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Valuing options to renew at future market value: the case of commercial property leases

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Abstract

In this study, we develop and empirically test a valuation model for a commonly encountered option in office leases: a tenant's option to renew at future market rent (a fair market value) with lease termination as the maturity date. The model integrates decision analysis with real options analysis and market risk with private risks. "Option value" is defined as the private value of the option to either party pre-contract, while "option price" assumes a fair agreement between transacting parties and can be positive (rental premium paid) or negative (rental discount offered). Without manifest expectations, an analysis of a sample of office leases supports the model's logic with price estimates in a practical range. The tenants' option price/value is shown to have a negative relationship with the original/renewal lease term; conversely, the landlords' option value is positively related to the original/renewal term. Comparative analyses show that transaction costs have a positive effect on tenants' option value and on prices, while vacancy costs and the vacancy period are both positively related to the landlords' option value and negatively related to price. Market rent is found to have a negative relationship with option price. Overall, this study provides a theoretical analysis and empirical tests of the value of a real option that allows option holders to renew/extend their contracts at a fair market value.

Keywords: Fair market value renewal, Commercial property leases, Real option, Valuation, Integrated method

Introduction

Research into financial options and other derivatives typically deals with standardized, exchange-traded securities with specific information for exercise prices, expiration dates, exercise styles, and settlement methods, largely because of their familiarity and the availability of price and other time-series data. Financial options are valuable to option holders when the asset price expected at the maturity date is higher (lower) than the strike price for a call (put) option. The strike price is generally predetermined and known when a financial option is traded. Although many non-standardized financial contracts with embedded and/or explicit option and contingent claim characteristics are routinely privately transacted in modern commerce, pricing such claims has received little research attention. While scores of "standard" option and contingent claims pricing methods have been developed, most analytical solutions are not practically useful

in private applications, including in pricing embedded options in transactions such as commercial property leases.

A popular option embedded in real estate transactions is the option to renew a lease contract or purchase property at its fair market value when the contract expires. In other words, it is an option with a strike price that is equal to the market price. In the standard financial options market, such options do not have value for option holders because the option value is zero at expiration. Investors can always buy assets at fair market values; hence, it is not necessary to hold an option in advance. However, the option to renew/purchase at a fair market value is frequently seen in practice; for instance, airline companies can place orders for aircraft and meanwhile hold an option to buy additional aircraft in the future.[1] In the lease market, tenants may be granted an option to renew their lease contract at a fair market rent when the contract ends; in some circumstances, they may also hold options to purchase the property in the future at a fair market price. Does this option to renew at a fair market value have value for tenants (or landlords)? Intuitively, the option to renew at market value should not be trivial, as the continued presence of options in this format indicates that it is valuable to landlords and/or tenants.[2] However, to the best of our knowledge, no studies have explored the value of options to renew at future market value that are embedded in option-like non-financial contracts. Our study fills this research gap by developing a real option valuation model and testing the model using typical office leases with options to renew at future market rent.

We develop a real option valuation model by considering practical factors that may affect the incentives of tenants and landlords when exercising the option to renew a lease contract.[3] The options embedded in commercial leases differ from standardized exchange-traded options in important ways. First, leases separate the rights of ownership and use and ownership from possession, so creating incentive conflicts between landlord and tenant from risks in bilateral contracting (e.g., asymmetric information, counterparty opportunism, etc.) is summarized in two main agency risks for leases: bilateral monopoly exploitation (neglected landlord-contracted maintenance) and asset residual value expropriation (neglected tenant-contracted care for the property). Options provide a private mechanism for managing such landlord/tenant agency risks. Second, in addition to market risks (e.g., rent), a landlord/tenant's specific private circumstances could influence the probability of exercising embedded options (e.g., tenants' business conditions), and thus could be of specific value to a landlord/tenant. Third, transaction costs for both parties may result if a lease is cancelled prematurely or is not renewed (vacancy, relocation). *Options could reduce these costs for either/both parties;* for example, exercising options to renew lease contracts allows tenants to avoid relocation costs, costs associated with business interruption, the loss of location goodwill, etc., while landlords can save search and screening costs for new tenants and the loss of rental income due to a vacancy between two lease contracts. Such factors introduce significantly richer option

[1] The option to buy another batch of aircraft is frequently embedded in aircraft purchase contracts. See, https://www.reuters.com/business/aerospace-defense/uks-iag-orders-37-airbus-a320neo-family-aircraft-2022-07-28/.

[2] If this option was not valuable, landlords and tenants would not spend time writing the clauses and the option details in lease contracts.

[3] Practical factors, such as institutional arrangements, market conditions, and financial constraints have been considered in option pricing studies (e.g., Shen 2012; Shen and Pretorius 2013; Yao and Pretorius 2014; Wong and Cheung 2017) and other studies in real estate (e.g., Shen et al. 2018; Shen et al. 2019).

exercise contexts compared to conventional financial options, where typically only the option holder benefits.

Within this context, we investigate a provision that is common in typical office leases in many jurisdictions, namely a tenant's option to renew at a future date, often at "*future market rent*." Our aim is to develop a valuation model for this provision that incorporates these seemingly intangible factors. We define "option value" as the private value of the option for either party pre-contract, which arises from avoiding transaction costs when the option to renew is exercised. "Option price" assumes a fair trade between transacting parties with private values (positive [rental premium paid] or negative [rental discount offered]). We assume a well-functioning office rental market where purchasing is a practical substitute to leasing and incomplete financial markets, while considering both market risk (arising from the property price and rent) and private risks (arising from tenants' private circumstances that can affect exercise decisions).[4] We draw on aspects of both landlord and tenant private strategic and economic circumstances and combine decision theory, real options analysis, and dynamic analysis techniques into one framework (the "integrated method") (see Smith and Nau 1995; Smith and McCardle 1998; Nugroho 2016; Trigeorgis and Tsekrekos 2018; Kou et al. 2021). The value of a renewal option for a landlord (tenant) is calculated as the difference in payoffs for the landlord (tenant) with and without this option.

We test the model on 25 typical Hong Kong Grade A fixed term, constant payment office leases drawn from the same expansionary phase in the asset and rental market. With no a-priori expectations, we show that a renewal option can have value for landlords and/or tenants, dependent on the initial states at the time it is granted and the specific cost/value to renew or not renew. The resulting range of option values for counterparties appear entirely practical, with option prices ranging from −6.85% to 26.63% of the total lease value. Most prices are positive, suggesting that tenants paid landlords for the option. Both the option value and price for tenants are negatively related to the original and renewal lease terms, likely the result of reduced amortization costs over a longer term. The option value for landlords is positively related to term, likely the result of balancing higher potential vacancy risk with longer terms.

Comparative analyses show that tenants' option value and price increase when transaction costs (and relocation costs) increase, consistent with previous studies (Wong and Cheung 2017; Armerin and Song 2019). For landlords, vacancy costs and the vacancy period both have a positive relationship with option value and a negative relationship with price, indicating that landlords have more incentive to offer an option when expected vacancy costs are higher. The probability of renewal under private risks is positively related to the option's value to both parties and to price. Option prices are also negatively related to rent, which is different from what traditional option price theory may suggest. Overall, our results reveal the relevance of tenants and landlords' private circumstances in pricing lease options and of complex differences between pricing

[4] Smith and McCardle (1998) define private risk as "risks that cannot be perfectly hedged by trading" (page 201). Following Borison (2005), we define private risk as project-specific risk that could affect tenants' exercise decisions on an option to renew lease contracts. These are private factors, including the growth and contraction of tenants' businesses, organization changes due to mergers and acquisitions, and special requirements from facilities. We identify private risks associated with tenants using a Delphi study.

financial options and embedded option-like characteristics in non-standard financial contracts.

This paper proceeds as follows. We review relevant literature on valuing lease options and then review the potential determinants of the value of a tenant's renewal option for both the landlord and tenant. We then develop the valuation model and present and analyze the empirical results.

Conceptual framework and review of empirical studies

In the literature on valuing property leases, researchers have attempted to develop theoretical valuation models for options to purchase, upward-only rent reviews, options to cancel, and options to renew. Grenadier (1996) evaluates a purchase option under default risk using a theoretical real options framework and finds that a tenant will maintain the asset and avoid default to retain the option. Several studies apply option pricing models such as the binomial (Cox et al. 1979) and Black–Scholes models (Black and Scholes 1973; Merton 1973) to evaluate upward-only rent review clauses in property leases; such clauses allow rental to reset to the market rate if the market rent exceeds the contract rent (e.g., Ward and French 1997; Ward et al. 1998; Booth and Walsh 2001; Clapham 2004). Ambrose et al. (2002) numerical analysis shows that the initial rent in a 10-year upward-only adjusting lease is significantly lower than the rent in similar leases that allow both upward and downward rent reviews. This discount is larger when the volatility of rents and interest rates is greater. Foo Sing (2012) prices the option for tenants to cancel a lease with a binomial pricing model, in which the value of a default option is estimated to be 1.08% for a hypothetical 3-year lease contract. Vimpari (2018) applies a real options pricing model to value a tenant's downscaling option in lease agreements and shows that both the landlord and tenant can benefit from the downscaling option by saving relocating costs (for the tenant) and re-renting costs (for the landlord). Recent studies also incorporate multiple flexibilities for tenants, like options to defer payment, cancel, or convert, (Liang et al. 2012; Triki and Abid 2022) or double-sided flexibility for tenants and landlords (Al sharif and Qin 2015) into option pricing models to assess their value. Overall, these studies suggest that options embedded in property leases are valuable (to the tenant and/or landlord), and option pricing techniques can be applied to evaluate them.

Prior studies also apply option pricing models to evaluate options to renew that are embedded in property leases, possibly the most popular type of lease option. Options to renew have taken different guises, with the strike price sometimes defined as a fixed amount, a fixed percentage of the market value at expiration, the current rent adjusted by the cumulative change in some index (for example, the inflation index), or the future market value at expiration. Amédée-Manesme et al. (2015) apply a binomial model to value a lease contract with the option to renew in the future at a better rate and show that this option is valuable to tenants. Their results suggest that the value of the option to renew increases as market rent becomes more volatile. Buetow and Albert (1998) apply a real option approach to explicitly price a renewal option where the strike price is related to a cumulative index; they likewise find that this option is valuable. They further demonstrate that the more negative (or less positive) the relationship between the market rent and indexed rent, or the larger the initial rent, the greater the option's value.

Buetow and Albert (1998) suggest that the option to renew at future market rent is valueless, as the option is always at-the-money and has zero intrinsic value. However, as Geltner et al. (2014) observe, options to renew at market value can be valuable, and their value may depend on transaction costs and/or other benefits. In spite of this, no research to explicitly price such options has been identified.

Our study addresses the research gap in evaluating options to renew at future market prices. Two existing studies are related to this research. Armerin and Song (2019) explore a rental option policy owned by tenants who can renew their current lease contract at the lower of the predetermined rent and market rent. They introduce two factors related to tenants—transaction costs and a moving threshold—into the option pricing model and show that the value of the rental option for tenants increases if the moving cost is larger and moving threshold is lower. This study also considers transaction costs tenants face if they do not exercise the renewal option. Our study differs from Armerin and Song (2019), as we consider transaction costs for both tenants and landlords, as well as tenants' private risks that may affect their decision to renew a lease. Wong and Cheung (2017) empirically explore whether landlords should offer a rent discount due to the cost incurred when searching for a new tenant (one type of transaction cost) or a rent premium because tenants incur large moving costs for a new lease. Their empirical findings suggest that rent discounts are larger when the property size is larger and the vacancy period is longer, while rent discounts are smaller if the lease is longer. Wong and Cheung (2017) differs from our study, as we propose a theoretical options pricing model to study the value of the option to renew and consider different factors that can affect that value, such as relocation costs and private risk.

Framework of the valuation model

We draw on counterparties' economic circumstances during lease negotiations and known real options pricing frameworks to develop a valuation model for a renewal option at future market rent. It is a common presumption that renewal options may be valuable for tenants by allowing them to avoid relocation and fit-out costs, as well as loss of location-derived goodwill (see Posner 1993; Buetow and Albert 1998; Rowland 2000; Geltner et al. 2014). Landlords may similarly benefit from reduced transaction costs associated with establishing a new lease and reduced agency costs, and by avoiding possible rental income loss between consecutive leases. The potential loss in rental income during a vacancy period and tenant substitution is, however, influenced by future market rent; thus, such decisions are not indifferent to market rent. We therefore propose that the key determinants of option value include transaction costs, vacancy costs, rental income, substitute property purchase (see below), and the probability of renewal.

We know that transaction costs play a significant role in property, although not symmetrically for all market participants in all situations and at all times. For our purposes, the costs of searching, screening, contracting, and post-contract agency conflict-monitoring activities in leases are expected to be reflected in option values, because the costs of renewal are a priori expected to be lower than costs for a new contract. Further, when a lease expires, a temporary vacancy may occur if the next lease does not commence immediately. If landlords must hold a vacant asset, they carry the actual holding cost and opportunity cost of the vacancy (rental income). Eliminating vacancies could eliminate

(or at least reduce) vacancy costs and interrupted cashflow. These costs thus are also part of the expected landlord payoffs in estimating option value; landlords may also benefit when a tenant exercises its option. Thus, the tenant's option to renew potentially has value for both parties.

In many office markets, lease vs. buy analyses may be academic (e.g., Wang et al. 2020). Hong Kong, however, is characterized by significant sectional titling of Grade A offices, and lease/buy substitution is thus an entirely practical consideration (although arguably, it happens infrequently for strategic reasons). For completeness, we assume that tenants can either lease or buy offices.[5] To simplify the analysis of purchasing a substitute property instead of exercising an option, we assume the same location, a similar grade, and functional substitution based only on rents (and locational substitution by purchasing is possible). This allows introducing the equivalent cost of owning compared to leasing/renewing into the valuation framework, both as a determinant of option value and a factor influencing exercise.

Independent of the above is the probability that a tenant will exercise its option, a key variable. Although the option is the tenant's right, when making renewal decisions, tenants are expected to consider the benefits and costs of renewing, moving, and a purchase substitute, all as discussed. Market risks—rent and price—that influence the lease vs. buy decision may affect exercise probability, but it is likely that private factors like the growth/contraction of a tenant's business, proximity to similar businesses, and other private circumstances also directly influence the exercise decision.

In fact, both market risks (rent, price) and private risks (tenant circumstances) influence the value of a tenant's renewal option. Conventional real options analysis may not be best suited to these circumstances, as its assumptions do not hold. With real options analysis, the existence of appropriate twin securities and the consequent validity of the no-arbitrage assumption are always problematic. These problems are often overcome through equilibrium assumptions and risk neutral valuation (see Grenadier 1995). However, the problem is often simply ignored; for example, Buetow and Albert (1998) use a variant riskless-hedge portfolio under a no-arbitrage assumption, without reference to the origin of such a portfolio. Where private risks and considerations are important and may dominate or the situation is dominated by private risks (Borison 2005), the real options approach may thus not be appropriate. We therefore consider the *integrated approach* pioneered by Smith and Nau (1995) to solve real options pricing problems, since it is more appropriate for valuing the option to renew at market rent. We focus on one integrated method: the combination of option pricing and decision analysis. Decision analysis is a systematic method used to analyze various decisions, but it seldom considers opportunities to hedge market risks. In contrast, option pricing values market opportunities but is not general enough and neglects non-financial risks like private risks (Smith and McCardle 1998). Integrating these two methods allows us to use the advantages of both methods to value a decision with mixed risks, without assuming market completeness and the existence of a perfect replication portfolio (Smith and McCardle 1998; Luenberger 2013; Trigeorgis and Tsekrekos 2018).

[5] . The main findings are similar if we assume that tenants only obtain the use of offices by leasing.

Table 1 Situations and value for counterparties

States	Market vacancy expected to decrease	Market vacancy expected to increase
Tenant	Transaction cost: reduce more as tenants find it more difficult to find suitable premises	Transaction cost: reduce less as tenants find it easier to find suitable premises
	Option value: larger as tenants benefit more from the renewal	Option value: smaller as tenants benefit less from the renewal
	Lease price: higher as tenants are willing to pay more for the renewal option	Lease price: lower as tenants are less willing to pay for the renewal option
Landlord	Transaction cost: reduce less as landlords find it easier to few tenants	Transaction cost: reduce more as landlords find it more difficult to find new tenants
	Option value: smaller as landlords benefit less from the renewal	Option value: larger as landlords benefit more from the renewal
	Lease price: higher as landlords are less willing to offer discounts for the renewal option	Lease price: lower as landlords are willing to offer more discounts for the renewal option

Option values and option price

In sum, a tenant's option to renew at future market rent may reflect benefits and costs for both landlords and tenants and could be influenced by both market and private risks. We argue that it may in some situations be more valuable for tenants, while in others, it may be more valuable for landlords. For example, when vacancy rates decline, it may be difficult for tenants to find suitable space but easier for landlords to find new tenants. In this situation, the tenant's option can insure against the risk of business interruption and higher than market rent while also saving transaction costs. If this state is expected at maturity, tenants are expected to pay a *premium* to include the option (an implicit rental premium). Likewise, if market vacancy is expected to increase toward maturity and if the tenant exercises the option at maturity, the landlord could benefit by reducing transaction costs incurred for a new lease, reducing agency costs related to the original and subsequent lease, and avoiding lost rental income between consecutive leases. In this situation, the option is expected to be more valuable to the landlord, who is expected to buy it with a *negative premium* (an implicit rental discount). Table 1 summarizes the typical states under which tenants and landlords, respectively, may consider the option valuable.

We therefore expect the option "price" to reflect a successful negotiation between counterparties with private values. We define *"option value"* as the private benefit that the tenant/landlord may obtain from the option, while *"option price"* is defined as the premium (positive) or discount (negative) paid at the time of transacting to obtain the option or have it accepted. Option value reflects the tenant/landlord's value of the option to renew at future market rent, while the option price is the consequence of negotiations between the tenant and landlord after they separately assess option values. The next key assumption is that a fair price reflects half the difference in value to either party, because only that will return parties to a "normal" (equilibrium) trading state. As a reference point, first consider a lease without a renewal option. Then, consider a tenant with a bid price for this lease and consider separately the option's value to the tenant. Also, consider a landlord with an asking price for the same lease, and then consider separately the option's value to the landlord. A fair lease price should be:

Table 2 Potential scenarios for counterparties at lease expiration

Scenarios	Tenant	Landlord
With option	Renew	Renewal lease
	Search for a new lease	Search for a new lease
	Purchase	
Without option	Search for a new lease	Search for a new lease
	Purchase	

$$\textit{Lease price for the contract without option} + (\textit{option value to tenant} - \textit{option value to landlord})/2.$$

This price is more complex but also more general than stock option prices, which may be seen as a special case of this concept.[6] To operationalize it, we must first estimate the option's value for both parties at the time of the original transaction, and then estimate its price from the difference. A positive difference indicates a premium paid by the tenant; a negative difference indicates a rental discount given by the landlord.

Decision trees

It is necessary to state assumptions and describe the potential actions/situations to which payoffs are linked to analyze the counterparty payoffs. Landlords are characterized as specialist intermediaries in renting out office space, while tenants can acquire offices by leasing or purchasing. Under any conditions, and ignoring tax, the lease vs. purchase choice depends mainly on comparing the user's costs of owning and leasing. With an option, tenants may face three potential scenarios at lease expiration: renew, search for a new lease, or purchase.

We begin at the lease's expiration for the base case (lease without renewal option). If the landlord offers a tenant the choice to renew, based on the tenant's intentions, a complex situation arises. The lease has to be negotiated, comparable to entering into a new lease; thus, transaction costs may not be significantly reduced compared to a new lease. A tenant could then face two scenarios when the lease expires: search for a new lease or purchase. However, if the option is included in the original contract, at expiration, the landlord's decision set is based on the tenant's exercise decision: renew the lease (if exercised) or search for a new lease (if not). The landlord may also have to search for a new tenant at expiration in the base case. Similar to the potential scenarios for a tenant, a landlord may offer an option if there is not one in the original contract. Again, this scenario is comparable to searching for a new tenant. In sum, five scenarios emerge for tenants with or without an option, with three corresponding scenarios for landlords, as shown in Table 2.

Based on these scenarios, we develop the decision trees and respectively analyze the tenant and landlord's detailed payoffs. The landlord's decision tree has three scenarios, with payoffs consisting of transaction costs, vacancy costs, and rental income.

[6] Assume the buyer's expected payoff for a call option in the stock market is X, so that the writer's payoff is $-X$. Following the pricing method proposed here, the option price is equal to $[X-(-X)]/2=X$, exactly the result from the traditional option pricing model.

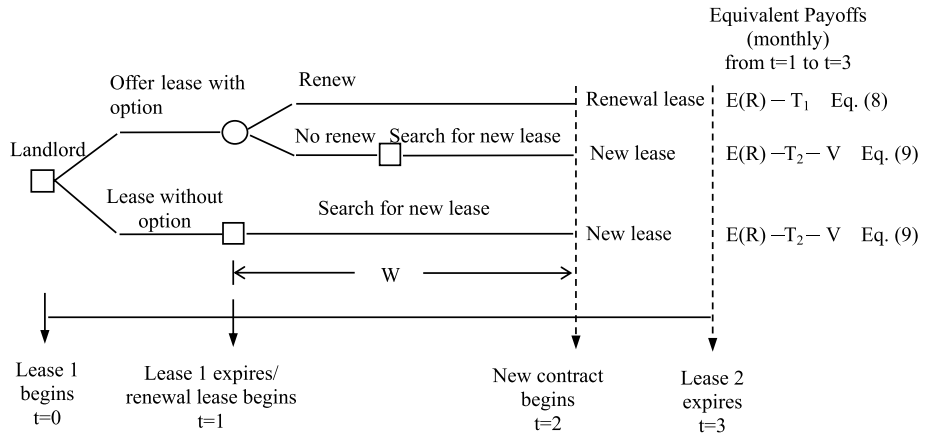


Fig. 1 Decision tree and landlord payoffs

If the tenant chooses to renew, the landlord still incurs transaction costs (T_1), such as contracting costs, but these are likely substantially reduced compared to those of a new lease ($T_1 < T_2$) (searching, screening, and most contracting costs). Further, the landlord benefits from uninterrupted rental income following termination of the original lease. However, if the tenant does not exercise the option or does not accept an offer to renew, the landlord possibly faces a vacancy period (waiting time) (W) before a new contract commences. During this period, the landlord receives no rental income and also incurs vacancy costs (V) (management, other tenant charges in net leases). Typically, the option requires tenant commitment within a stipulated period before expiration. Because it is unknown whether the tenant will renew, a landlord will not have early knowledge about whether/when to begin searching.

In sum, the landlord's payoffs for a renewal lease and for a new lease can be termed the *expected equivalent income of the renewal lease* and the *expected equivalent income of a new lease*, respectively, which involves expected rental income ($E(R)$), transaction costs (T_1/T_2), and possible vacancy costs for a new lease (V). A decision tree with equivalent payoffs for the landlord is presented in Fig. 1. There are three periods after lease commencement at $t=0$; $t=1$ is the end of the lease; the potential vacancy period is from $t=1$ to $t=2$, when a new contract begins; and $t=3$ is the end of the renewed/new lease. The landlord's option value can be estimated by calculating the equivalent discounted payoffs of a renewed lease and the equivalent discounted payoffs of a new lease in different situations. The difference between the equivalent lease payoffs with and without the option are then compared, from the end of the original lease to the end of the next contract ($t=1$ to $t=3$).

In similar fashion, key costs and benefits that affect the tenant's option value may be summarized as the *expected equivalent user costs of leasing* (for the renewal lease and a new lease, which include transaction costs and rental payments) or the expected *user cost of owning the asset*. There are only two periods in the decision tree for the tenant, without a vacancy period. If the tenant does not have an option, there may be losses from business interruption in a landlord's market due to the search time and costs required. In this situation, beginning a search early will mitigate the risk. Similar to the landlord's situation, the tenant's transaction costs to renew the contract

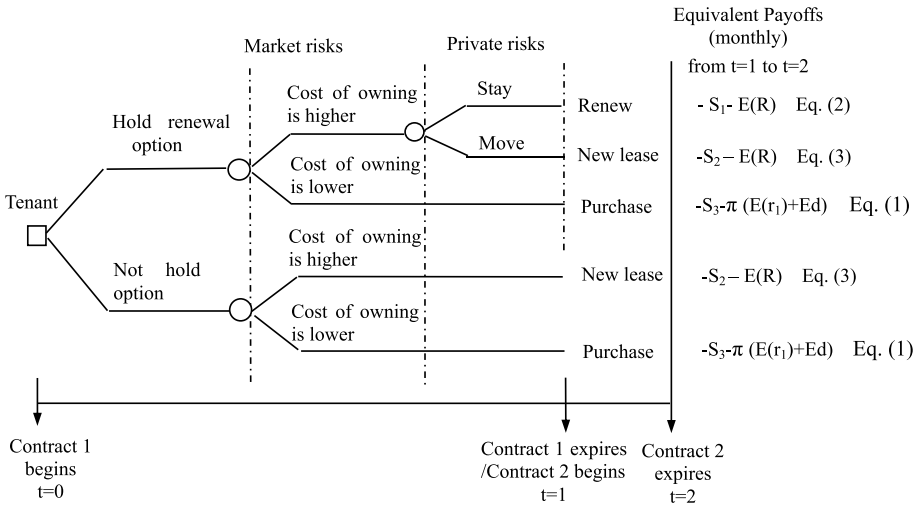


Fig. 2 Decision tree and tenant payoffs

(S_1) will be significantly less than those for a new lease (S_2). The costs of owning the asset include transaction costs in purchasing (S_3) and user costs (with capital investment π). We base the user costs of owning on Miller and Upton's (1976) approach, which is expressed as the sum of the opportunity cost of capital ($E(r_1)$) and the asset's depreciation ($E(d)$). The tenant's decision tree with associated payoffs is shown in Fig. 2. There are two periods in the decision tree: from the beginning of the initial lease at $t=0$ to the end of the lease at $t=1$, and from $t=1$ to the expiration of the renewed/new lease at $t=2$. The tenant's option value can thus be expressed as the difference between the tenant's equivalent payoffs (costs) of the leases with and without the option. As is to be expected, the probability that the tenant will exercise may be affected by both market risks and private risks, also shown in Fig. 2.

Valuation model

We now combine real options and decision analysis logic to complete the valuation model. We use option pricing methods to estimate the risk neutral probability of purchasing under market risks and decision analysis to estimate the subjective probability of renewing under the influence of private risks. Decision analysis is then used to calculate the payoffs and obtain an option price using the risk-free discount rate. We proceed as follows:

1. The binomial method and decision analysis are used to calculate the tenant's probability of renewal under market risks and private risks, respectively, at $t=1$. The integrated method is then used under different scenarios to obtain the probabilities of renewing, contracting a new lease, and purchasing.
2. The payoffs at the end nodes of the tenant's decision tree are estimated at $t=1$, and the estimated probabilities and risk-free discount rate are used to roll back through the decision tree and calculate the tenant's option value at $t=0$.

Table 3 Definition of terms

R	Initial rental payment
P	Initial property price of office with a lease
T_1/T_2	Expected transaction costs for a renewal lease/new lease for the landlord at the end of Lease 1
$S_1/S_2/S_3$	Expected transaction costs for a renewal lease/new lease/office purchase by the tenant at the end of Lease 1
$E(R)$	Expected rental payment of a lease (renewed or new) at the end of Lease 1
V	Expected vacancy costs in one period between two consecutive contracts
W	Expected vacancy period/waiting time between the two consecutive contracts
$E(r_1)/E(r_2)$	Expected opportunity cost of capital invested by tenant/landlord in the office asset
$E(d)$	Expected depreciation rate of the office
π	Expected capital investment (property price) in the office at the end of Lease 1
q_1/q_2	Tenant's probability to purchase under the influence of market risk with a renewal option/no renewal option
q_3	Tenant's probability to renew under the influence of private risks
q_4	Probability of renewal faced by the landlord (to estimate payoffs with the option)
$ECO/ECL_1/ECL_2$	Expected equivalent user cost of owning/leasing for the renewed lease/leasing for the new lease at the end nodes of the tenant's decision tree
ECE	Expected certainty equivalent; equivalent payoffs that are projected from ECL_1 and ECL_2
EPL_1/EPL_2	Expected equivalent rental income from renewal/a new lease at the end nodes of the decision tree for the landlord

3. The expected vacancy period for the landlord and the payoffs at the end nodes of the landlord's decision tree are estimated, combined with the estimated exercise probability, and rolled back at the risk-free rate to obtain the landlord's option value at $t=0$.
4. Finally, the option price is estimated as half the difference between the tenant and landlord's option values at $t=0$.

We summarize the notation used in the study in Table 3.

Tenant's option value

The probability that a tenant may exercise its option is a key variable that affects the option's value for both parties. Two risks—market and private—influence the probability. The binomial method is used to estimate the probabilities of different potential scenarios. This is achieved by comparing the corresponding expected equivalent tenant user costs of owning and leasing (for renewal or a new lease) per unit of time at the original lease's expiration, for each situation-value combination of rent and price, following Luenberger's (2013) double-lattice approach. Appendix 1 outlines the details of estimating the probabilities to purchase under the influence of market risk with (q_1) and without (q_2) an option. We conducted a Delphi study to identify key private risks that may influence the tenant's decision to renew and estimate the probability of renewal affected by these factors (q_3). The process is summarized in Appendix 2.

The office's expected rent and price can also be derived using the double-lattice approach. We estimate the payoffs in one period at the end nodes of the tenant's decision tree, that is, the expected equivalent user costs of leasing and owning. The expected equivalent user cost of leasing is estimated as the sum of the corresponding expected rental payments and transaction costs. The expected equivalent user cost of owning is

the sum of the tenant's opportunity cost of capital, the asset's depreciation, and the transaction costs. The expected equivalent user costs of owning (ECO), renewing (ECL_1), and obtaining a new lease (ECL_2) in one period can be expressed as:

$$ECO = -[E(r_1) + E(d)]\pi - S_3 \frac{r_1(1+r_1)^n}{(1+r_1)^n - 1} \tag{1}$$

$$ECL_1 = -E(R) - S_1 \frac{r_1(1+r_1)^l}{(1+r_1)^l - 1} \tag{2}$$

$$ECL_2 = -E(R) - S_2 \frac{r_1(1+r_1)^l}{(1+r_1)^l - 1} \tag{3}$$

where $E(r_1)$ is the expected opportunity cost of capital, $E(d)$ is the expected office depreciation rate with capital investment π at the original lease's expiration, and n is the office's residual economic life.[7] π is measured using the expected market price of similar offices. $E(r_1)$ can be obtained from the capital asset pricing model (CAPM). $E(R)$ is the expected rental payment in one period, and l is the term of the new lease. S_1, S_2, and S_3 are the corresponding transaction costs for the tenant. The discount rate (r_1) used to determine the equivalent transaction costs in one period is $E(r_1)$. We first use the equivalent costs in one period for comparison and calculation. The option value and price based on one period's costs are also estimated and then multiplied by the term of the new lease to obtain the total value.

Following Smith and McCardle (1998) and Luenberger (2013), we then use the integrated method to project the problem in an incomplete market into an equivalent problem with complete markets, the estimated risk neutral probabilities of renewal influenced by market risks, and the estimated probabilities influenced by private risks. The main step is calculating the expected certainty equivalent $ECE(x)$. Still following Luenberger (2013) and Smith and McCardle (1998), we assume that if utility is a function of total wealth, most investment decisions involve relatively small increments to that wealth. We also assume that the tenant's preference is for constant risk aversion, which can be represented by an exponential utility function $U(x) = -e^{-x/a}$, where a is the constant risk aversion coefficient and $\boldsymbol{x}$ is a random wealth/consumption variable. For our present purpose, we determine a subjectively (see Luenberger 2013). Given the corresponding equivalent user costs of renewing and a new lease and the probability of renewal influenced by private risks (q_3), the expected certainty equivalent $ECE(x)$ can be expressed as:

$$ECE(ECL_1, ECL_2) = -a \times \ln\left[q_3 \times e^{-ECL_1/a} + (1 - q_3) \times e^{-ECL_2/a}\right]. \tag{4}$$

[7] The opportunity cost of capital is the return foregone on the capital invested to own the property. Depreciation is the loss of property value due to internal or external factors, such as physical, functional, and economic obsolescence. Detailed discussions on depreciating the property value can be found in Geltner et al. (2014).

Thus, the payoffs have been projected into an equivalent one under complete markets. We can then follow the binomial method to calculate tenant payoffs with and without the option.

$$\text{Payoff for tenant with option } = ECO \times q_1 + ECE(ECL_1, ECL_2) \times (1 - q_1) \tag{5}$$

$$\text{Payoff for tenant without option } = ECO \times q_2 + ECL_2 \times (1 - q_2) \tag{6}$$

We then roll the payoffs back through the decision tree with the risk neutral probability and risk-free interest rate to obtain the present value of the payoffs with and without the option. The tenant's option value can be obtained by calculating their difference, following Amédée-Manesme et al. (2015).[8] The tenant's total option value can be obtained by multiplying the one-period option value by the lease term.

Landlord's option value

We turn now to estimating the landlord's payoffs and option value. One key issue is estimating the expected vacancy period, viewed in principle as a queuing problem following Kenyon and Tompaidis's (2001) approach (see Appendix 3). This method is used to estimate the expected idle time (Xiao et al. 2021; Kou et al. 2022).

Another key issue in the landlord's decision tree is the probability that the original lease with option will be renewed (q_4). This can be calculated given the tenant's payoffs under the three different scenarios—renewing, new lease, and purchase—and given the tenant's payoff from Eq. (5) above. The following equation can be solved to derive q_4.

$$ECL_1 \times q_4 + ECL_2 \times (1 - q_1 - q_4) + ECO \times q_1 = ECO \times q_1 + ECE(ECL_1, ECL_2) \times (1 - q_1) \tag{7}$$

Given the rental income, transaction costs and vacancy costs, the landlord's expected equivalent income from renewing (EPL_1) and a new lease (EPL_2) in one period at the end nodes of the decision tree can be expressed as:

$$EPL_1 = E(R) - T_1 \frac{r_2(1 + r_2)^l}{(1 + r_2)^l - 1} \tag{8}$$

$$EPL_2 = E(R) \frac{(1 + r_2)^l - 1}{(1 + r_2)^{l+w} - 1} - (T_2 + V \times W) \frac{r_2(1 + r_2)^l}{(1 + r_2)^l - 1} \tag{9}$$

where $E(R)$ is the expected rental income in one period, l is the term of the new lease, T_1 and T_2 are the landlord's transaction costs for renewal and for a new lease, respectively. V represents the vacancy costs in one period. W is the vacancy period or idle time between the end of the original lease and the beginning of the following new lease (two consecutive contracts). Here, r_2 is the landlord's expected opportunity cost of capital

[8] The tenant's value of a lease contract with a renewal option equals the value of an option-free lease contract plus the value of the renewal option. Traditional option pricing studies value only call/put options because options are detachable from the underlying assets and are traded independently in the exchange market. The option to renew is embedded in the lease contract. Thus, the value of the option to renew is determined by the value of the lease with the renewal option minus the value of a straight lease with similar terms (Amédée-Manesme et al. 2015). The same practice is applied to value convertible bonds (see the discussions on convertible bond values in Finnerty (2015).

$E(r_2)$. Due to the potential vacancy period, the rental income received by the landlord in a new lease is spread to cover the period of lost rental income.

Thus, the payoffs for the landlord with and without the option are presented as follows:

$$\text{Payoff for landlord with option} = EPL_1 \times q_4 + EPL_2 \times (1 - q_4), \text{ and} \tag{10}$$

$$\text{Payoff for landlord without option} = EPL_2. \tag{11}$$

We can obtain the landlord's option value at lease commencement in one period by discounting the payoff with an option at the risk-free rate, discounting the payoff without an option at the landlord's opportunity cost of capital, and then calculating their difference. The landlord's total option value can be obtained by multiplying the one-period option value by the term of the new lease.

Option price—premium/discount

Finally, the option price (premium/discount) can be estimated by taking half the difference between the tenant's and landlord's estimated values. Thus, the option price is expected to be:

$$\begin{matrix}\text{Premium} \\ (+)/\text{Discount}\,(-)\text{ at} \\ t = 0\end{matrix} = 0.5 \times \left(\begin{matrix}\text{Option value to the} \\ \text{tenant at } t = 0\end{matrix} - \begin{matrix}\text{Option value to the} \\ \text{landlord at } t = 0\end{matrix} \right) \tag{12}$$

Data and empirical analysis

Data and key parameters for the valuation model

An option to renew at future market rent is common in Hong Kong's office lease market. According to office lease records from the Land Registry in Hong Kong, registered leases between 1991 and 2000 varied between one to six year terms; 30% of such leases include the option, and most have renewal periods of two years.[9] To test the model, we extracted the details by content-analyzing 25 practically similar Grade A Office leases for 12–36 month periods, in the core commercial district of Hong Kong, and registered with the Hong Kong Land Registry.[10] To calculate the volatility of the returns to office rent and price and their correlation, we use the rental and price indices for Grade A offices in Hong Kong, and calculate the property's capital return (based on the price index) and rental return (based on the rental index).[11] The Hong Kong Monetary Authority's 30-day Exchange Fund Bill is used as a proxy for the risk-free rate. To calculate the tenant's expected opportunity cost of capital, we use the average beta (β_1) for the tenant's industry and the average beta (β_2) for the landlord's industry to estimate the opportunity cost of capital. To estimate average β_1 and β_2 we use appropriate Hang Seng Sub-indices

[9] A standardized version abstracted from a sample of thirty leases is as follows: "It is agreed that upon expiry of the term, the tenant has an option whereby the agreement can be extended for a further stated term at the then market rent, with the same conditions as the present agreement, excluding the option to renew." Tenants are typically required to inform the landlord within three or six months of the expiration of their intention to renew.

[10] Comparability details include tenancies in high-rise buildings above ground level, varying business sectors, varying rental values, and with only one embedded option: to renew at future market rent. We only selected contracts entered into and terminated in the economically expansionary period between 1992 and 1997 to mitigate otherwise secular private risks.

[11] The data are extracted from: https://www.rvd.gov.hk/en/publications/property_market_statistics.html.

that include the tenant and landlord's industries, and the Hang Seng Index to represent the market.

To simplify the estimates for transaction costs, we concentrate on institutional costs (commission fees, legal fees, stamp duty) and relocation costs (moving, decoration/fit-out), which can be obtained or estimated from market participants, comparable cases, and with the expert assistance from representatives of the Hong Kong Estate Agents Authority. We presently ignore other unquantifiable or minor transaction costs and assume that in a well-functioning market with good information, the cost of agreeing on a new rent is negligible. To avoid further complexity, we assume that depreciation is constant straight-line.

We follow Luenberger (2013) and use the tenant's assumed initial wealth in the exponential approximation to estimate its risk aversion coefficient. Another reasonable assumption is that the tenant must at least have the initial capital to pay market rent upon entering the leasing market. We therefore use the monthly market rent compared to the monthly payoff to approximate the risk aversion coefficient. To calculate the probability that a tenant will purchase rather than lease, we use quarterly periods (in testing, monthly data did not alter the general results). Price and rental data for calculating the probability of purchasing is drawn from the historical Grade A Office Price and Rental data series published by the Hong Kong Government's Rating and Valuation Department. We assume further that the landlord's transaction costs primarily include commission fees, stamp duties, and legal fees, which can be obtained in the same way as for tenants. We further assume that initially, the vacancy cost per unit area (mainly management costs) is constant and can be investigated using documented leases. The data required to calculate the vacancy period include the estimated expected market vacancy rates, which are also calculated from the historical data series published by the Hong Kong Government's Rating and Valuation Department.

Model demonstration

To demonstrate the model and valuation process, one lease case among the sample of office leases (Case 10) is chosen to price the option to renew at future market rent. The premises for this lease contract are located in a Grade A office building with a net area of 5500 square feet. The lease was contracted in 1997 and started on March 1, 1997, running to the end of February 2000, a total of 36 months. The actual rent is HK$231,660 per month and the management fee is HK$28,600 per month. The building was five years old in 1997. The landlord offered the tenant an option to renew for another three years at the future market rent after this lease expired, with notification required from the tenant not more than six months but not less than four months before the lease expiration date. No other options are embedded in this lease. We assume that the term (l) of a new lease after the original lease expires is also 36 months, as the renewal lease and purchased asset have building characteristics that are similar to those of the present leased asset with the same remaining years (n).

To calculate the option price, we first need to obtain model parameters consistent with the market and the specific case at the proper time.

Transaction costs Transaction costs are important for calculating the tenant and landlord payoffs. For tenants, the transaction costs investigated are mainly from four practical sources: commission fees, stamp duty, legal fees, and relocation costs (including moving costs and fitting-out costs) for contracting a new lease. For landlords, we only consider the first three parts. The transaction costs for the renewal lease are expected to involve only stamp duty and legal fees.

Commission fee In terms of the information from the Estate Agents Authority in Hong Kong, we assume the leasing commission fee for each party is a half-month's rent, and the commission fee for purchasing an office asset is 1% of the total price.

Stamp duty In Hong Kong, the stamp duty in lease transactions is usually shared by the two parties but the stamp duty payable in a conveyance is usually paid by the purchaser. The standard rates for different price ranges are collected from the Estate Agents Authority and the Inland Revenue Department in Hong Kong.

Legal fees After consulting with legal practitioners, we assume the legal fee for both the tenant and landlord for leasing is about 1% of the annual rent, and the legal fee for the purchaser (tenant) is 1% of the purchase price.

Relocation costs We only consider the main components of relocation costs, namely the internal fitting-out costs and the cost of hiring a mover to move to a new place, although relocation costs can include other relatively minor costs. The relocation costs for each lease are estimated after consulting several property agency companies. After consulting these companies, we assume that the relocation costs are about HK$800 per square foot, including fitting-out costs and moving costs. If the tenant chooses to purchase, the prospective asset should be different property and therefore, relocation costs will also be incurred.

Based this information, the transaction costs can be summarily calculated in Table 4.

Cost of capital and depreciation Following CAPM principles, market returns (r_m) can be estimated from stock market data. We use the Hang Seng index as a proxy for the Hong Kong stock market. The average historical monthly return on the Hang Seng index from 1987 to February 1997 is 0.01354. We assume the monthly rate of market return is 0.01354 in February 2000 and that it remains constant. The yield of Hong Kong's 30-day Exchange Fund Bill is chosen as the proxy for the risk-free interest rate. It was 4.86% in February 1997.

We can use the average $ß_1$ for the tenant's industry as an estimate of the tenant's opportunity cost of capital and likewise, use the average $ß_2$ for the landlord's industry (property) as an estimate of the landlord's opportunity cost of capital. For our present purposes, we choose to use the sectoral index compiled as part of the Hang Seng index service. The tenant's industry belongs to the Commerce and Industry sector. We regress the Commerce and Industry Sector Index return and the Property Sector Index return on the Hang Seng index return, respectively, from January 1987 to January 1997. Before regressing, all returns are net of the corresponding risk-free interest rates. We obtain $ß_1 = 1.06$ and $ß_2 = 1.18$. If we assume these are stable during the next three to six years, we can estimate the opportunity cost of capital for the two parties. Thus, we can calculate the tenant's and landlord's opportunity costs of capital, which are 1.41% and 1.52% per month, respectively.

Table 4 Calculation of transaction costs

Items	Cost (HK$)	Notes
Transaction costs (*S*) for tenants		
Renewal lease (S_1)		
Stamp duty	R*5500*12*0.0025	Annual rent*0.5%*0.5
Legal fee	R*5500*12*0.01	Assume 1% of annual rent
Subtotal	R*5500*0.15	
New lease (S_2)		
Commission fee	R*5500*0.5	Half month rent
Stamp duty	R*5500*12*0.0025	Annual*0.5%*0.5
Legal fee	R*5500*12*0.01	Assume 1% annual rent
Relocation costs	800*5500	Assume 800HK$ p.s. f
Subtotal	R*5500*0.65 + 4,400,000	
Purchasing: (S_3)		
Commission fee	P*5500*0.01	1% of price
Stamp duty	P*5500*0.0375	3.75% of price
Legal fee	P*5500*0.01	Assume 1% purchase price
Relocation costs	800*5500	Assume 800 HK$ p.s. f
Subtotal	P*5500*0.0575 + 4,400,000	
Transaction costs (*T*) for landlords		
Renewal lease: (T_1)		
Stamp duty	R*5500*12*0.0025	Annual rent*0.5%*0.5
Legal fee	R*5500*12*0.01	Assume 1% of annual rent
Subtotal	R*5500*0.15	
New lease: (T_2)		
Commission fee	R*5500*0.5	Half month rent
Stamp duty	R*5500*12*0.0025	Annual rent*0.5%*0.5
Legal fee	R*5500*12*0.01	Assume 1% of annual rent
Subtotal	R*5500*0.65	

Depreciation is a very complex subject and is poorly understood for real estate and other fixed assets. For simplicity, we assume the distribution of the economic depreciation rate for the leased office asset is stationary over time, and the residual value is zero. Following accounting conventions, we assume the economic life of an office building in Hong Kong is 40 years; this results in an annual straight line depreciation rate of 2.5%.

Vacancy cost and vacancy rate To simplify the problem, we consider a management fee as the only vacancy cost (V). According to the original lease contract, this cost is HK$28,600 per month in this case. To calculate idle time, we need to estimate the market vacancy rate in 2000. Based on the historical data, we use a (5-year weighted) double moving average to estimate the market vacancy rate in February 2000 as 10.89%.

Estimation of variables The estimation of the probability of leasing or purchasing is influenced by market risks and follows the methodology introduced in Appendix 1. The lease term in this case is 36 months or 12 quarters. We take one quarter as the calculation unit step and then calculate the quarterly volatility of rental returns and property returns based on the returns estimated from quarterly rental and capital value indices from January 1986 to February 1997. We obtain volatilities of 0.0656 and 0.1009 for rental and capi-

tal values, respectively. We calculate the correlation coefficient of the quarterly returns of rents and capital values based on the historical indices from January 1986 to February 1997 and obtain $\rho = 0.4647$. After calculating the parameters for the binomial double lattice (e.g., up and down factors and risk-neutral probabilities) and solving Eq. (A1) in Appendix 1, we obtain the probabilities $p_{11} = 0.4234$, $p_{12} = 0.1535$, $p_{21} = 0.1120$, and $p_{22} = 0.3111$.

The expected equivalent user costs of owning (ECO), renewing (ECL_1), and a new lease (ECL_2) can be calculated based on Eqs. (1)–(3) and the estimated parameters (e.g., the opportunity cost of capital, depreciation rate, expected rental, and transaction costs). We then compare the expressions with only rent and price variables—ECO and ECL_1—under each possible combination situation of rent (R) and price (P) following the double lattice logic. If max (ECL_1, ECO) = ECO, the tenant will purchase instead of renewing. Summing up all the probabilities to purchase under all situations at the end of the lease, the probability of purchasing while the tenant holds a renewal option, $q_1 = 0$, can be obtained. Using a similar method and logic, by comparing EPL_2 and ECO, the probability of purchasing when there is no renewal option, $q_2 = 0.000019$, can also be estimated. The Delphi method is used to estimate the probability of renewing or moving influenced by private risks. The experts' consensus opinion about the probability of renewal given the private risks for this case is $q_3 = 0.7$.

The market rent in the first quarter of 1997 is HK$ 43 per square foot (psf)/month and the market price in that quarter is HK$12,045 psf. We suppose the market rent and price in February 1997 are at the same level. We then use returns from the rental and capital value indexes to calculate the volatility, up and down movement, and corresponding probabilities. Starting with the rent or price in February 1997, we can obtain the expected market rent and price at the end of the original lease (February 2000) using the binomial tree calculation, that is, HK$ 49 psf/m and HK$13,940 psf, respectively.

To calculate the vacancy period/idle time, we need to estimate the office asset's utilization rate to calculate the effective arrival rate λ first based on the model developed in Appendix 3. Taking the expected market utilization rate as the average utilization rate of an office asset, we then determine the average utilization rate as 0.891, given the expected office market vacancy rate of 10.89%. We assume that the lease contract needs at least one month for preparation before it can start and observe that the landlord requires a maximum 6-month notification time. Therefore, we obtain the preparation time, $a = 1$ month, and the notification time period, $n = 6$ months. Since the lease term in the case is 36 months, l=36 months. Replacing $E[u]$, a, n, l with 0.891, 1, 6, and 36 in Eq. (A8) in Appendix 3, the effective arrival rate λ can be calculated as 0.095. Replacing λ and a with 0.095 and 1, respectively, in Eq. (A4) in Appendix 3, the idle time obtained is 6.5 months.

Option value and price In terms of the expected rent and price and the parameters, the tenant's payoffs for owning (ECO), renewing (ECL_1), and a new lease (ECL_2) are calculated as HK$ −1,366,042/month, HK$ −272,403/month, and HK$ −433,940/month from Eqs. (1)–(3), respectively. To combine the private risk and project the problem in incomplete markets into an equivalent problem with complete markets, we calculate the certainty equivalent $ECE(x)$ using the probability of renewal influenced

by private risks $q_3 = 0.7$. It is reasonable to assume that the tenant has at least the initial capital to pay the market rent of the office asset when they enter into the office leasing market. Comparable to the monthly payoff, we therefore assume a is equal to the monthly market rent. From Eq. (4), we have $ECE = -333{,}485$.

Based on ECE and the expected equivalent user costs of owning ECO and the probability of purchasing influenced by market risks when there is a renewal option, that is, $q1 = 0$, we obtain the payoff for the tenant holding the renewal option as HK$ −333,485 from Eq. (5). Given the probability of purchasing when there is no renewal option $q2 = 0.000019$, the payoff for the tenant without a renewal option is HK$ −433,958 from Eq. (6). We then use the risk-free rate to discount the payoffs back to the original lease origination date. The resulting option value for the tenant is HK$3,129,045 (or HK$86,918 per month).

To obtain the landlord's option value, we first input the parameters, expected rent, and vacancy period/idle time estimated above into the landlord's payoff equations (Eqs. 8 and 9). We obtain the equivalent payoffs of renewing $EPL_1 = 269{,}481$ and of a new lease $EPL_2 = 203{,}942$. Through the tenant's payoffs under different scenarios (ECO, ECL_1, and ECL_2) and the probability influenced by market risks, the probability of renewal, $q_4 = 0.6219$, can be calculated using Eq. (7). The landlord's payoff when there is a renewal option is HK$244,699 from Eq. (10). The landlord's payoff when there is no renewal option is HK$203,942 from Eq. (11). We can obtain the landlord's option value at the lease commencement date by discounting the payoff with the option using the risk-free rate and discounting the payoff without the option using the landlord's opportunity cost of capital. The option value is HK$3,326,944 (or HK$92,415 per month).

Based on the tenant and landlord's option values, the option price is: $0.5 \times (3{,}129{,}045 - 3{,}326{,}944) = -HK98,949.

This result is negative, which means the option to renew is more valuable to the landlord than the tenant. Given that the lease's total market value on March 1, 1997 is HK$8,428,860, the option price equals −1.17% of the total lease value. The landlord must offer a 1.17% discount (HK$98,949) to the tenant for the tenant to accept the option.

Tenant and landlord option prices and option values

Based on these assumptions, Table 5 presents the option values for both parties and option prices for all 25 cases.

To compare this with different lease values, we use option prices and option values for both parties, expressed as percentages of the lease value. Option prices vary from 0 to 26.63% of the lease value in situations that yield a premium to the landlord, and from −0.52 to −6.85% in situations where the tenant receives a discount. The average percentage is 4.77% with a standard deviation of 0.0889; the absolute value of the percentages is 7.21%, both less than 10%. Most price estimates are considered to be in a reasonable price range, Therefore, from a practical viewpoint, the model's performance is considered acceptable.

Table 5 Option price and option value for both parties (HK$ thousand)

Cases	Option price	Option price (% of lease value)	Value to tenant	Value to landlord	Original rental (/ month)	Original lease term (month)	Renewal lease term (month)
1	421.40	12.66	1312.91	470.10	79,326	24	12
2	352.20	3.26	2193.60	1489.20	210,000	36	12
3	0.00	0.00	2021.80	2023.07	334,165	24	24
4	− 528.45	− 6.64	1950.90	3007.80	217,061	36	36
5	− 593.28	− 6.62	2982.19	4168.76	245,448	36	36
6	44.28	1.33	790.65	701.44	173,810	24	24
7	− 274.09	− 2.89	3178.58	3726.76	289,198	36	36
8	309.71	10.75	1004.34	385.19	106,766	24	12
9	323.94	7.89	1198.15	549.82	96,148	24	12
10	− 98.95	− 1.17	3129.05	3326.94	231,660	36	36
11	148.69	26.63	371.49	74.11	52,920	12.6	12
12	121.39	2.94	1196.93	953.33	75,000	24	24
13	226.07	4.33	929.25	476.67	116,864	36	12
14	183.58	13.56	528.64	161.67	53,521	24	12
15	445.50	6.52	2734.78	1842.59	205,000	24	24
16	340.36	6.30	3024.05	2345.11	129,216	24	36
17	121.02	1.79	2838.22	2597.32	153,828	36	24
18	− 13.62	− 0.52	937.22	964.45	61,347	36	36
19	− 249.26	− 6.85	838.07	1336.59	204,720	36	36
20	− 337.03	− 3.61	3106.99	3782.80	173,000	36	36
21	171.60	20.99	635.63	292.42	70,528	13	36
22	806.34	10.96	2667.40	1054.72	282,255	24	12
23	− 201.96	− 2.22	1366.20	1770.12	190,890	36	24
24	32.35	0.17	4361.98	4299.78	291,760	36	24
25	242.98	19.69	669.02	183.05	39,500	24	12

In general, option prices are positive in these cases, indicating that, in most cases, tenants are willing to pay a premium to hold a renewal option. The premiums paid by tenants are on average larger than the discounts offered by landlords. On the one hand, during an economic expansion period, reallocation costs such as decorating costs, losses due to business interruption, the loss of location goodwill, and other costs paid by tenants after moving to a new office are relatively high. The tenants also have high probabilities of renewing the lease contract due to their private business circumstances (private risks) and thus are willing to pay for the renewal option. On the other hand, landlords' transaction costs could be lower because of low vacancy rates and low vacancy risk during the economic expansion period. The benefits of the renewal option are thus smaller for landlords than for tenants, leading to positive option prices. The findings imply that due to transaction costs, landlords have relatively greater bargaining power than tenants in negotiating lease contracts when economic conditions are good.

From the basic case information and observing and analyzing the results, it is clear that there is a relationship between option price/option value for both parties and the original lease term/renewal term. All leases with negative prices have longer terms (36 months), a notable result even without a-priori expectations. No observable special

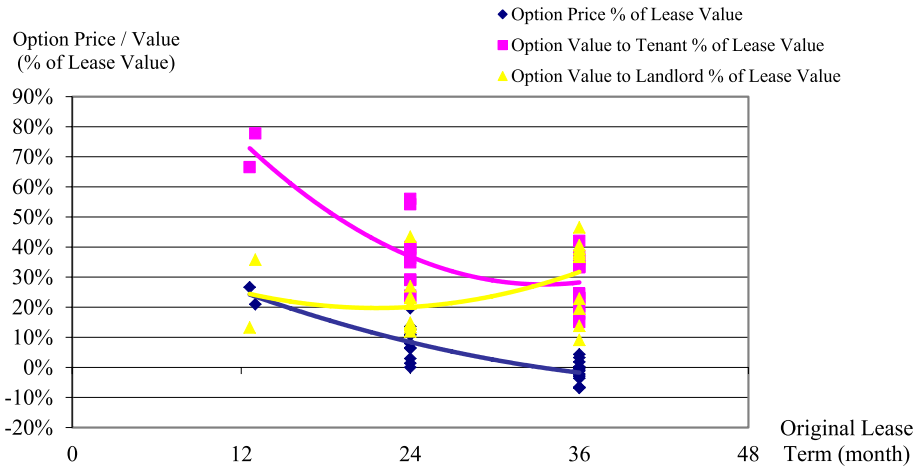


Fig. 3 Option value/price with original lease term

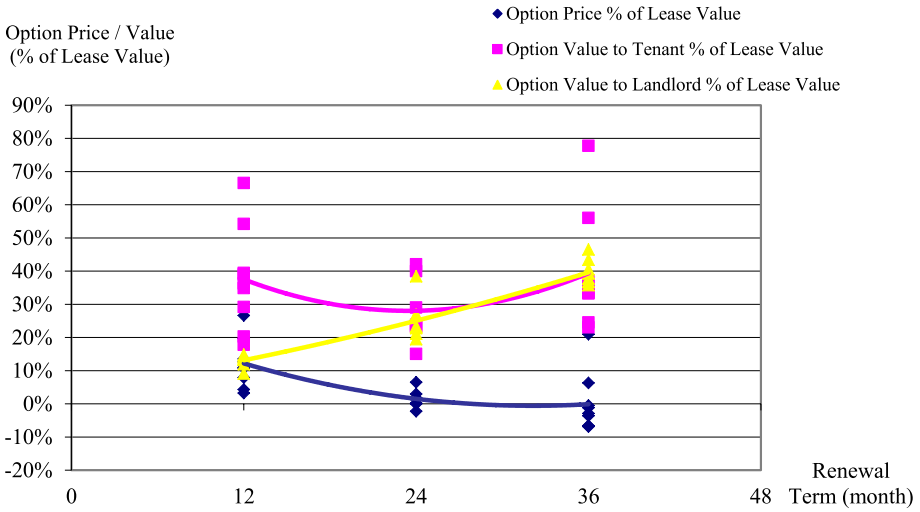


Fig. 4 Option value/price with renewal lease term

relationship was observed between option prices and the contracted term. To observe their relationship with terms and trends, the percentages of value and price for both parties for all cases are presented with the lease and renewal terms separately in Figs. 3 and 4, and combined in Fig. 5.

Figures 3 and 4 generally show a negative relationship between option price/value for both parties and the original/renewal lease term. Figure 3 shows that the tenant's option value has a negative relationship with the original lease term: the longer the term, the lower the value. The landlord's option value generally increases as the original lease term increases, but it is marginally less for 24-month leases/renewal leases than for 12- and 36-month leases, exhibiting a slightly concave shape.

Figure 4 shows that the landlord's option value has a clear positive relationship with the renewal lease term: the longer the term, the greater the value. However, the tenant's option values show a concave shape with the tenant's value for 24-month renewal leases lower than the values for 12- and 36-month renewal leases. This may be because the

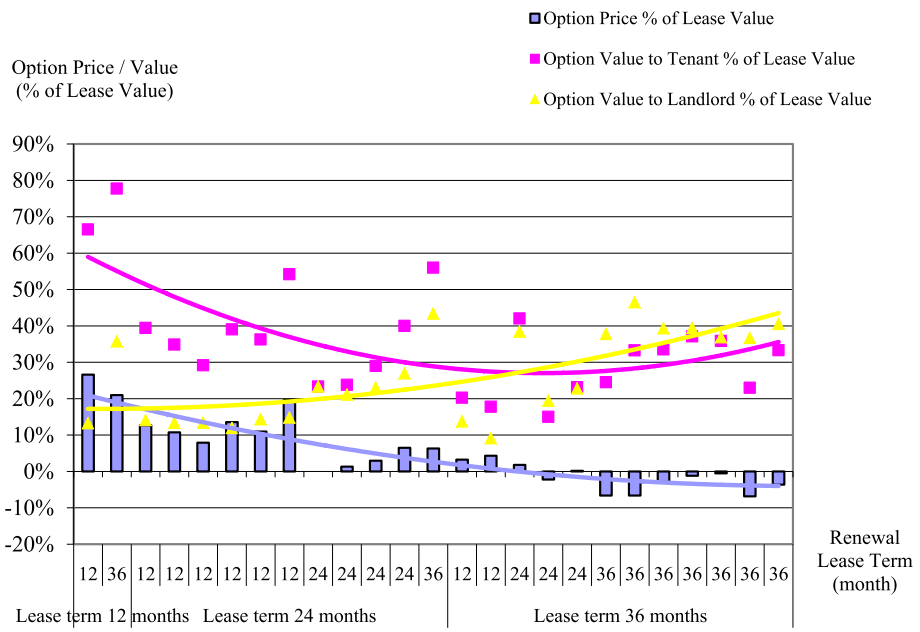


Fig. 5 Option value/price with original and renewal lease terms

original term may not be the same as the renewal term in some cases, resulting in a slightly concave shape for the landlord's option value in Fig. 3 and for the tenant's option value in Fig. 4. If the effect of such a difference is excluded, the landlord's option value has a clear relationship with the original term, and the tenant's option value has a negative relationship with the renewal term. We conclude from these analyses that the tenant's value and the price have negative relationships with the lease term (original term/renewal term), and the landlord's value has a positive relationship with the lease term (original/renewal term). This differs from a common stock option where the option price is positively related to the term.

The tenant's option value with a changed term may be explained by the negative relationship between unit transaction costs and the term. The shorter the term, the greater the distributed costs over one period, the more the tenant could save, and thus the more valuable the option is. Given the composition of payoffs, transaction costs may have less influence on the landlord's payoffs than on the tenant's. Instead, rental income is more important to the landlord. The longer the term, the greater the potential vacancy risk may be, and the more the costs associated with a loss of rental income that may be avoided with the option. The benefit of avoiding the rental income loss may exceed the negative effect of transaction costs, and thus, the landlord's option value is shown to be greater in longer leases. Different renewal strike price assumptions yield different results than shown here; for example, Grenadier (1995) shows the premium of the option to renew *at a fixed rent* has a positive relationship with the renewal lease term, and Clapham (2003) shows that the term structure of the option premium for *an indexed renewal option* is hump-shaped. Both results are theoretical and not drawn from actual case applications and so are not directly comparable.

When integrating original terms and renewal terms, we see similar results in Fig. 5 as those found in Figs. 3 and 4. In addition, Fig. 5 shows in general that given the same

original lease terms, the longer the renewal term, the lower the option price but the higher the option value for the landlord. This apparently negative term structure of option prices could possibly be explained by agency theory. Pretorius et al. (2003) argue that two parties have the greatest incentive conflicts in short term leases, leading to the relatively highest expected equivalent rent for short term leases. However, the incentive conflicts reduce with the lease term, and therefore, expected rents for longer terms may be comparatively lower. The same logic can be applied for the renewal option price.

Overall, most option prices are positive, and the absolute values of negative prices are also small (compared to the corresponding lease value). This suggests that, during the study period, tenants had to pay landlords or were offered a minor discount to obtain the option, reflecting a market generally favorable for landlords during that time. Intuition thus suggests that option price/value may also have some relationship with market cycles, a topic for further research.

Comparative analyses

We conducted comparative analyses to examine the impact of important factors on price and value for both parties, following previous studies in the real options literature (e.g., Zhang et al. 2015; Pasricha et al. 2022). These factors include variables from the perspective of traditional options: rent, price, risk-free interest rate, volatility of office rent and price, the asset's depreciation rate, and tenant's beta. Each factor was varied within a reasonable range, holding the other factors constant, to observe the resulting variance in the option price/value for each case.

We examine the effect of transaction costs on option price and option value for the landlord and tenant by first simultaneously varying the overall transaction costs (T) under different scenarios for the two parties from the 20% level ($0.2T$) to the 200% level ($2T$), with 20% as the incremental change unit. Figure 6 shows the corresponding option prices and value for both parties.

Figure 6 shows that, overall, transaction costs are positively related to option price. It also shows that, overall, transaction costs are positively related to the tenant's option value, and negatively related to the landlord's option value. The results show that as transaction costs increase, the overall probability of renewal (q_4) slightly declines instead of increasing. With the presence of private risk, the tenant may not be more likely to renew when transaction costs increase. The landlord's payoff changes slightly with transaction costs, and the negative effect of the probability of renewal exceeds the positive effect of transaction costs. The landlord's option value thus declines with increased transaction costs. When transaction costs decline below around T, the option price becomes negative, and the landlord must offer a discount, which then increases as transaction costs decline. Conversely, when transaction costs increase above around T, the option price becomes positive, and the tenant must pay a premium, which increases with greater transaction costs.

We thus see that higher transaction costs result in a higher tenant option value, all else equal. This reflects potential increased savings when an option is included, thus making it more valuable to the tenant. Furthermore, Fig. 6 shows that the tenant's option value is more sensitive to transaction costs than that of the landlord. This may be because the tenant's transaction costs are comparatively greater and potentially more disruptive,

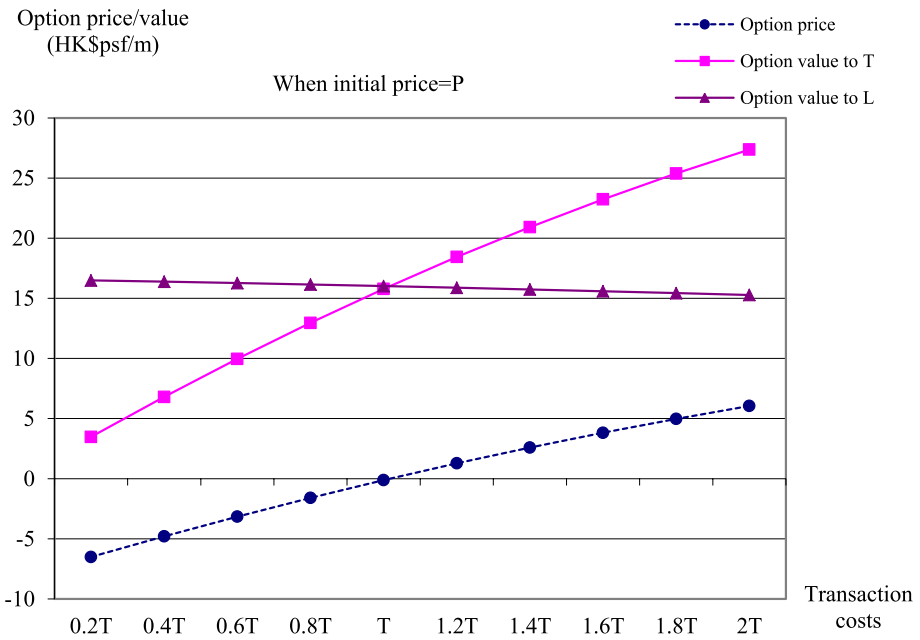


Fig. 6 Effect of transaction costs on option price and option value for both parties

while the effect of a loss of rental income in a vacancy period may be relatively more important to the landlord. The same rates of change in transaction costs may therefore cause greater changes in a tenant's benefit or costs compared to those of the landlord. We disaggregated transaction costs in further testing and found that relocation cost (probably the largest part of the tenant's transaction costs) is positively related to the tenant's option value and option price, but negatively related to the landlord's option value.

Vacancy is one of the most important concerns of the landlord, as it directly affects potential rental income and vacancy costs. A main advantage of the option is that it may help the landlord avoid a vacancy period if it is exercised. Figure 7 shows the relationship between the vacancy period and option price/value for both parties, with a vacancy period varying from 0 to 18 months, holding other factors constant.

Figure 7 shows that the vacancy period clearly has a positive relationship with the landlord's option value, consistent with our intuition. The longer the expected vacancy period, the more valuable the option and the more willing the landlord is to grant the option to hedge the possible rental income loss and vacancy costs. The longer the expected vacancy period, the greater the landlord's potential benefit. Vacancy has no direct impact on the tenant, and the tenant's option value remains unchanged. Therefore, all other factors constant, the option price declines with an increase in the vacancy period. When the expected vacancy period is longer, the landlord has greater incentive to offer the option. The same effect is shown in the relationship between vacancy cost and option price/values.

Figure 8 shows the relationship between the vacancy period and option price/value for two parties by varying the probability to renew influenced by private risk from 0.1 to 1. The probability has no impact on the tenant and landlord payoffs, but influences the option value for both parties when the payoffs are rolled back through the payoff

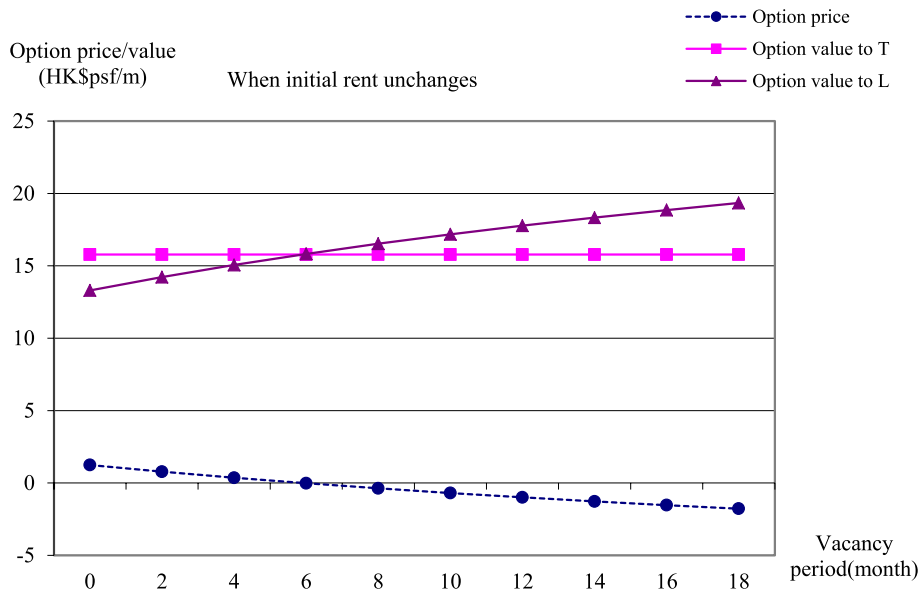


Fig. 7 Effect of vacancy period on option price and option value for both parties

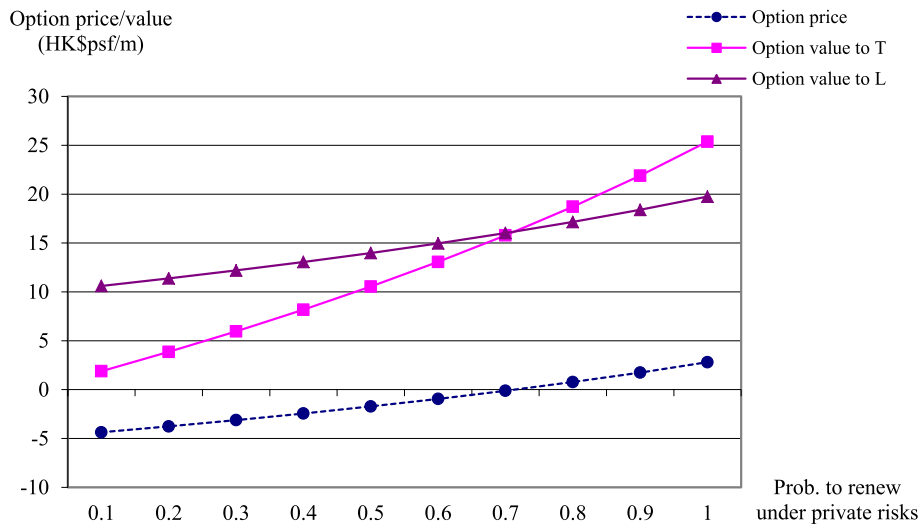


Fig. 8 Effect of probability of renewal under private risks on option value/price

trees. The probability of renewal under private risks thus influences the overall probability of renewal throughout the tree. The result shows that the probability of renewal has a positive relationship with option value for the two parties and the option price.

The expected office rent directly influences the option's value and price. In the model, however, its influence is estimated based on the initial rent. Therefore, to observe the impact of rent on price and value for both parties, we hold the initial price at its original level (P) and other factors constant, and vary the initial unit rent from HK$5psf/m to HK$100psf/m. Figure 9 presents the relationship between initial rent and option price/values.

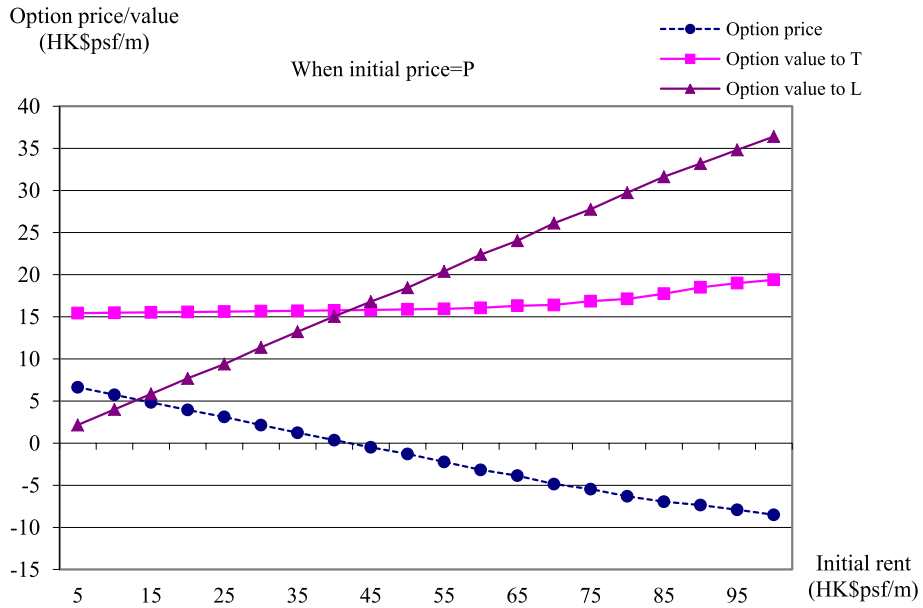


Fig. 9 Effect of initial rent on option price and value for both parties

The results show that the initial market rent has a significant impact on option value/price. The impact is exerted through the transaction costs and loss of rental income linked to payoffs and probability of renewal; although the option value/price is not directly reflected in the difference between future market rent and strike rent, as traditional financial options would suggest. The option price has a negative relationship with rent, unlike the positive relationship in traditional options price theory. The greater the initial rent, the more valuable the option is to both parties. Nonetheless, the landlord's value is more sensitive to the initial rent than the tenant's value; thus, the initial rent has a negative relationship with price. The different sensitivity of value to initial rent for the two parties also implies that a change in the initial rent has a greater impact on the landlord's payoff. This is mainly attributed to a substantial increase in the loss of rental income during the possible vacancy period, compared to other value determinants in the two parties' payoffs. With an increase in the initial rent, the landlord would be more willing to offer the option to hedge the greater risk of a loss of rental income. At the same time, transaction cost savings for the landlord also increase with increased initial rent, further increasing the impact of the initial rent on the landlord's option value.

When the initial rent is quite low compared to office asset prices, the probability that a tenant will choose to purchase is zero or very low and thus can be ignored. The tenant's option value now mainly reflects the transaction cost savings. Relocation costs form the largest part of the transaction costs and effectively do not change with a change in the initial rent; thus, they have a marginal effect on the tenant's option value. This is reflected as a relatively flat line in Fig. 9. If the gap between the rent and price is small due to higher initial rent, the probability of purchasing could increase significantly while the probability of renewal drops. In such cases, the tenant's option value would also decline, indicating that the option has no value for the tenant when rent is high compared to purchasing.

The results of comparative analyses also confirm that the risk-free rate exerts its effect mainly through the time value of money. It thus has a negative relationship with the option

values for both parties, and a positive relationship with the option price, similar to a common stock call option. However, when varied within a reasonable range, the rent volatility, price volatility, depreciation rate, and tenant's beta do not show clear relationships with the option price/value of either party. The non-monotonous relationship between the volatility of rent/price and the option price/value may be due to the impacts on option price/value of multiple variables and the correlation between rent and price. The impact of private risk may also be a reason. Nonetheless, the depreciation rate has a negative impact on the probability of purchasing and a small positive effect on the probability of renewal.

On the whole, the comparative analyses confirm the effect of the determinants of option value and thus support the model's logic. Comparatively, transaction costs, particularly relocation costs, are shown to be more important in determining the tenant's option value, while hedging the loss of rental income including the effects of market rent and vacancy period is shown to be more important in determining the landlord's option value. The probability of renewal influenced by private risks is shown to have a positive relationship with the option's value for both parties and with the option price.

Conclusion

In this study, we develop a valuation model for a tenant's option to renew a constant rental office lease at future market rent under an incomplete market assumption. The model combines the real options approach and decision analysis (the *integrated method*), assumes both market and private risks, and incorporates the economic characteristics of conventional office leases. The empirical results and comparative analyses show that the valuation model generates practically reasonable option values when tested with a sample of actual office leases. It generally shows a negative term structure of option price/value for tenants and positive term structure of option value for landlords. The original lease terms and renewal lease terms in general also seem to have an impact on option price and option value for both parties, albeit not a very strong one. Unfortunately, no data of actual option prices exist to judge the accuracy and robustness of the results, a recurring problem in studies of embedded contingent claims.

Our study is the first to apply an option pricing approach to evaluate the tenant's option to renew at future market rent and appears to be the first to empirically estimate option values for both tenants and landlords. The model and empirical results indicate that the boundaries and prices of a renewal option at future market rent can be positive or negative and depends on tenants' private circumstances, vacancy costs, and transaction costs. This study thus contributes to the option pricing literature. Our valuation model differs from real option models in previous studies (e.g., Buetow and Albert 1998; Foo Sing 2012; Nugroho 2016) in the following aspects. First, it considers the transaction costs for both parties in the lease contract. Second, we show that private risk (related to a tenant's business conditions) and both parties' transaction costs could also affect the option value. Last, we develop a flexible valuation framework that incorporates market risks, private risks, and different scenarios of option holders and counterparties. This model can be easily extended to price embedded options in transactions such as fair market value renewal options in different types of lease contracts, options to purchase in the future (e.g., in the aircraft market), and options to automatically renew employment contracts. The model can be used to calculate option value/price for both parties in the transactions.

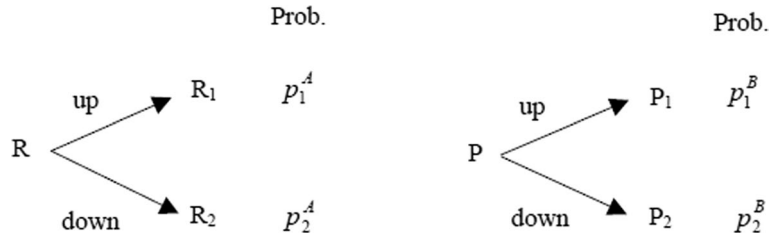
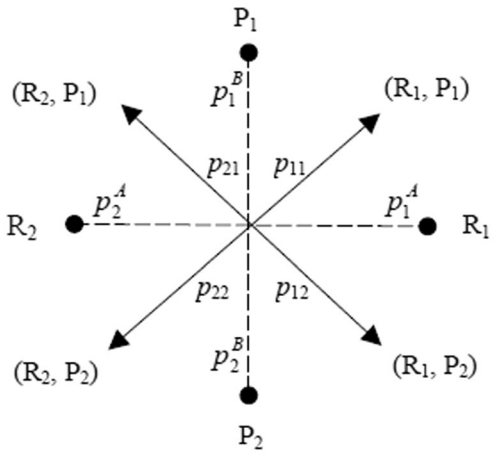


Fig. 10 Individual binomial movements of rent and price

Appendix 1: Assessing the probability of lease renewal influenced by market risks

Normally, when considering the decision to purchase or lease an office, tenants will compare the equivalent user costs of owning and leasing where rent and price are two main variables. Since rent and price are both random variables and expected to be correlated, we use Luenberger (2013) double lattice approach to estimate the probability (Prob.) to purchase. Separately, rent (R) and price (P) have the following binomial movement in one time period (Fig. 10).

Suppose A represents rent and B represents price, then R_1 in up movement and R_2 in down movement have corresponding probabilities p_1^A and p_2^A. Likewise, P_1 in up movement and P_2 in down movement have corresponding probabilities p_1^B and p_2^B. When two variables change simultaneously, there will be four possible successor movement combinations: (R_1, P_1)—(rent up, price up), (R_1, P_2)—(rent up, price down), (R_2, P_1)—(rent down, price up) and (R_2, P_2)—(rent down, price down) and corresponding probabilities: $\boldsymbol{p_{11}}$, $\boldsymbol{p_{12}}$, $\boldsymbol{p_{21}}$, and $\boldsymbol{p_{22}}$. The first notation in the double subscript is for the movement state of rent and the second one is for the movement state of price. **1** means it will be up in the next step and 2 means it will move downward in the next step. Thus a picture of the lattice for the movement combinations of rent and price and corresponding probabilities is shown (Fig. 11).



Source: Developed from Luenberger (1998, p. 453)

Fig. 11 Nodes and probabilities of the combination

Following Luenberger (2013), the probabilities for the four successor movements of two correlated variables must satisfy:

$$\begin{aligned} &p_{11} + p_{12} = p_1^A \\ &p_{21} + p_{22} = p_2^A \\ &p_{11} + p_{21} = p_1^B \\ &(p_{11} - p_1^A p_1^B)U^A U^B + (p_{12} - p_1^A p_2^B)U^A D^B \\ &\quad + (p_{21} - p_2^A p_1^B)D^A U^B + (p_{22} - p_2^A p_2^B)D^A D^B = \sigma_{AB} \end{aligned} \tag{A1}$$

where $U^A = \ln u^A$, $D^A = \ln d^A$,$U^B = \ln u^B$, and $D^B = \ln d^B$, σ_{AB} is the covariance of the logarithm of the two returns $\sigma_{AB} = \rho\sigma(A)\sigma(B)$, ρ is the corresponding correlation coefficient of the returns of rent and price of Grade A office assets.

Thus with the necessary parameters, the probabilities for the four successor movements—$p11$, $p12$, $p21$, and $p22$.can be calculated. These probabilities are further used to derive the probabilities to purchase (q_1/q_2).

Appendix 2: Assessing the probability of renewal influenced by private risks

To estimate the probability to renew that is influenced by private risks, we conducted a Delphi study (e.g., Dixon et al. 2009). The first step in the Delphi study required expert participants to prioritise a given list of factors considered to be important private factors that will influence a tenant's decision to exercise the option, through asking the participants to scale the importance of the factors. Participants were also invited to propose any additions to the list, if thought important. These opinions were combined into a "group consensus opinion" about the priority of private factors which were communicated to participants, and reassessment was requested. This step provided the basis for participants to estimate the probability for actual cases. Table 6 lists key private factors and their priority.

In the second step, each participant was asked to estimate the probability that a tenant will renew her office lease at the option exercise date for each case, based on the prioritized private factors and key information about the tenant and the leased office for each

Table 6 Key private factors and priorities

Priority	Key private factors
1	Growth/contraction of the tenant's business
2	Organizational change (acquisition, merger, etc.)
3	Special requirement from facilities, e.g., high IT and telecom specification
4	Transportation accessibility
5	Proximity effect: direct business purposes, e.g., location close to other organizations in a similar business
6	Quality of building management
7	Building prestige
8	Amenities/facilities, e.g., gymnasia, parks, restaurants, shopping
9	Proximity effect: indirect business purposes, e.g., location close to other organizations for marketing purposes, for example for visibility
10	Car parking
11	Age of building

Table 7 Panel estimates for the probabilities to renew for 25 cases

Cases	Range	Median	Cases	Range	Median
Case 1	0.3–0.8	0.7	Case 14	0.5–0.8	0.6
Case 2	0.3–0.8	0.7	Case 15	0.3–0.75	0.7
Case 3	0.3–0.8	0.5	Case 16	0.3–1	0.7
Case 4	0.4–0.8	0.65	Case 17	0.4–0.8	0.8
Case 5	0.4–0.8	0.75	Case 18	0.4–0.8	0.8
Case 6	0.3–0.8	0.4	Case 19	0.3–0.8	0.5
Case 7	0.2–0.75	0.6	Case 20	0.4–0.75	0.7
Case 8	0.4–0.8	0.65	Case 21	0.2–0.8	0.6
Case 9	0.4–0.8	0.6	Case 22	0.3–1	0.7
Case 10	0.4–0.8	0.7	Case 23	0.4–0.5	0.4
Case 11	0.6–0.8	0.7	Case 24	0.3–0.8	0.6
Case 12	0.4–0.8	0.5	Case 25	0.3–0.8	0.75
Case 13	0.3–0.6	0.4			

case and excluding the market risks. The participants were requested to reassess until arriving at a "consensus" opinion. The median of the group result was used as the consensus expected probability (q_3) that the tenant will exercise given the private risks. During the procedure, the participants completed the exercise without communicating with each other and were required to make estimates with consideration given to an expansion phase in the Grade A office market.

Table 7 provides the panel estimates for the probabilities to renew for 25 lease cases, including the range and the median of the estimated probabilities for each case.

Appendix 3: Vacancy period for the landlord

Assume the idle time between the expiration of the original lease and the commencement of the following lease is w. When there is a renewal option in the lease, the tenant will make the decision to renew or not during the notification period prior to the termination of the original lease. Since the landlord will not begin to search for a lease too early; in practice we assume when the tenant does not accept the option the time to start searching for a new lease is similar to the commencement of the notification period of the option decision. Assume there are no extra search requirements if the tenant does not exercise the option, compared with a lease without an option. Then we assume the vacancy time between these two contracts will be is similar. Queuing theory may be used in this circumstance, following in principle the method developed by Kenyon and Tompaidis (2001). In the general leasing research area, Kenyon and Tompaidis (2001) developed a model to value those lease options whose exercise decisions are independent of lease rental such as options to renew at future market rent. They apply probability and queuing theory with idle time and the asset utilization rate as key factors to establish a model to price the extension options and termination options.

For our purpose, we assume the effective demand for leasing a single asset follows the Poisson arrival process with constant average arrival rate during the period from the exercise decision time to the commencement of a new lease. In that period, the property

leasing market is relatively stable. No queuing is allowed. If a contract request arrives at time t, the contract may start at a minimum proper period of time $a \geq 0$ after t. This time period a is the minimum preparation time for searching, screening and contracting a new lease and can also be assumed as the minimum time period ahead of lease expiration for the renewal decision. We use l to denote the length of the original lease and L to denote the end of date of the original lease. Thus, n denotes the time period ahead of the lease expiration when the landlord starts to search for a new lease at L-n. We suppose L-n is also the start of the notification time period, which is the length of time period for the tenant to decide whether he will exercise the option with certainty before the expiration time. Thus n is the time period for the landlord to search, screen and contract a new contract before lease expiration irrespective of the presence of a renewal option and can be assumed as the maximum preparation time before a lease can start. The minimum and maximum preparation time period to some extent explains the setting of the notification time period in a lease with a renewal option. Here we call n "*notification time period*" only for the convenience of description, although it differs from the normal notification time period. Then the idle time can be denoted as w (l, n, a). For this research we assume $l \geq n \geq a$. Assume the market utilization rate u, which is equal to one minus market vacancy rate, keeps constant during the idle time and is not significantly affected by a single asset. Let L denote the end date of the original lease and define the following two events:

A: at least one contract request arrives in the period between L–a and L.
B: at least one contract request arrives in the period between L–n and L–a.

We also define $\overline{A}$ and $\overline{B}$ as complementary events to A and B respectively. Then,

- A means the landlord can find a new tenant before the lease expiration after the tenant decides not to renew. Due to preparation time, there may be a little idle time between two consecutive contracts.
- $\overline{A}$ means the landlord cannot find a new tenant before lease expiration. Therefore, there will be idle time before commencement of a new lease.
- B means the following contract can begin immediately after the original lease expires. There is also no idle time between two consecutive contracts.
- $\overline{B}$ means the following contract cannot begin immediately after the original lease expires. There may be some idle time between two consecutive contracts.

Therefore, in event B, the expected idle time between two consecutive contracts is zero, that is $E[w \mid B] = \mathbf{0}$. Since the asset will be occupied without vacancy, the utilization rate in event B is expected to be one, namely $E[u \mid B] = \mathbf{1}$. To make the situation simple, assume the landlord will accept as soon as an effective lease request arrives. Assume $\boldsymbol{\lambda}$ is the effective demand arrival rate. Figure 12 show the framework for different events.

From the properties of Poisson processes,

$$\mathrm{P\{A\}} = 1 - e^{-\lambda a}, \text{ and} \tag{A2}$$

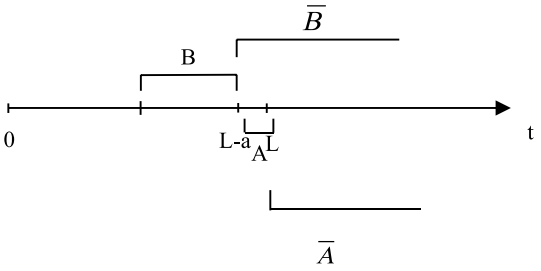


Fig. 12 Idle time

$$\mathrm{P\{B\}} = 1 - e^{-\lambda(n-a)}. \tag{A3}$$

The expected idle time between two consecutive contracts and the expected utilization rate can be given by,

$$\begin{aligned} E[\mathrm{w}] &= \mathrm{P\{B\}}E[\mathrm{w|B}] + \mathrm{P}\{\overline{B}\}E[\mathrm{w}|\overline{B}] = \mathrm{P\{\overline{B}\}(P\{A\}}E[\mathrm{w}\,|\overline{B} \cap \mathrm{A}] + \mathrm{P}\{\overline{A}\}E[\mathrm{w}\,|\overline{B} \cap \overline{A}] \\ &= e^{-\lambda(n-a)}\left[e^{-a\lambda}\left(a + \frac{1}{\lambda}\right) + (1 - e^{-a\lambda})\left(a - \frac{a}{2}\right)\right] \end{aligned} \tag{A4}$$

and

$$\begin{aligned} E[\mathrm{u}] &= \mathrm{P\{B\}}E[\mathrm{u} \mid \mathrm{B}] + \mathrm{P}\{\overline{B}\}E[\mathrm{u}\,|\overline{B}] \\ &= \mathrm{P\{B\}} + \mathrm{P}\{\overline{B}\}(\mathrm{P\{A\}}E[\mathrm{u}\,|\overline{\mathrm{B}} \cap \mathrm{A}] + \mathrm{P}\{\overline{A}\}E[\mathrm{u}\,|\overline{B} \cap \overline{A}] \\ &= 1 - e^{-\lambda(n-a)} + e^{-\lambda(n-a)}\left[(1 - e^{-\lambda a})\int_0^a \frac{1}{a}\frac{l}{l+x}dx + e^{-\lambda a}\int_0^\infty \lambda e^{-\lambda x}\frac{l}{l+a+x}dx\right]. \end{aligned} \tag{A5}$$

The expression E[u] can be transformed to

$$\begin{aligned} \mathrm{E[u]} &= 1 - e^{-\lambda(n-a)} + e^{-\lambda(n-a)}\left[(1 - e^{-\lambda a})\int_0^a \frac{1}{a}\frac{l}{l+x}dx + e^{-\lambda a}\int_0^\infty \lambda e^{-\lambda x}\frac{l}{l+a+x}dx\right] \\ &= 1 - e^{-\lambda(n-a)} + \frac{1}{a}e^{-\lambda(n-a)}(1 - e^{-\lambda a})[\ln(l+a) - \ln l] + e^{-\lambda n}\int_0^\infty \lambda e^{-\lambda x}\frac{l}{l+a+x}dx. \end{aligned} \tag{A6}$$

Assume $\mathrm{z} = l + \mathrm{x} + \mathrm{a}$, λ is constant, then $e^{-\lambda n}\int_0^\infty \lambda e^{-\lambda x}\frac{l}{l+a+x}dx$ can be transformed to

$$l\lambda e^{\lambda(l+a-n)}\int_{l+a}^\infty e^{-\lambda z}\frac{1}{z}dz,$$

where the exponential integral $\int_{l+a}^\infty \frac{e^{-\lambda z}}{z}dz$ can also be transformed to $\int_{(l+a)\lambda}^\infty \frac{e^{-x}}{x}dx$. According to the function $Ei(x) = \int \frac{e^{-x}}{x}dx$ and its series expansion (See Tuma 1998),

$$Ei(x) = \int_{+\infty}^x \frac{e^{-x}}{x}dx = c + \ln x - x + \frac{1}{2}\frac{x^2}{2!} - \frac{1}{3}\frac{x^3}{3!} + \cdots \tag{A7}$$

then,

$$E[u] = 1 - e^{-\lambda(n-a)} + \frac{1}{a}e^{-\lambda(n-a)}\left(1 - e^{-a\lambda}\right)[\ln(l+a) - \ln l] + l\lambda e^{(l+a-n)\lambda}\left[\ln(l+a)\lambda - (l+a)\lambda + \frac{(l+a)^2 * \lambda^2}{2*2!} - \frac{(l+a)^3 * \lambda^3}{3*3!} + \cdots\right] \tag{A8}$$

It can be seen that the expected utilization rate and the expected idle time are the variables that depend on the demand arrival rate **λ**. Set the expected utilization rate *E[u]* of the asset equal to the market expected utilization, namely one minus the expected average actual vacancy rate in the market at $t=1$, then λ can be calculated. In this way, expected idle time w can be estimated. Data required to calculate the expected utilization rate are expected market vacancy rates which we can estimate from the historical data series also published by the Hong Kong Government's Rating and Valuation Department.

Abbreviations

CAPM	Capital Asset Pricing Model
P	Price
Prob.	Probability
psf	Per square foot
R	Rent

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Author contributions
JW collected data regarding the commercial leases. JW and FP constructed the model and run the empirical analysis. JW, JS and FP wrote the manuscript. All authors read and approved the final manuscript.

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Availability data and materials
The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Competing interests
The authors declare that they have no competing interests.

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