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A novel stochastic modeling framework for coal production and logistics through options pricing analysis

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Abstract

We propose a novel stochastic modeling framework for coal production and logistics using option pricing theory. The problem of valuing the inherent real optionality a coal producer has when mining and processing thermal coal is modelled as pricing spread options of three assets under the stochastic volatility model. We derive a three-dimensional Fast Fourier Transform ("FFT") lower bound approximation to value the inherent real optionality and for robustness check, we compare the semi-analytical pricing accuracy with the Monte Carlo simulation. Model parameters are estimated from the historical monthly data, and stochastic volatility parameters are obtained by matching the Kurtosis of the low-ash diff data to the Kurtosis of the stochastic volatility process which is assumed to follow Cox–Ingersoll–Ross ("CIR") model.

Keywords: Stochastic volatility, Real option analysis, Fast Fourier transform method, Coal, Monte-Carlo, Closed-form solution

Introduction

Coal is a combustible black or dark brown rock, mainly consisting of altered plant matter, inorganic matter and water. It is predominantly used for the production of electricity and in the steel making process. Whilst there is a strong political will against the use of fossil fuels like coal, commentators point out that the economical and socio-demographic drivers remain strong, see for instance (Schernikau 2010). The International Energy Agency's World Energy Outlook 2021 has a forecast based on the stated policies of governments, both those implemented as well as in development. This only predicts a reduction in coal demand between 2020 and 2030 of less than 5% (IEA 2021) suggesting that Coal will remain an important commodity globally for several more years.

The thermal coal market places more value on coal with a higher calorific value and a lower content of inorganic matter. Coal with a higher calorific value is less costly to transport and often more suitable for the power station boilers currently in existence. The inorganic mineral matter mainly consists of metal oxides such as silicon and aluminium which are incombustible and so this dilutes the thermal energy content of the coal. Furthermore, the inorganic matter forms ash and clinker in the furnace of a coal fired power station, which can be costly to dispose of. The industry commonly refers to the amount of inorganic mineral matter as the ash content of the coal.

During the production of thermal coal the miner/producer can make various choices such as how the coal is mined, and how it is prepared which affect the quality and the amount of coal produced. For example, in a long-wall underground mine, the height of the section being mined can be modified. Typically, mining a narrow section allows the miner to select for the better quality coal but this is at the expense of not recovering all the resource in the seam. Similarly, industrial techniques such as density separation methods can be used to reduce the ash content of the coal; a process commonly referred to as washing. A sample of raw coal contains particles of all relative density values in a continuous range from the lowest to the highest value. The ash percentage of coal particles increases as their relative density increases, Nicol (1997). Therefore, the density of the medium that is used in the separation method can be varied to achieve different levels of ash content in the final coal product. Whilst washing harder removes more ash, the downside is that more organic combustible matter is also removed. The ratio of the weight of product coal to weight of the coal fed into the wash plant is known as the *yield*.

Historically, the difference in price between high and low quality coals was relatively small and static. The decision on how to mine and wash the coal only needed to be periodically evaluated in a deterministic setting using tools that optimised based on discounted cash-flow analysis ("DCF"). However, more recently, the spread in prices between different grades of coal has become much more volatile and so the optimal configuration for mining would frequently change. One could seek to run DCF based optimisers more frequently, however, the use of DCF models for mine planning has been the subject of criticism (Davis and Newman 2008) given markets follow a stochastic process; not deterministic. Furthermore, the use of DCF optimisers does not allow for the economic valuation of the flexibility inherent in the mining operation. The ability to value such flexibility would allow mining producers to make better investment decisions on plant, machinery upgrades and process change.

A different approach to DCF analysis is required in order to value the flexibility within a mining operation. We have therefore proposed to consider the application of Real Option Analysis ("ROA") on the aforementioned mining and washing flexibilities—the most flexible parts of the coal production process—along with consideration for the stochastic process of coal prices. Which will be the first study of its kind (ROA) in regards to washing flexibilities at a coal mine. With the hypothesis that the value that could be monetized is at a level significant to warrant a change in business practises within the coal mining community.

Real Option is the name given to assets or managerial flexibility that allow for choice and have payoffs similar to financial options such as calls or puts. Real Option Analysis is the application of mathematical finance techniques to value and risk manage these Real Options. A lot of the literature on Real Options focuses on managerial flexibility or long term investment decisions. Myers (1977) Pioneered real option pricing methodology and introduced ROA as a decision opportunity for a corporation or an individual. Likewise, Leslie and Michaels (1997) advocated the use of ROA with a hypothetical example applied to oil field extraction choices. Studies on ROA being applied to investment decisions also include the likes of Amusan and Adinya (2021) for the investment timing and value of iron ore mining projects; Chen et al. (2018) for the investment timing and value of gas storage; and also in the coal arena with Krisna and Faturohman (2021) for coal mining project investments. Recently (Alexander and Chen 2021) introduced a general decision-tree framework to manage model risk for real option to divest in a project.

An ROA study on Power Generation Assets, Eydeland and Wolyniec (2002) shows how one can value and monetize flexibility within an asset or piece of equipment that has asymetric monetary pay-offs. This is very similar in nature to the asset flexibility inherent in a coal wash-plant, whereby the choice to generate electricity or not is akin to the choice of whether to wash coal or by-pass. Therefore, the first contribution of this paper is that we add to this literature which lacks a comprehensive study on stochastic modelling of coal production using real options in conjunction with option pricing theory. Secondly, we formulate a three-dimensional version of Fourier transform method for the flexible computation of the real option prices and performed a numerical experiment to show substantial computational gain compared to the Monte Carlo method. Thirdly, we carry out empirical analysis to provide important insights of using real options analysis in coal production to advance financial risk management for a hypothetical coal mine and this will be critical for risk assessment and business evaluation. Lastly, to the best of our knowledge, this paper is the first to adopt option pricing theory to model real options with stochastic volatility component.

The remainder of this paper is organised as follows: in "Real options for investment projects" section, we describe the nature of the problem and reduce the valuation problem to a spread option pricing problem. "The model and methods" section discusses the model. Here we consider a 4-dimensional stochastic volatility model driving the modeling variables. We derive the numerical scheme based on a lower bound approximation for pricing a three-asset spread options. "Data and model parameters" section focuses on data and data analysis. We also discuss the methodology to estimate the model parameters. In "Empirical analysis" section, we implement the model; we compare the semi-analytical formula with the Monte Carlo Simulation. "Conclusion" section concludes.

Real options for investment projects

A hypothetical coal mine is considered, that is similar to those found in the Hunter Valley region of New South Wales, Australia, whereby the coal is exported through the Port of Newcastle. The run of mine coal (the coal prior to any processing) is high in ash (inorganic non-combustible matter) and a wash plant is situated at the mine site. The mine has the ability to either wash the run of mine coal ("Washed Scenario") to produce a "low ash" coal or bypass the wash plant ("Bypass Scenario") to produce a "high ash" coal.

In both scenarios, the mining costs are the same. The Washed Scenario, carries extra processing costs over the Bypass Scenario due to the operational costs of the wash plant and the cost of disposing of the rejects from the wash plant (mainly inorganic matter) back into the pit. Logistics costs of getting the coal from the mine to the port and onto the vessel for export is considered to be the same on a per tonne basis. Ad val royalites are applied at the same rate for both scenarios with the exception that the differential in wash and bypass costs includes the value of being able to deduct the beneficiation costs (Ian Macdonald and Mf 2008) from the final royalty payments. The low ash and high ash coal are considered to be of a quality with a typical ash content of 14.2% and 22.5% (air dried) respectively. The former is a grade of coal that would typically be exported to Japanese customers and the latter is of a quality that would typically go to the markets of China and India. For simplicity the wash plant is considered to only have one setting, with the resultant yield being between 65 and 80%. Logistical constraints such as stockyard space are considered to not impact the ability for the producer to elect whether to wash or bypass the coal. Coal sold into the export market is usually sold in US Dollars. The differential in costs between the two scenarios are in Australian Dollars. The two scenarios can therefore be modelled as a spread option with a strike that follows a stochastic process; mainly due to the strike being in Australian Dollars and the underlying coal prices being in US Dollars. This problem construction is solved for a lower bound, however, if one was to also consider the various settings of the wash plant, the financial mathematical modelling would be that of a rainbow option. Furthermore, given there are also choices in how the coal is mined, the application of Real Option Analysis can be expanded to modelling the full production, processing and logistics chain as compound options.

Model variables

The problem parameters and variables, and their associated assumptions are defined below:

- u: underlying, with historical prices used for low and high ash coal delivered on a Free on Board ("FOB")[1] basis out of the Port of Newcastle, Australia.
- y: yield being between 65 and 80% for the washed coal (low ash) and 100% for bypass coal (high ash),[2]
- x: royalty (8.2% has been used as per the royalty rate applied in NSW for open-cut coal mines, therefore $x = 91.8\%$), Ian Macdonald and Mf (2008)
- a: United States Dollar ("USD")/ Australian Dollar ("AUD") exchange rate
- l: logistics costs. 10.55 AUD/mt (rail) + 2.50/mt AUD (port) to give 13.05 AUD/mt, Naess (2015)
- ω: wasted rejects costs at 3 AUD/mt,
- c: cost of mining
- PL_A: profit or loss of mining and selling coal under Washed Scenario to produce a low ash. A, i.e.,

$$PL_A = (u^A y_A x) - a((l y_A) + (1 - y_A)\omega + c) \tag{1}$$

- PL_B: profit or loss of mining and selling coal under Bypass Scenario to produce a high ash coal B, i.e.,

$$PL_B = (u^B y_B x) - a((l y_B) + (1 - y_B)\omega + c) \tag{2}$$

[1] Free on Board means that the seller delivers when the goods pass the ship's rail at the named port of shipment. Ramberg et al. (1999).

[2] The range of yield is based on expert opinion provided by Dave Porteus, Principal Consultant (Managing Director) at DFP Solutions Pty Ltd.

Spread option payoff

The payoff at expiry date T is given by

$$P = \max\{PL_A - PL_B, 0\} \tag{3}$$

$$= \max\{(u^A y_A x) - a((ly_A) + (1 - y_A)\omega) - (u^B y_B x) + a((ly_B) + (1 - y_B)\omega), 0\} \tag{4}$$

$$= \max\{(u^A y_A x) - (u^B y_B x) - (a(ly_A - ly_B + (y_B - y_A)\omega)), 0\} \tag{5}$$

$$= \max\{(u^A y_A x) - (u^B y_B x) - a(\omega - l)(y_B - y_A), 0\}. \tag{6}$$

Let $u^A = (u^B + u^C)6000/5500$ where $u^C =$ the differential in price basis 5500 kcal/kg NCV between the underlyings:

$$P = \max\{((6000/5500(u^B + u^C))y_A x) - (u^B y_B x) - a(\omega - l)(y_B - y_A), 0\}. \tag{7}$$

In Eq. (7), we assume the differential in price between the low ash coal and high ash coal u^C, and high ash coal u^B, and foreign exchange, all evolve stochastically and the forward dynamic under a risk-neutral measure will be discussed in "The model and methods" section. The payoff in Eq. (7) can be seen or easily translated as the payoff of spread option with three underlying assets with strike equal to zero.[3]

Under classical assumptions of Black and Scholes Kirk (1995)[4] derived an approximation formula for spread options with two underlying assets, which is widely applied in practice but not as accurate as desired. Departing from log-normality assumptions[5] means one has to resolve to numerical methods. Although Monte Carlo method is always an alternative solution to solving many problems where no closed-form solutions are available, it has its drawback on computational speed especially in cases where a pricing model has to be calibrated to liquid pricing data. Therefore there is a trade-off that has to be tackled between numerical accuracy and computational heaviness. The widely used numerical method is the fast Fourier transform (FFT) method developed by Carr and Madan (1999). This method is applicable as long as the characteristic function for asset return is known. Lot more extensions to Carr and Madan were suggested and implemented to price different kind of options with different payoff functions. Many of these examples appear in the study by Eberlein et al. (2010). For multi-assets cases, as mentioned above the first Fourier Transform implementation is due to Dempster and Hong (2002) and which was then extended by Hurd and Zhou (2010) to multidimensional FFT method for spread options.

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[3] The cost of mining c is the same in both scenarios and so cancels out. This leads to pricing a spread option on three assets with strike price $K = 0$, just as in the case for Margrabe option (Margrabe 1978).

[4] Assuming modeling variables follow a Geometric Brownian motion.

[5] In fact it will be hard to derive close-form approximation under some model such as Lévy based models.

Option type

As we are solving for a lower-bound Real Option, we only have two states. The first state is to wash the coal. This would typically be the default for any mine with a wash plant. The second state would be to *exercise* and elect to bypass the coal.

The strike of the option is Australian Dollar denominated and so in US Dollar terms it behaves stochastically. We therefore consider such in the model.

The Monte-Carlo method is then run with starting prices taken at times in history whereby the option would have been Out-Of-The-Money ("OTM").

The simulations assume the expiry of the option is three months prior to the mining of the subject coal which is to allow for changes to mine planning. The coal considered is to be mined in approximately fifteen months time and so the time to expiry is twelve months.

The yield is also varied so the sensitivity to the yield can be explored. The lower the yield the more likely that market may move to a place whereby the option would become In-The-Money ("ITM") and so the producer would elect to by-pass the coal. See Model Implementation and numerical results section.

The model and methods

In this section we introduce a stochastic model. We describe all model parameters and constraints. We outline how such model is estimated from real data. We will also do a little exercise on pricing such a model using Monte Carlo method or semi-analytical method.

We model 3 correlated assets in the multivariate (Heston 1993) framework. The forward prices for these assets under the forward pricing measure $\mathbb{Q}$ are:

$$\frac{dF_1(t,T)}{F_1(t,T)} = \sigma_1 \sqrt{V_t} dW_1(t), \tag{8}$$

$$\frac{dF_2(t,T)}{F_2(t,T)} = \sigma_2 \sqrt{V_t} dW_2(t), \tag{9}$$

$$\frac{dF_3(t,T)}{F_3(t,T)} = \sigma_3 \sqrt{V_t} dW_3(t), \tag{10}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_V(t), \tag{11}$$

where

$$\begin{aligned}
\mathbb{E}_{\mathbb{Q}}[dW_1 dW_2] &= \rho_{12} dt, \\
\mathbb{E}_{\mathbb{Q}}[dW_1 dW_3] &= \rho_{13} dt, \\
\mathbb{E}_{\mathbb{Q}}[dW_2 dW_3] &= \rho_{23} dt, \\
\mathbb{E}_{\mathbb{Q}}[dW_1 dW_V] &= \rho_{1V} dt, \\
\mathbb{E}_{\mathbb{Q}}[dW_2 dW_V] &= \rho_{2V} dt, \\
\mathbb{E}_{\mathbb{Q}}[dW_3 dW_V] &= \rho_{3V} dt.
\end{aligned}$$

Model definitions :

- Equation (8) is the forward process for low-ash diff, $F_1(t,T) = (6000/5500(u^B(t) + u^C(t))e^{r(T-t)}$, r is the risk-free rate,
- Equation (9) is the forward process for high-ash, i.e., $F_2(t,T) = u^B(t)e^{r(T-t)}$
- Equation (10) is the forward process for foreign exchange, i.e., $F_3(t,T) = a(t)e^{(r_d-r_f)(T-t)}$, r_d and r_f are the domestic and foreign spot rate respectively,
- Equation (11) is the instantaneous stochastic volatility assumed for follow CIR process [refer to Cox et al. (1985)].

Now working with log forward prices, $f_i = \log F_i$ for $i = 1, 2, 3$ one gets the following pair of stochastic differential equations:

$$df_1(t) = -\frac{1}{2}\sigma_1^2 V_t dt + \sigma_1 \sqrt{V_t} dW_1(t) \tag{12}$$

$$df_2(t) = -\frac{1}{2}\sigma_2^2 V_t dt + \sigma_2 \sqrt{V_t} dW_2(t) \tag{13}$$

$$df_3(t) = -\frac{1}{2}\sigma_3^2 V_t dt + \sigma_3 \sqrt{V_t} dW_3(t). \tag{14}$$

By applying Itô's Lemma we get the characteristic function:

$$\chi(u_1, u_2, u_3) = \mathbb{E}^{\mathbb{Q}}\left[\exp\left(\mathrm{i}\sum_{\mathrm{j}=1}^{3} \mathrm{u_j f_j(T)}\right)\right] \tag{15}$$

$$= \exp\left(\mathrm{i}\sum_{\mathrm{j}=1}^{3} \mathrm{u_j f_j(0) + A(T) + B(T)V(0)}\right), \tag{16}$$

where[6]

$$\mathrm{i} = \sqrt{-1} \tag{17}$$

$$A = -\frac{\kappa\theta}{\sigma_V^2}\left[2\log\left(\frac{2\varrho - (\varrho-\gamma)(1-e^{-\varrho T})}{2\varrho}\right) + (\varrho-\gamma)T\right] \tag{18}$$

$$B = \frac{2\zeta(1-e^{-\varrho T})}{2\varrho - (\varrho-\gamma)(1-e^{-\varrho T})} \tag{19}$$

[6] Here

$$\tilde{F}_1(t,T) = \mathcal{C}_1 F_1(t,T),\ \tilde{F}_2(t,T) = \mathcal{C}_2 F_2(t,T),\ \tilde{F}_3(t,T) = \mathcal{C}_3 F_3(t,T)$$

with the scaling constants:

$$\mathcal{C}_1 = y_A x, \mathcal{C}_2 = y_B x, \mathcal{C}_3 = (\omega - l)(y_B - y_A.).$$

$$\zeta = -\frac{1}{2}\left[\left(\sum_{j=1}^{3}\sigma_j^2u_j^2 + 2\rho_{12}\sigma_1\sigma_2u_1u_2 + 2\rho_{13}\sigma_1\sigma_3u_1u_3 + 2\rho_{23}\sigma_2\sigma_3u_2u_3\right)\right.$$
$$\left. + \mathrm{i}(\sigma_1^2\mathrm{u}_1 + \sigma_2^2\mathrm{u}_2 + \sigma_3^2\mathrm{u}_3)\right] \tag{20}$$

$$\gamma = \kappa - \mathrm{i}(\rho_{1\mathrm{V}}\sigma_1\mathrm{u}_1 + \rho_{2\mathrm{V}}\sigma_2\mathrm{u}_2 + \rho_{3\mathrm{V}}\sigma_3\mathrm{u}_3)\sigma_{\mathrm{V}} \tag{21}$$

$$\varrho = \sqrt{\gamma^2 - 2\sigma_V^2\zeta}. \tag{22}$$

Our valuation problem in Eq. (7) can be translated in the following form:

$$p = e^{-rT}\mathbb{E}_{\mathbb{Q}}[(\tilde{F}_1(T) - \tilde{F}_2(T) - \tilde{F}_3(T))^+] := e^{-rT}\mathbb{E}_{\mathbb{Q}}\left[\left(e^{\tilde{f}_1} - e^{\tilde{f}_2} - e^{\tilde{f}_3}\right)^+\right]^6 \tag{23}$$

where

$$x^+ = \max\{x, 0\}.$$

FFT method

The first application of closely related to multidimensional FFT pricing of spread options was proposed by Dempster and Hong (2002) who derived FFT algorithms for correlation options and spread options. In the case for spread options, Dempster and Hong approximated the exercise region through a combination of rectangular strips thereby attempting to account for singularities in the transform variables and then they applied FFT techniques on a regularized region to derive the upper and lower bounds for spread options value. As mentioned above, spread options have exercise region with non-linear edge and applying the methodologies of Dempster and Hong can be computationally expensive. A workaround to deriving the analytic approximation of the 2-dimensional exercise region, Hurd and Zhou (2010) proposed an alternative and the most suitable version for FFT algorithms to pricing options in two and higher dimensions (while preserving the simplicity) which is based on square integrable integral formulae for the payoff function [see also Alfeus and Schlögl (2019)]. As in Dempster and Hong (2002), we consider the following modified exercise region:

$$\Omega_\lambda := \left\{(\tilde{f}_1,\tilde{f}_2,\tilde{f}_3) \in \mathbb{R}\times\mathbb{R}\times\left[-\frac{1}{2}N\lambda, \frac{1}{2}N\lambda\right)|e^{\tilde{f}_1} - e^{\tilde{f}_2} - e^{\tilde{f}_3} \geq 0\right\}.$$

Equation (23) can be rewritten as:

$$p = e^{-rT}\int\int\int_{\Omega_\lambda}\left(e^{\tilde{f}_1} - e^{\tilde{f}_2} - e^{\tilde{f}_3}\right)q_T(\tilde{f}_1,\tilde{f}_2,\tilde{f}_3)d\tilde{f}_3d\tilde{f}_2d\tilde{f}_1, \tag{24}$$

where q_T represents the risk-neutral density function at time T. Define the spread option lower bound as:

$$\underline{\Pi}(k_1,k_2,k_3) = \int_{k_1}^{\infty}\int_{k_2}^{\infty}\int_{k_3}^{\infty}\left(e^{\tilde{f}_1} - e^{\tilde{f}_2} - e^{\tilde{f}_3}\right)q_T(\tilde{f}_1,\tilde{f}_2,\tilde{f}_3)d\tilde{f}_3 d\tilde{f}_2 d\tilde{f}_1. \tag{25}$$

Let $\alpha_1,\alpha_2,\alpha_3 > 0$.[7] Define the modified integral as:

$$\underline{\pi}(k_1,k_2,k_3,\alpha_1,\alpha_2,\alpha_3) = e^{\alpha_1 k_1+\alpha_2 k_2+\alpha_3 k_3}\underline{\Pi}(k_1,k_2,k_3). \tag{26}$$

Notice that the characteristic function can be obtained via Fourier transform of Eq. (26) and it is computed as follows:

$$\begin{aligned}
&\chi(u_1,u_2,u_3,\alpha_1,\alpha_2,\alpha_3)\\
&= \int_{\mathbb{R}}\int_{\mathbb{R}}\int_{\mathbb{R}} e^{\mathrm{i}(\mathrm{u}_1\mathrm{k}_1+\mathrm{u}_2\mathrm{k}_2+\mathrm{u}_3\mathrm{k}_3)}\underline{\pi}(k_1,k_2,k_3,\alpha_1,\alpha_2,\alpha_3)dk_3dk_2dk_1\\
&= \int_{\mathbb{R}}\int_{\mathbb{R}}\int_{\mathbb{R}} e^{(\alpha_1+\mathrm{iu}_1)k_1+(\alpha_2+\mathrm{iu}_2)k_2+(\alpha_3+\mathrm{iu}_3)k_3}\int_{k_1}^{\infty}\int_{k_2}^{\infty}\int_{k_3}^{\infty}\left(e^{\tilde{f}_1} - e^{\tilde{f}_2} - e^{\tilde{f}_3}\right)q_T(\tilde{f}_1,\tilde{f}_2,\tilde{f}_3)d\tilde{f}_1d\tilde{f}_2d\tilde{f}_3dk_1dk_2dk_3\\
&= \int_{\mathbb{R}}\int_{\mathbb{R}}\int_{\mathbb{R}}\left(e^{\tilde{f}_1} - e^{\tilde{f}_2} - e^{\tilde{f}_3}\right)q_T(\tilde{f}_1,\tilde{f}_2,\tilde{f}_3)\int_{-\infty}^{\tilde{f}_1}\int_{-\infty}^{\tilde{f}_2}\int_{-\infty}^{\tilde{f}_3} e^{(\alpha_1+\mathrm{iu}_1)k_1+(\alpha_2+\mathrm{iu}_2)\mathrm{k}_2+(\alpha_3+\mathrm{iu}_3)\mathrm{k}_3}dk_3dk_2dk_1d\tilde{f}_3d\tilde{f}_2d\tilde{f}_1\\
&= \int_{\mathbb{R}}\int_{\mathbb{R}}\int_{\mathbb{R}}\left(e^{\tilde{f}_1} - e^{\tilde{f}_2} - e^{\tilde{f}_3}\right)q_T(\tilde{f}_1,\tilde{f}_2,\tilde{f}_3)\frac{e^{(\alpha_1+\mathrm{iu}_1)\tilde{f}_1+(\alpha_2+\mathrm{iu}_2)\tilde{f}_2+(\alpha_3+\mathrm{iu}_3)\tilde{f}_3}}{(\alpha_1+\mathrm{iu}_1)(\alpha_2+\mathrm{iu}_2)(\alpha_3+\mathrm{iu}_3)}d\tilde{f}_3d\tilde{f}_2d\tilde{f}_1\\
&= \frac{\phi_T(u_1-\alpha_1\mathrm{i},u_2-(\alpha_2+1)\mathrm{i},u_3-(\alpha_3+2)\mathrm{i})-\phi_T(u_1-(\alpha_1+2)\mathrm{i},u_2-(\alpha_2+1)\mathrm{i},u_3-\alpha_3\mathrm{i})}{(\alpha_1+\mathrm{iu}_1)(\alpha_2+\mathrm{iu}_2)(\alpha_3+\mathrm{iu}_3)}.
\end{aligned} \tag{27}$$

Define an $N\times N\times N$ equally space grid $\Lambda_1\times\Lambda_2\times\Lambda_3$, where

$$\begin{aligned}
\Lambda_1 &:= \{k_{1,p}\} := \left\{\left(p-\frac{1}{2}N\right)\lambda_1 \in \mathbb{R} | 0 \le p \le N-2\right\}\\
\Lambda_2 &:= \{k_{2,q}\} := \left\{\left(q-\frac{1}{2}N\right)\lambda_2 \in \mathbb{R} | 0 \le q \le N-2\right\}\\
\Lambda_3 &:= \{k_{3,s}\} := \left\{\left(s-\frac{1}{2}N\right)\lambda_3 \in \mathbb{R} | 0 \le s \le N-2\right\},
\end{aligned}$$

λ_1,λ_2 and λ_3 are chosen such that

$$\lambda_1\Delta_1 = \lambda_2\Delta_2 = \lambda_3\Delta_3 = \frac{2\pi}{N},$$

where Δ_1, Δ_2 and Δ_3 denote the integration step size.

For each $p = 0,\ldots,N-1$, define

$$\begin{aligned}
\underline{k}_3(p) &:= \min_{0\le q\le N-1}\left\{k_{3,s}\in\Lambda_3 | e^{k_{3,s}} - e^{k_{2,p+2}} - e^{k_{1,p+1}} \ge 0\right\}\\
\underline{k}_2(p) &: = \min_{0\le q\le N-1}\left\{k_{2,q}\in\Lambda_2 | e^{k_{3,p+2}} - e^{k_{2,q}} - e^{k_{1,p+1}} \ge 0\right\}.
\end{aligned}$$

The price is now computed via inverse FFT:

$$\Pi(k_{1,p},k_{2,q},k_{3,s},\alpha_1,\alpha_2,\alpha_3) \tag{28}$$

[7] As pointed out in Carr and Madan (1999), we multiply the option price lower bound expression in (25) by an exponentially decaying term so that it is square-integrable in k_1, k_2 and k_3 over the negative axes.

$$
\begin{aligned}
&= \frac{e^{-\alpha_1 k_{1,p}-\alpha_1 k_{2,q}-\alpha_3 k_{3,s}}}{(2\pi)^3} \int_{\mathbb{R}} \int_{\mathbb{R}} \int_{\mathbb{R}} e^{-\mathrm{i}(\mathrm{u}_1 \mathrm{k}_{1,\mathrm{p}}+\mathrm{u}_2 \mathrm{k}_{2,\mathrm{q}}+\mathrm{u}_3 \mathrm{k}_{3,\mathrm{s}})} \\
&\chi(u_1, u_2, u_3, \alpha_1, \alpha_2, \alpha_3) du_3 du_2 du_1 \\
&\approx \frac{e^{-\alpha_1 k_{1,p}-\alpha_1 k_{2,q}-\alpha_3 k_{3,s}}}{(2\pi)^3} \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} e^{-\mathrm{i}(\mathrm{u}_{1,\mathrm{l}} \mathrm{k}_{1,\mathrm{p}}+\mathrm{u}_{2,\mathrm{m}} \mathrm{k}_{2,\mathrm{q}}+\mathrm{u}_{3,\mathrm{n}} \mathrm{k}_{3,\mathrm{s}})} \\
&\chi(u_{1,l}, u_{2,m}, u_{3,n}, \alpha_1, \alpha_2, \alpha_3) \Delta_3 \Delta_2 \Delta_1,
\end{aligned}
\tag{29}
$$

where

$$
\lambda_1 \Delta_1 = \lambda_2 \Delta_2 = \lambda_3 \Delta_3 = \frac{2\pi}{N},
$$

and

$$
u_{1,l} = \left(l - \frac{N}{2}\right)\Delta_1, u_{m,2} = \left(m - \frac{N}{2}\right)\Delta_2, u_{3,n} = \left(n - \frac{N}{2}\right)\Delta_3.
$$

As in Dempster and Hong (2002), Eq. (23) is approximated as follows:

$$
\begin{aligned}
e^{-rT}\mathbb{E}_{\mathbb{Q}}[\left(e^{\hat{f}_1} - e^{\hat{f}_2} - e^{\hat{f}_3}\right)^+] = e^{-rT} \sum_{p=0}^{N-2} \Pi(k_{1,p}, \underline{k}_2(p), \underline{k}_3(p), \alpha_1, \alpha_2, \alpha_3) \\
- \Pi(k_{1,p+1}, \underline{k}_2(p), \underline{k}_3(p), \alpha_1, \alpha_2, \alpha_3).
\end{aligned}
\tag{30}
$$

Equation (30) is the lower bound approximation of spread option with three assets. We can compute this quickly using Riemann sums[8] or three-dimensional FFT methods. We compare the semi-analytical solution in Eq. (30) with the Monte Carlo simulation in the next section.

Data and model parameters

Historical market data of coal markets for both low and high ash coal; and foreign exchange rates are used:

- Low-ash Spot Coal Price (USD/mt basis 6000 NAR): "low-ash".
- High-ash Spot Coal Price (USD/mt basis 5500 NAR): "high-ash".
- Low-ash Spot Coal Price (USD/mt converted to basis 5500 NAR less High-ash Spot Coal Price (USD/mt basis 5500 NAR): "low-ash diff".
- Low-ash first full calendar forward contract Price (USD/mt basis 6000 NAR): "low-ash cal".
- Spot foreign exchange rate for Australian Dollars to US Dollars: "AUD/USD".

The low-ash and high-ash coal markets will and have rarely ever inverted i.e. the price for low ash coal is always greater than high-ash coal. This is as a result of the ability for the market to arbitrage if the spread was ever inverted: buying low-ash and delivering into high-ash coal contracts. As a result, rather than model using a typical spread option,

[8] The integral in Eq. (28) can be solved easily using three dimensional numerical integration schemes. In MATLAB, we have used *integral3*.

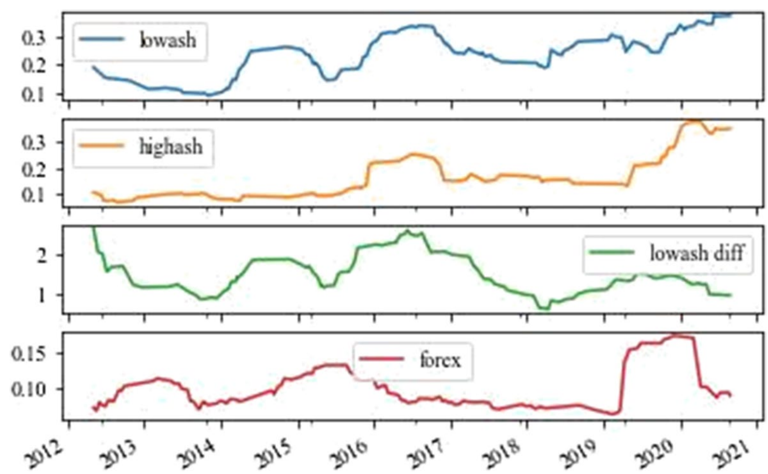

Fig. 1 Rolling historical volatility of the historical market returns

which allows for prices to invert, we model the markets as a high ash coal price, and generate a low-ash coal price by adding an always positive differential that behaves stochastically: the *low ash diff*.

Whilst there are financial options traded in the coal market, few are ever spread options and liquidity can be poor for vanilla options. We therefore do not have implied volatilities and correlation between any of the assets, rather we use conservative estimates based on historical returns.

Data analysis

The historical volatilities of the spot contracts with a rolling window of 50 trading days are shown below. In Fig. 1 each chart shows that the volatility behaves stochastically, and can exhibit significant step changes.

The historical correlation between the spot contracts with a rolling window of 50 trading days are shown in Fig. 2. The correlations for each relationship would appear to have noise and/or follow a stochastic process.

For each asset, the returns exhibit fat tails of varying degrees and this is shown in Fig. 3. This is comparable to other commodity markets.

The historical kurtosis of the returns with a rolling window of 50 trading days is shown in Fig. 4. It is broadly consistent to the QQ plots with distinct periods of high kurtosis at various places in time for each of the assets.

Model parameters

In this section we discuss a methodology of obtaining model parameters. We estimate the volatility and correlations parameters from historical data.[9] We are only left with instantaneous volatility CIR model parameters that are unknown. Our approach is to estimate these unknown parameters through moment matching, i.e., we impose that the

[9] i.e. $\sigma_1, \sigma_2, \sigma_3, \rho_{12}, \rho_{13}, \rho_{23}$ are all estimated from the monthly historical data as introduced in "Data and model parameters" section.

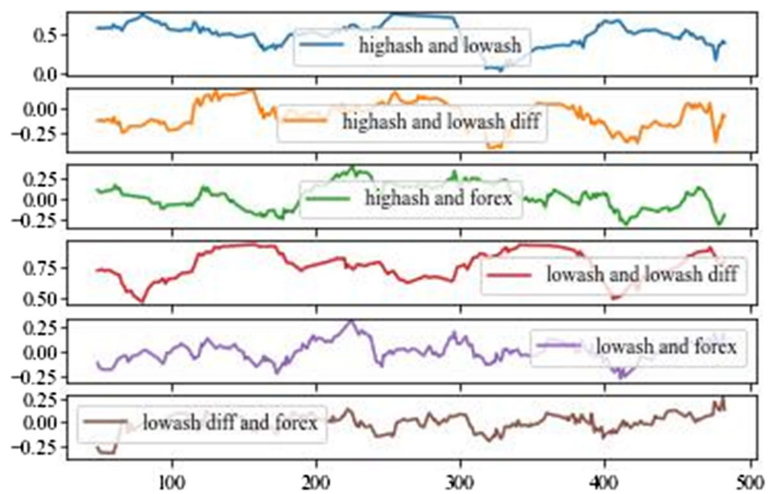

Fig. 2 Rolling correlation of the historical returns

Kurtosis of the instantaneous volatility process must be the same as the Kurtosis of the differential of the low ash and high ash.

The Kurtosis[10] of the volatility process is computed as in Jafari and Abbasian (2017) and also given in Eq. (31).

$$\mathbb{E}\left[V_t^4\right] = \sum_{j=0}^{2} \binom{4}{j} A_t^{4-2j} B_t^{2j} \left[\frac{1}{2\kappa}\left(e^{2\kappa} - 1\right)\right]^{2j}, \tag{31}$$

where

$$A_t = e^{-\kappa t} V_0 + \theta(1 - e^{-\kappa t}) \quad \text{and} \quad B_t = \sigma_V e^{-\kappa t}.$$

Table 1 depicts model parameters that will be used in "Empirical analysis" section to price spread options. In Table 2, we give correlations among modeling variables.

Empirical analysis

A series of monte-carlo simulations have been run with different wash plant yields. The initial market prices have been taken from times during the last 15 years where the options started off as OTM. The mean undiscounted option premiums in USD/mt for each yield are in Table 3 below:

In Table 4 and Fig. 5, we compare the semi-analytical solution with the Monte Carlo simulations. We ran 1 million simulations over 1000 time steps. Results are very close, justifying the correctness of the implementation of the closed-form solution. Looking at Fig. 5, the bounds are very tight and one can observe that the semi-analytical price is within the 95% confident level of Monte Carlo bound. For semi-analytical method,

---

[10] We compute

$$\mu_2 = \mathbb{E}[V_t - \mathbb{E}[V_t]^2] \text{ and } \mu_4 = \mathbb{E}[V_t - \mathbb{E}[V_t]^4].$$

Then

$$\text{Kurtosis } = \frac{\mu_4}{\mu_2^2}.$$

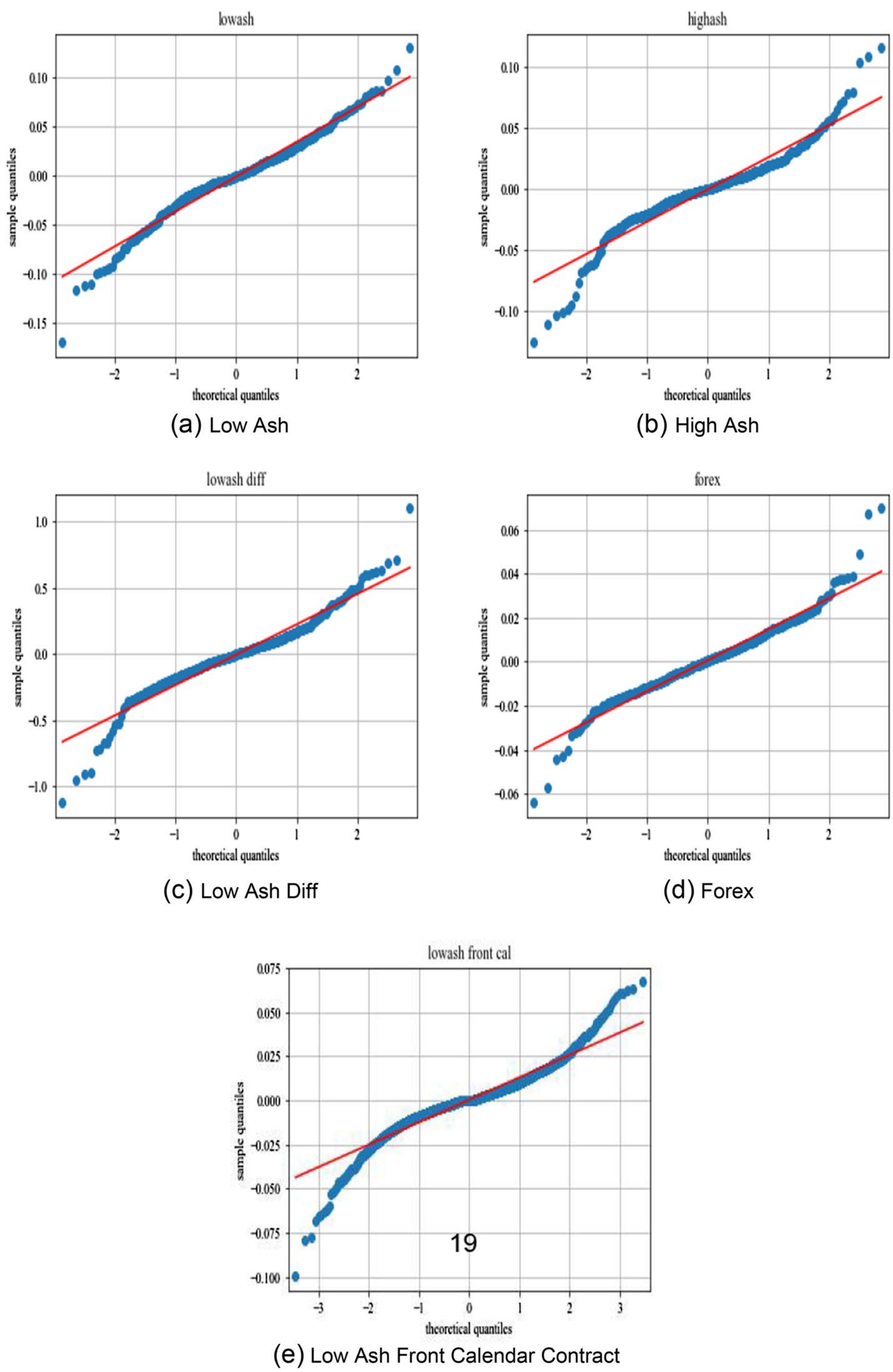

Fig. 3 **a** Low ash. **b** High ash. **c** Low ash diff. **d** Forex. **e** Low ash front calendar contract. QQplots analysis

we chose $\alpha_1 = 0.65, \alpha_2 = 1.4, \alpha_3 = 0.25$.[11] Next, in Fig. 6 we investigate the effect of the correlation between the forward prices and the instantaneous stochastic volatility, and these correlation have impacts on option prices. Finally, in Fig. 7 we show the distribution of the Monte Carlo payoff for the spread option. As the time to expiry increases the payoff becomes asymptotically normal distributed.

[11] For efficient approach of computing these damping factor see Bayer et al. (2022).

Table 1 Model parameters

κ	1.21780019	σ_1	1
θ	0.93593908	σ_2	0.2
V_0	0.36863154	σ_3	0.24
σ_V	0.36881736		

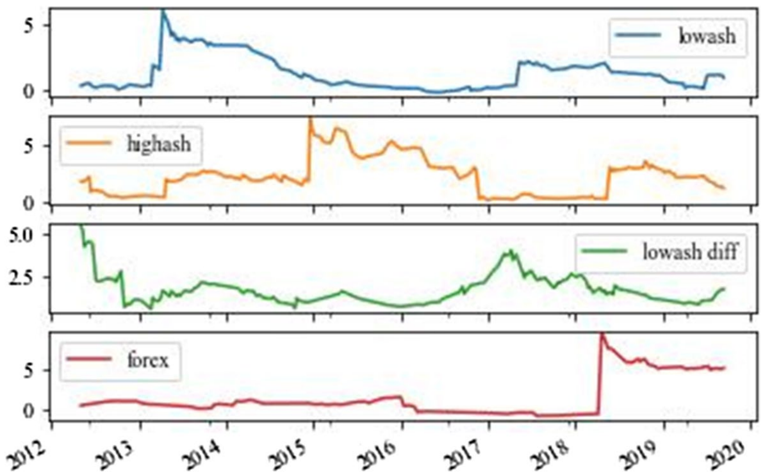

Fig. 4 Rolling historical kurtosis of the historical market returns

Table 2 Correlation parameters

	Highash	Lowash diff	Forex	Vol of vol
Highash	1	0.2	0.4	0.5
Lowash diff	0.2	1	0	0.5
Forex	0.4	0	1	0.5
Vol of vol	0.5	0.5	0.5	1

Table 3 Undiscounted option premiums basic statistics

	Yield of 65%	Yield of 70%	Yield of 75%	Yield of 80%
Count	119.00	119.00	119.00	119.00
Mean	9.64	5.44	2.34	0.86
Std	2.37	1.52	0.82	0.31
Min	5.25	2.14	0.84	0.13
25%	7.99	4.31	1.79	0.66
50%	9.51	5.22	2.22	0.80
75%	10.71	6.21	2.85	1.07
Max	15.76	9.80	5.03	1.51

Conclusion

The key findings show that an application of Real Option Analysis, with consideration of the stochastic nature of coal prices suggests that there is significant value that can be monetised from the washplant/bypass asset flexibility of between almost 1 USD to over 9 USD for each ROM tonne. See Table 3 above.

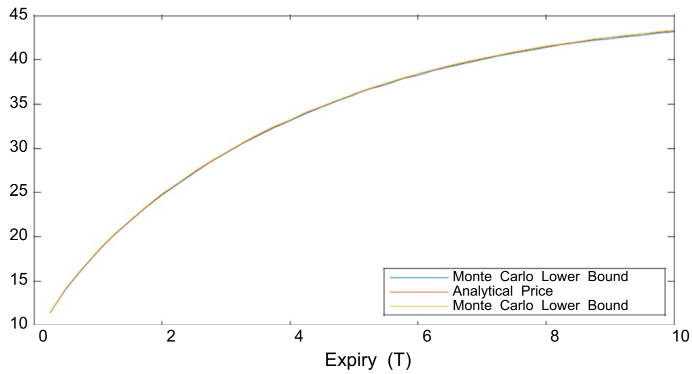

Fig. 5 Monte Carlo bounds versus semi-analytical prices

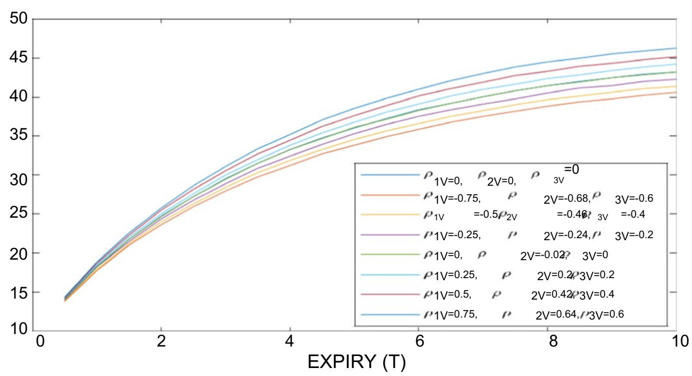

Fig. 6 Correlation effect on option prices

Table 4 Semi-analytical pricing compared with Monte Carlo

T	In-the-money			At-the-money			Out-of-the-money		
	Analytics	Monte Carlo		Analytics	Monte Carlo		Analytics	Monte Carlo	
	Semi-analytical	Price	Std errors	Semi-analytical	Price	Std errors	Semi-analytic	Price	Std errors
0.5	9.655	9.680	0.00005	9.682	9.690	0.00006	10.050	10.054	0.00007
1.0	13.887	13.923	0.00014	14.160	14.145	0.00014	15.827	15.827	0.00019
1.5	17.263	17.285	0.00022	17.731	17.689	0.00023	20.425	20.458	0.00031
2.0	20.124	20.129	0.00030	20.758	20.715	0.00032	24.342	24.393	0.00041
2.5	22.601	22.629	0.00037	23.381	23.410	0.00039	27.849	27.809	0.00051
3.0	24.770	24.788	0.00043	25.676	25.644	0.00045	30.822	30.805	0.00059
3.5	26.679	26.632	0.00047	27.698	27.694	0.00050	33.456	33.445	0.00065
4.0	28.365	28.367	0.00051	29.483	29.526	0.00054	35.831	35.781	0.00070
4.5	29.857	29.897	0.00055	31.064	31.059	0.00058	37.857	37.851	0.00075
5.0	31.177	31.155	0.00057	32.465	32.416	0.00060	39.661	39.687	0.00078
CPU	35	126		34	124		36	128	

For semi-analytical method [refer to Eq. (30)], we chose $\alpha_1 = 0.65, \alpha_2 = 1.4, \alpha_3 = 0.25$, CPU times in secs

In previous studies, such as Ajak and Topal (2015), the application of Real Option Analysis in practise at an operational level is a greater issue than proving there is value to utilising the Real Option methods. Ayodele (2019) performed a study on factors which influence the adoption of real option analysis in emergent markets, and found that firm/

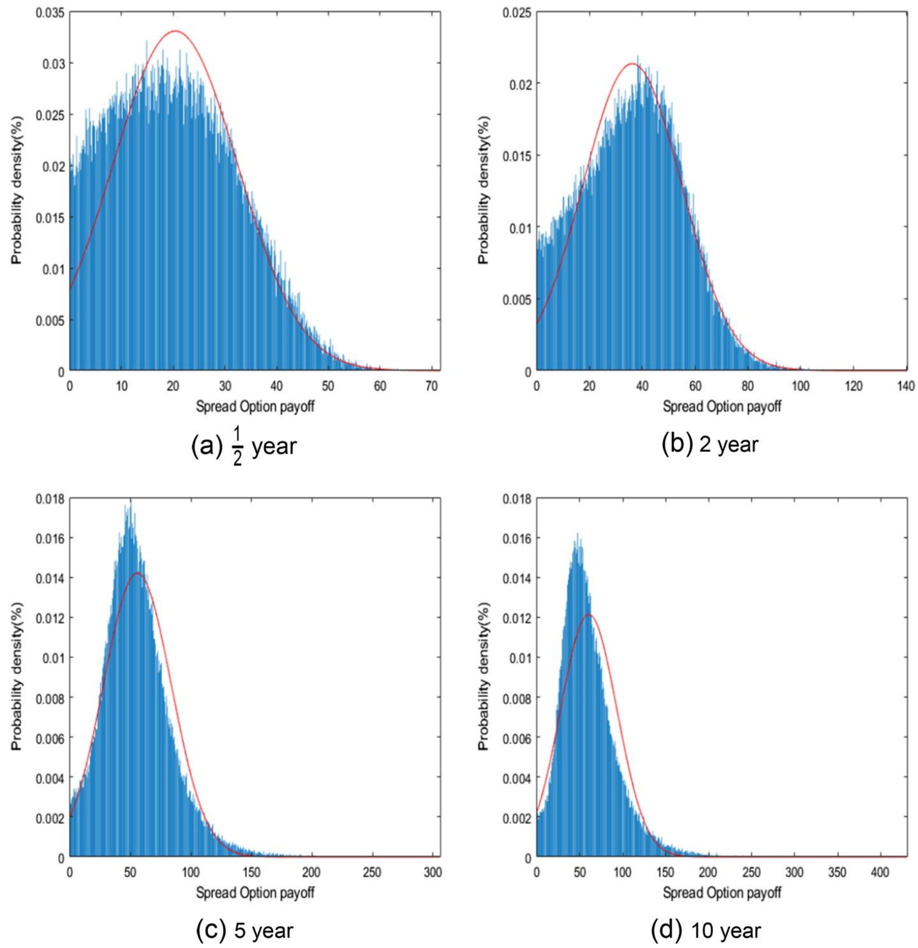

Fig. 7 **a** $\frac{1}{2}$ year, **b** 2 year, **c** 5 year, **d** 10 year. Spread option Monte Carlo payoff distribution

management constraint was a major factor influencing the choice of appraisal techniques for assets. Furthermore, the authors referenced (Andalib et al. 2016) whereby they found that most firms expect strict adherence to laid down standard practice. Horn et al. (2015) surveyed the chief financial officers of companies within Scandinavia, less than 10 % used real option analysis. The authors found that larger companies and companies with higher research and development intensity and capital expenditures are more likely to use real option analysis. The dominant reason for non-use is a lack of familiarity, where 70% of respondents report to not be familiar with real option concepts and techniques. However, Ajak and Topal (2015) demonstrated that a Real Option method can be applied to decision making at a mine's operational level. In the authors' experience, firms willing to make the investment in human resource expertise and real option projects have typically found success and often gained a comparative advantage over firms who did not take those steps. The author would also surmise that the knowledge gained by a business in the pursuit of monetising assets via ROA would also benefit the firm's understanding of the underlying market, in a similar fashion to that found by Li (2021) whereby trading in the option market induces informed trading and thus reduces information asymmetry.

Nevertheless, the hurdle of a business having the will and ability to restructure its business to monetise such real options would appear to be the main limitation to this study.

The hypothesis, that an application of Real Option Analysis on coal production would yield enough value to prompt a miner to change its business practise in order to monetise such flexibility is partially answered, whereby we now have a theoretical value for such optionality. However, a coal miner would need to judge whether such value is sufficient to justify applying human and financial resources to apply Real Option Analysis to their business. Which would suggest that future research into this area would be best related to how businesses successfully apply Real Option Analysis.

Appendix: literature review

The relevant literature in relation to ROA is reviewed and categorised in the Table 5.

Table 5 Literature review

Paper	Category
Myers (1977) Pioneered real option pricing methodology	Introduced Real option as a decision opportunity for a corporation or an individual. The real option value (ROV) is the value of this decision opportunity to buy or sell the project
McDonald and Siegel (1986), Quigg (1993) and Kulatilaka (1998)	First to consider real option with stochastic strike
Capozza and Sick (1991), Trigeorgis (1993), Benaroch and Kauffman (2000), Boer (2000), Yeo and Qiu (2002)	They assume that forward prices follow geometric Brownian motion
Oil exploration, Leslie and Michaels (1997)	Advocating the use of Real Option Analysis with a hypothetical example applied to oil field extraction choices
Oil refineries whereby management has some flexibility to switch operating process units in response to a change in prices, Imai and Nakajima (2000)	Development of multinomial lattice model to value the flexibility of the inherent real options within an oil refinery with recommendation that management incorporate such valuations when evaluating oil refinery projects
Power Generation Assets, Eydeland and Wolyniec (2002)	Description of the mathematical problem of optimising and valuing power generation and storage assets
Valuing the option to switch between the dry bulk market and wet bulk market for a combination carriers, Sødal et al. (2008)	Uses real option analysis to investigate market efficiency in switching a dry bulk to the wet tanker market and vice versa
Evaluate investments in pump storage plants, Muche (2009)	Application of Real Option Analysis in order to value the flexibility of pump storage plants, taking into consideration power price volatility and price spikes
Investment strategy for underground gas storage facilities based on real option model considering gas market reform in China, chen2018investment	Real option application for the investment timing and value of gas storage
Valuing investment decisions of renewable energy projects considering changing volatility, Zhang et al. (2020)	Real option application to model solar power generation investment
Coal Mining Project, krisna2021economic	Application of real option analysis to coal mining project investments
Economic feasibility of forest biomass thermal energy facility, An and Min (2021)	Real Option approach on the use of forecst biomass investments
Assessment of Iron Ore Project, Amusan and Adinya (2021)	Real option application for the investment timing and value of iron ore mining projects
Re-processing of mine tailings, Araya et al. (2021)	Application of real option analysis on investment decisions related to re-processing of mine tailings

Abbreviations

A	Low ash coal
B	High ash coal
C	Difference between low ash and high ash
u	Underlying asset
y	Yield being between 65 and 80% for the washed coal (low ash) and 100% for bypass coal (high ash)
x	Royalty (8.2% has been used as per the royalty rate applied in NSW
a	USD/AUD exchange rate
l	Logistics costs
ω	Wasted rejects costs
c	Cost of mining
PL	Profit or loss
T	Maturity of the option
r_d	Domestic spot rate
r_f	Foreign spot rate
σ_1	Volatility of low-ash diff C
σ_2	Volatility of high-ash B
σ_3	Volatility of foreign exchange
V_t	Instantaneous volatility process assumed to follow Cox–Ingersoll–Ross (1985) dynamics
κ	Mean reversion speed of the volatility
θ	Long term volatility
σ_V	Volatility of volatility
ρ	Correlation of the driving Brownian motions
FFT	Fast Fourier transform
CIR	Cox–Ingersoll–Ross
DCF	Discounted cash-flow analysis
ROA	Real option analysis
FOB	Free on board
OTM	Out-of-the-money
ATM	At-the-money
ITM	In-the-money
USD	United States Dollar
AUD	Australian Dollar

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Author contributions
All authors read and approved the final manuscript.

Declarations

Competing interests
The authors declare no competing interests.

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References

Ajak AD, Topal E (2015) Real option in action: an example of flexible decision making at a mine operational level. Resour Policy 45:109–120
Alexander C, Chen X (2021) Model risk in real option valuation. Ann Oper Res 299(4):1025–1056
Alfeus M, Schlögl E (2019) On spread option pricing using two-dimensional Fourier transform. Int J Theor Appl Finance 22:1950023
Amusan AR Adinya I (2021) Real option technique for an assessment of the Itakpe iron ore project. In: Journal of physics: conference series, vol 1734, IOP Publishing, p 012047
An H, Min K (2021) Economic feasibility of forest biomass thermal energy facility using real option approach. J Korean Soc Forest Sci 110(3):453–461
Andalib S, Tavakolan M, Gatmiri B (2016) Analyzing the barriers influencing the application of real options in the construction industry, pp 1823–1833
Araya N, Ramírez Y, Kraslawski A, Cisternas LA (2021) Feasibility of re-processing mine tailings to obtain critical raw materials using real options analysis. J Environ Manage 284:112060
Ayodele TO (2019) Factors influencing the adoption of real option analysis in red appraisal: an emergent market perspective. Int J Construct Manag 22:1042

Bayer C, Hammouda CB, Papapantoleon A, Samet M, Tempone R (2022) Optimal damping with hierarchical adaptive quadrature for efficient fourier pricing of multi-asset options in Lévy models. arXiv preprint arXiv:2203.08196
Carr P, Madan B (1999) Option valuation using the fast Fourier transform. J Comput Finance 2(4):61
Chen S, Zhang Q, Wang G, Zhu L, Li Y (2018) Investment strategy for underground gas storage facilities based on real option model considering gas market reform in china. Energy Econ 70:132–142
Cox J, Ingersoll J, Ross S (1985) A theory of the term structure of interest rates. Econometrica 53(2):385–407
Davis A, Newman AM (2008) Modern strategic mine planning. Technical report, Colorado School of Mines
Dempster M, Hong S (2002) Spread option valuation and the fast Fourier transform. Math Finance Bachelier Congr 2000:203–220
Eberlein E, Glau K, Papapantoleon A (2010) Analysis of Fourier transform valuation formulas and applications. Appl Math Finance 17:211–240
Eydeland A, Wolyniec K (2002) Energy and power risk management: new developments in modeling, pricing, and hedging, Wiley
Heston SL (1993) A closed-form solution for options with stochastic volatility with applications to bond and currency options. Rev Financ Stud 6:327–343
Horn A, Kjærland F, Molnár P, Steen BW (2015) The use of real option theory in Scandinavia's largest companies. Int Rev Financ Anal 41:74–81
Hurd T, Zhou Z (2010) A Fourier transform method for spread option pricing. SIAM J Financ Math 1(1):142–157
Ian Macdonald MLC, Mf MR (2008) Mining act 1992: determination under section 283(5). Technical report, NSW Government
IEA (2021) World energy outlook 2021. https://www.oecd-ilibrary.org/content/publication/14fcb638-en
Imai J, Nakajima M (2000) A real option analysis of an oil refinery project. Financ Pract Educ 10:78–91
Jafari MA, Abbasian S (2017) The moments for solution of the cox-ingersoll-ross interest rate model. J Finance Econ 5(1):34–37 (http://pubs.sciepub.com/jfe/5/1/4)
Kirk E (1995) Correlation in the energy markets. In: Managing energy price risk, 1st edn. Risk Publications and Enron, London, pp 71–78
Krisna OS, Faturohman T (2021) Economic analysis of coal mining project using real option valuation method. Rev Integr Bus Econ Res 10:450–470
Kulatilaka N, Perotti EC (1998) Strategic growth options. Manage Sci 44(8):1021–1031
Leslie KJ, Michaels MP (1997) The real power of real options. McKinsey Q 3:4
Li K (2021) The effect of option trading. Financ Innov 7:1
Margrabe W (1978) The value of an option to exchange one asset for another. J Financ 33:177–186
McDonald R, Siegel D (1986) The value of waiting to invest. Q J Econ 101(4):707–727
Muche T (2009) A real option-based simulation model to evaluate investments in pump storage plants. Energy Policy 37(11):4851–4862
Myers SC (1977) Determinants of corporate borrowing. J Financ Econ 5(2):147–175
Naess C (2015) Port of newcastle operations: submission in response to Glencore's application to the national competition council, figure 9 and 10
Nicol S (1997) The principles of coal preparation. Preparation Society, Brisbane
Quigg L (1993) Empirical testing of real option-pricing models. J Finance 48(2):621–640
Ramberg J et al (1999) ICC guide to incoterms 2000, ICC
Schernikau L (2010) Economics of the international coal trade: the renaissance of steam coal, Springer, Cham. https://books.google.com.au/books?id=2s4oiQMY650C
Sødal S, Koekebakker S, Aadland R (2008) Market switching in shipping—a real option model applied to the valuation of combination carrier. Rev Financ Econ 17(3):183–203
Zhang M, Liu L, Wang Q, Zhou D (2020) Valuing investment decisions of renewable energy projects considering changing volatility. Energy Econ 92:104954

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