of individual option contracts is around 5 times. In addition, the article also analyzes the sensitivity of L. Different parameters with portfolio results are presented in Table 10.

All indicators except the Sharpe ratio show a monotonic relationship with the parameter L in Table 10. When the upper limit of leverage is in the range of 5 to 10, the Sharpe ratio of the portfolio varies less. However, the maximum retracement for $L = 10$ is about

Table 10 Parameter sensitivity analysis of L in intelligent option model

L	Mean	Std	SpR	MDD	Max	Min
1	0.0142	0.0307	0.4084	0.0669	0.1540	− 0.0316
2	0.0317	0.0613	0.4899	0.1218	0.3063	− 0.0649
3	0.0492	0.0920	0.5171	0.1746	0.4587	− 0.0982
4	0.0667	0.1226	0.5307	0.2254	0.6110	− 0.1314
5	0.0838	0.1525	0.5385	0.2742	0.7634	− 0.1647
6	0.1018	0.1839	0.5443	0.3211	0.9157	− 0.1980
7	0.1193	0.2146	0.5480	0.3662	1.0680	− 0.2312
8	0.1367	0.2452	0.5507	0.4094	1.2204	− 0.2645
9	0.1543	0.2759	0.5534	0.4507	1.3727	− 0.2978
10	0.1720	0.3065	0.5555	0.4903	1.5251	− 0.3311

This table presents the sensitivity analysis of the upper leverage limit L in the Model (5), where L varies from 1 to 10 in equal intervals. The results compare the mean, standard deviation, Sharpe ratio, MDD, and maximum and minimum values of returns for different L

Table 11 Abbreviation summary

Abbreviations	Full name
CSRC	China Securities Regulatory Commission
ATM	At-the-money
OTM	Out-the-money
IV	Implied volatility
EL	Embedded leverage
SpR	Sharpe ratio
MDD	Maximum drawdown rate

78% greater compared to $L = 5$. This means that a single increase in the upper leverage limit brings high returns while increasing the risk of the portfolio.

Combining the real market situation and the sensitivity analysis of the parameters, L is set to 5 in the experiment. Of course, for investors with higher risk appetite, the level of L can be increased appropriately.

Appendix 5: Abbreviations

Table 11 mainly presents the abbreviations mentioned in the article.

Acknowledgements
Not applicable

Author contributions
This paper is co-authored by Professor Fengmin Xu and his Ph.D. student Jieao Ma from the School of Economics and Finance, Xi'an Jiaotong University. FX is the main contributor to the innovative ideas in the paper, providing support on the mathematical model and optimization methods. JM is responsible for the numerical experimental part of the related paper and the writing of the paper. All authors read and approved the final manuscript.

Funding
This work is supported by the National Natural Science Foundation of China (Nos. 11631013, 11571271, 11971372).

Availability of data and materials
The data in the article mainly includes the public trading data of options contracts, SSE 50 index data, and CSI 300 index data. All data are from Wind database and no private data are involved.

Declarations

Competing interests
The authors declare that they have no competing interests.

Received: 26 August 2022 Accepted: 25 March 2023
Published online: 19 April 2023

References
Alexander S, Coleman TF, Li Y (2006) Minimizing CVaR and VaR for a portfolio of derivatives. J Bank Finance 30(2):583–605
Avramov D, Cheng S, Metzker L (2021) Machine learning versus economic restrictions: evidence from stock return predictability. Manag Sci (accepted)
Buechner M, Kelly BT (2022) A factor model for option returns. J Financ Econ 143(3):1140–1161
Cho K, Van Merriënboer B, Gulcehre C, et al. (2014) Learning phrase representations using RNN encoder-decoder for statistical machine translation. arXiv preprint arXiv:1406.1078
Constantinides GM, Czerwonko M, Perrakis S (2017) Mispriced index option portfolios. NBER working papers
Constantinides GM, Jackwerth JC, Savov A (2012) The puzzle of index option returns. Soc Sci Electron Publis 3(2):229–257
Dautel AJ, Hrdle WK, Lessmann S et al (2020) Forex exchange rate forecasting using deep recurrent neural networks. Digital Finance 2(1):69–96
Dert C, Oldenkamp B (1997) Optimal guaranteed portfolios and the casino effect. Research Memorandum Vu Amsterdam 25(16):273–276
Fama EF (1970) Efficient capital markets: a review of theory and empirical work. J Finance 25:383–417
Faias JA, Santa-Clara P (2017) Optimal option portfolio strategies: deepening the puzzle of index option mispricing. J Financ Quant Anal 52(1):277–303
Gu S, Kelly B, Xiu D (2020) Empirical asset pricing via machine learning. Rev Financ Stud 33(5):2223–2273
Guasoni P, Mayerhofer E (2020) Technical note-options portfolio selection. Operat Res 68
Gupta U, Bhattacharjee V, Bishnu PS (2022) StockNet-GRU based stock index prediction. Expert Syst Appl 207:117986
Han CH et al (2015) Robust hedging performance and volatility risk in option markets: application to Standard and Poor's 500 and Taiwan index options. Int Rev Econ Finance 40:160–173
He K, ZhangX, Ren S, Sun J (2015) Delving deep into rectifiers: surpassing human-level performance on imagenet classification. In: Proceedings of the 2015 IEEE international conference on computer vision, 1026–1034. IEEE Computer Vision Society, Washington, DC
Jackwerth JC (2000) Recovering risk aversion from option prices and realized returns. Rev Financ Stud 13:433–451
Jensen MC, Benington GA (1970) Random walks and technical theories: Some additional evidence. J Finance 25(2):469–482
Kang J, Kwon KY (2020) Volatility-managed commodity futures portfolios. J Futures Mark 41(2)
Khodamoradi T, Salahi M, Najafi AR (2020) Portfolio optimization model with and without options under additional constraints. Math Probl Eng
Korn R, Trautmann S (1999) Optimal control of option portfolios and applications. Or-Spektrum 21(1–2):123–146
Liu J, Pan J (2003) Dynamic derivative strategies. J Financ Econ 69
Markowitz HM (1952) Portfolio selection. J Financ 7(1):77
Merton RC (1977) An analytic derivation of the cost of deposit insurance and loan guarantees an application of modern option pricing theory. J Bank Finance 1(1):3–11
Rockafellar TR, Uryasev SP (2002) Conditional value-at-risk for general loss distributions. J Bank Finance 26(7):1443–1471
Topaloglou N, Vladimirou H, Zenios SA (2011) Optimizing international portfolios with options and forwards. J Bank Finance 35(12):3188–3201
Wang X, Huang X (2019) A risk index to model uncertain portfolio investment with options. Econ Model 80:284–293
Wang W, Li W, Zhang N et al (2020) Portfolio formation with preselection using deep learning from long-term financial data. Expert Syst Appl 143:113042
Weng B, Lu L, Wang X et al (2018) Predicting short-term stock prices using ensemble methods and online data sources. Expert Syst Appl 112:258–273
Xue W, Ma L, Shen H (2015) Optimal inventory and hedging decisions with CVaR consideration. Int J Prod Econ 162:70–82
Yu J, Chang KC (2020) Neural network predictive modeling on dynamic portfolio management—a simulation-based portfolio optimization approach. J Risk Financ Manag 13(11):285
Yuan S, Rieger MO. Diversification with options and structured products. Rev Derivat Res 2020(1).
Zeng Y, Klabjan D (2018) Portfolio optimization for American options. J Comput Finance 22(3):37–64

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