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# Non-fungible tokens: a bubble or the end of an era of intellectual property rights

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## Abstract

The viability of exponentially growing non-fungible token (NFT) market is evaluated by identifying potential value-generating mechanisms that can be rationalized. After identifying the value-generating mechanisms underlying the positive values of NFTs, this study establishes a pricing model for NFTs that follows a continuous-time financial framework. As NFTs are claimed to securitize “ownership rights short of use”, and as such they may potentially serve as a substitute for the need to rely replace the reliance on the legal protection provided by intellectual property rights (IPRs). Considering this issue, this study evaluates the likelihood that NFTs will replace existing mechanisms that protect producers’ rightful claim to use their assets or the need to apply the legal code that governs IPRs. The financial condition for this potential shift is derived for a category of assets whose use or consumption does not reduce supply as the notion of scarcity does not apply.

**Keywords:** Non-fungible tokens, Intellectual property rights, Status

**JEL Classification:** G12, K11

## Introduction

The recent exponential growth<sup>1</sup> of non-fungible tokens (NFTs) raises a question about the viability of their marketplace, that is, whether a “value-generating” mechanism may not be identified. While opponents who suspect that the NFT market is a bubble argue that NFTs have no underlying rational mechanism, advocates are quick to point out that market participants have been willing to pay over one million dollars for a single NFT. Asking the same question, Marsh (2019) compared the viability of the cryptocurrency market with that of the tulip mania and identified a real underlying value. Bloomfield and O’Hara (2000) investigated whether transparent markets can survive when faced with direct competition from less transparent markets, such as the market for NFTs that did not exist as of 2000. However, as opposed to cryptocurrency markets, in which the value-generating mechanism stems from the benefits derived from the platform’s efficiency, the viability of the NFT market must be explained by a different mechanism. If the NFT market is substantiated as viable, it may efficiently replace the need for a legal code that governs intellectual property rights (IPRs) regime in the long run.

<sup>1</sup> The cryptocurrency market currently has a 4.5 billion USD monthly volume according to OpenSea, the largest NFT marketplace.

A value-generating mechanism is a sub-group of opinion-generating mechanisms conceptualized in the literature as opinion dynamics, that is, the evolution of opinions through social interaction between a group of agents. A recent literature review was provided by Zha et al. (2020).

An NFT is a digitally created token that represents a digital asset that digitally replicates, perhaps without infringing IPRs,<sup>2</sup> a potentially value-generating underlying asset, such as a good or service, commodity, privileged information, or anything else that may generate value. It is recorded as an indelible ledger on a blockchain and is currently on the Ethereum blockchain. The ERC-721 protocol is used to create NFTs that can represent just about anything. Meanwhile, the “miners” who assist in the creation of NFTs are compensated for the verification process. To economize on the costs of creating NFTs, creators typically use numbers, addresses, and URLs. The number identifies the NFT, the address identifies the owner of the NFT, and the URL identifies the underlying asset associated with the NFT. The process is irreversible, and one can only “burn” an NFT by rendering it unusable. The term “non-fungible” indicates that an NFT is unique and cannot be substituted for an equivalent token.<sup>3</sup>

### Literature review

NFTs have been a research topic in various academic tangent disciplines, such as finance, real estate economics, law (intellectual property), and information technology. However, their analyses of this recent market innovation are conducted from different perspectives. For example, finance scholars are intrigued by the valuation issue, whereas the law scholars are concerned about the issue of property rights. Meanwhile, the information technology literature deals with technological issues arising from the uniqueness of NFTs.

Sunyaev et al. (2021) explained that there are two options for creating tokens in an existing distributed ledger. The first is to use a private distributed ledger in which only authorized agents can join, read transactions from, and append new transactions to the distributed ledger. The second option allows agents to decide only on an existing distributed ledger for the token economy instantiation and create custom tokens by using a smart contract. For a survey of the technical aspects of the token creation process, see Kannengießner et al. (2020) and Frigden (2021).

The microstructure of the NFT marketplace generally has two forms. In one form, the seller may fix the NFT price for a certain period, usually up to 180 days. An interested buyer can directly purchase the NFT in USD or the Ethereum equivalent amount. In the other form, the NFT can be offered under a fixed-time auction. For further discussion, see Mukhopadhyay and Ghosh (2020a, 2020b) and Fortnow (2021).

An NFT provides some form of ownership, but it does not grant the owner exclusive ownership and control of the use of the underlying asset; that is, someone else may legitimately own and use the underlying asset, which is defined as a scarce, potential cash

<sup>2</sup> Law scholars debate this issue and some are under the opinion that NFTs do infringe IPRs. For the purpose of this paper, this is hardly a critical issue.

<sup>3</sup> In economics, fungibility is the property of an asset, consumption good, or commodity whose individual components are indistinguishable from each other and essentially interchangeable. Examples of fungible assets include financial securities commodities and medium of payments.

flow producer. The distinction between ownership and the ability to control and use an underlying asset seems to be artificial as financial economists rarely make this distinction but rather assign a value to the outcome derived from the use of the asset irrespective of the identity of the owners.<sup>4</sup> This case does not apply to an NFT as the “identity” of the owner is a critical factor. Frye (2021) coins the term “pwnership,” which reflects the notion that an NFT involves clout, status, power, and fame rather than control. This observation may lend itself to the definition of a new concept; rather than producing potential cash flows, an asset may produce potential “intangible flows,” thereby generating positive pecuniary value.<sup>5</sup> The notion of intangible flows may include any type of benefit derived from holding an NFT, such as fame, status, knowledge, and expected monetary gain. Nadini et al. (2021) provide an overview of some central NFT features that span the six main NFT categories, including art, games, and collectibles. The current study adopts this view by incorporating various sources of benefits into the underlying process that governs the price behavior of NFTs.

Thus, if an NFT is an intangible asset, one may suspect that it provides some form of economically motivated protection, if not a legal one, such as the protection provided by IPRs or other types of exclusive legal property rights. Intellectual property is defined as a category of properties that include intangible creations of the human intellect.<sup>6</sup>

On the other hand, the economically motivated protection of NFTs may provide some potential gain; therefore, an important issue is whether the expected gains of NFTs may provide a substituted course of action for IPRs. For example, NFTs in real estate markets, or “title tokens” in the language of Konashevych (2020), provide proof of the unique exclusive ownership of real estate that may replace the need to record ownership in a centralized, public administration database and may even render redundant the commonly used title insurance. This is obviously a legal prospective of the issue, but the broad potential economic implications must also be considered as NFTs may entangle any type of asset (i.e., real, financial, or intangible). In real estate, the actual creation of NFTs has been evident, given the possibility of applying NFTs to a broader spectrum of tangible and intangible assets, such as firms’ reputation, security issuance, privileged information (as discussed below), and many others.

Some of the above applications are under a well-established set of regulations or are governed by IPRs. However, one can create NFTs of assets that are in the public domain and even create NFTs of others’ assets that are protected by IPRs. Some NFT marketplaces may avoid the so-called “infringement” of NFTs, but NFTs still exist and can be sold as decentralized blockchains tend to evade control regardless of the nature of the NFTs. Moreover, no one can argue that the creation of NFTs violates these regulations or infringes on IPRs as long as they do not rival the use of the underlying assets (e.g., reducing the ability of the underlying asset to generate value). Thus, the NFT financial arena may exist side by side with the legal world, and we may be so bold as to hypothesize that under certain conditions,

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<sup>4</sup> One possible exception where the distinction between ownership and control is maintained is in control theory.

<sup>5</sup> The notion of intangible flows may be broadened, as pointed out by one of the reviewers, to include for example, that one can gain value from the widespread knowledge of a “product” being simply named as owner, identified by the NFT.

<sup>6</sup> World Intellectual Property Organization (WIPO) (2016).

NFTs may render these regulations redundant and may even replace the need for the legal umbrella of IPRs.

A rational agent will produce an asset (and be able to control the use of a good, service, real, financial, or intangible asset) if and only if this agent can secure a potential reward that will compensate for the costs, time, and effort expended in the process of production. In financial terms, the net present value (NPV) must be non-negative. The absence of a mechanism that ensures the scarcity of assets and a clear-cut identity of ownership, thereby protecting the rights of producers, may lead to market failure. Producers of potentially value-generating assets can secure their rights under several sets of mechanisms, such as (i) prepaid production; (ii) production contracts subject to transfer of ownership, right of use, and payment upon completion of production; and (iii) production under implicit IPR contracts.

Although the protection provided by the legal code that governs IPRs may prevail, the infringement of IPRs is still possible. In such a case, the asset will become a public good while the producers will be unable to enforce their rights, or doing so will not make economic sense. In addition, the producers may voluntarily choose to produce an unprotected asset that becomes a public good. We should not dismiss the rationality of the latter case as the Internet has recently enabled the introduction of platforms that offer “quasi-public goods” (e.g., YouTube, mobile applications, Spotify). In these platforms, the use of underlying assets is not protected under IPRs, yet there exists an economic rationale for the value-generating mechanisms underlying these types of assets.

Thus, if otherwise IPRs, which are governed and enforced by law to prevent market failure, remain as an option to protect producers’ rightful claim to use their assets, NFTs may become a complementary and possibly a real option.

As pointed out above, Nadini et al. (2021) provide an overview of some central NFT features that span the six main NFT categories. The findings show that past sales history is the best predictor of NFT price.

Bao and Roubaud (2022) provide a summary of the empirical literature that analyzes the behavior of NFTs. Specific studies such as those of Maouchi et al. (2021), Aharon and Demir (2021), Dowling (2022a, b), Umar et al. (2022), Ko et al. (2022), and Yousaf et al. (2022), report empirical findings on the behavior of NFTs, adhering to asset pricing models while allowing for empirical estimation of the underlying parameters.

The broader aspects of digitalization of the business environment or the analysis of the vitality or likelihood of success of cryptocurrency markets are beyond the scope of this paper. A good comprehensive survey of the relevant literature is given in the work of Fang et al. (2022). Suggestions for value drivers are given in various studies, such as those of Pástor and Veronesi (2009), Howell et al. (2019), Li and Mann (2020), Sockin and Xiong (2020), Cong et al. (2021), and Dowling (2022a, b), as well as the empirical work of Karim et al. (2022). By contrast, this study aims to identify the potential value-generating mechanisms underlying NFTs and the implications for the need for IPRs.

### **Goal of this study**

This study aims to fill the gap in the academic literature by providing a theoretical financial framework for the valuation of NFTs. By incorporating the parameters empirically estimated in the literature into the derivations established in this study, we can calculate

the rational values of NFTs. As we also consider the tangent issues raised in other disciplines, we analyze the financial ramifications of the likelihood that NFTs may serve as substitutes for IPRs.

Therefore, this study has two goals. First, by establishing value-generating mechanisms, this study aims to establish NFT price. Second, this study hypothesizes that NFTs may potentially serve as a substitute for IPRs and establishes a financial condition for this hypothesis to materialize. We realize that law scholars may be skeptical about the likelihood of the above scenario occurring by raising legal constraints that are not discussed in this paper.

This study begins by establishing potential value-generating mechanisms for NFTs on the basis of the notion of “intangible flows,” which we define below. Subsequently, the study sets the conditions under which NFTs may rival the paradigm underlying IPRs. For this purpose, we recognize two types of NFTs. The first type comprises NFTs whose underlying asset is a paradigmatic and unique public good and whose limitless consumption does not reduce supply as the notion of scarcity does not apply (e.g., reputation, art, patents, and privileged information). In all these cases, the benefits of consumption are limitless. The second type comprises NFTs whose underlying asset is a scarce resource with multiple exemplars. Our analysis focuses on the first type as additional assumptions are required for the second type. The study then evaluates the notion of diversification of a portfolio of NFTs. Finally, the study applies NFTs to the issue of privileged information.

### **Modeling the value-generating mechanism of NFTs**

Scholars tend to evaluate NFTs in traditional terms, where an asset may rationally generate a positive value if there is a positive probability that it will generate periodic pecuniary income (e.g., dividends, rents, interest) and/or that the expected growth in undistributed income will generate capital appreciation. In general, the lack of an actual periodic distribution does not ipso facto turn asset prices into bubbles. However, what if an asset (e.g., a digitally created NFT) is precluded from the use of periodic pecuniary distributions and expected undistributed income? Can we envision a mechanism that may rationally justify the expected capital appreciation of NFTs? To revisit Marsh's example, in the case of the cryptocurrency market, scholars tend to attribute the value-generating mechanisms to the efficiency of the specific crypto-platforms. In the case of the tulip mania, one may argue that the tulips served as a commonly accepted medium of exchange. However, neither of these examples applies to NFTs, which produce intangible flows and capital gains that stem from the growth of these flows.

Our economy consists of the following three types of participants. The analysis of the various types of agents also lays down our assumptions.

Agent 1 is the classical economic producer who is the present value maximizer. This agent is capable of producing a scarce resource that may be in demand by others, such as real or financial assets, goods or services, pieces of art, patents, or intangible assets (e.g., privately owned pieces of undisclosed information). This agent will definitely refrain from producing the asset if the probability of pecuniary compensation is zero. The horizon of this agent may very well be finite, that is, it terminates when the asset ceases to produce a cash flow, as in the case of assets that are perishable, obsolete or out

of fashion, economically depreciable, or protected by a limited-time IPR. Subsequently, the asset becomes a public good.

Agent 2 is a potential buyer of an existing NFT. Similar to a typical rational economic agent, Agent 2 is motivated by expected pecuniary gains, representing a typical rational economic agent, but in addition, agent 2 is motivated by the expected value of intangible flows such as status, power, clout, and fame, which are associated with the ownership of NFTs. This agent does not wish to control the underlying asset. Rather, this agent wishes to acquire a status derived from the increasing demand for the underlying asset. That is, a positive correlation exists between the successful growth of the underlying asset and the agent's valuation of the status associated with the ownership of the NFT.

Scholars in the field of sociology have thoroughly analyzed the concept of the value of status. Mark et al. (2009) states,

*“No individual or category-level differences with respect to any exogenously status-relevant variable are necessary for consensual status beliefs to emerge. According to previously offered explanations, the emergence of consensual status beliefs requires individual and category-level differences with respect to some exchangeable resource or some other consensually valued object or characteristic. [...] The status value attached to a nominal characteristic can be completely arbitrary.”*

Thus, NFT creates a market for “ownership short of use,” thereby creating a parallel market for intangible assets such as status, clout, and power. Let  $S$  be an oceanic population of market participants, and let  $s \in S$  be a subpopulation consisting of type 2 agents, as defined above, each endowed with an “equal status” and equal cash position. All participants in subpopulation  $s$  share the same belief that  $P(t)$ , the underlying asset, is a positive signal concerning the expected value of status, but they differ in their beliefs about an additional set of nominal characteristics  $L$ . By contrast, the participants in subpopulation  $s' \notin S$  do not form any beliefs. The relative ordinal ranking of status by the participants in subpopulation  $s$  is denoted by  $W(L)$ . We endogenize two parameters of  $L(\cdot)$ : (1) the individual risk preference  $\pi$ , which determines that agent 2 takes the risky action  $q$ ,  $q = [0,1]$  depending on her risk preferences; and (2) the size of subpopulation  $s$ . We initially set a fixed size of subpopulation  $s$ , thereby assuming that  $L(\pi, s, \delta) = L(\pi)$ . We relax this assumption and incorporate  $(s, \delta)$  in the following sections, where  $\delta$  is an assumed behavioral constant, as specified below.

Additionally, we are uncertain about the use of a discount factor  $\mu^w$  to assess the present value of  $v(W)$  as it is not clear whether the future status is less valuable than the present status. Nevertheless, the risk of a future loss of status may lend itself to a positive discount factor. Thus,  $E[v(W(P, L))] = E'[v(W(P, L) - \pi_{s_i})]$  is the risk-adjusted pecuniary value of status that is cumulative over time; here,  $\pi_{s_i}$  is the premium that member  $s_i$  is willing to pay to avoid risk. By taking an exclusive action,  $q = [0,1]$ , which cannot be mimicked by others, member  $s_i$  may increase her ranking depending on the density function associated with the value of action  $q$ , the size of subpopulation  $s$ , and her risk preferences.

Let  $P(t)$  be the value and market price of the value-generating underlying asset that is continuously traded over time,  $t$ , up to a finite horizon  $T$ . Let  $F(\cdot)$  be the current value of the NFT. Finally,  $v(W_t)$  is cumulative over the entire holding period, that is,

$$v(W_t) = \int_t^T dv(W(t))e^{-\mu^w t} q dt. \quad (1)$$

As explained previously, the reward for the ownership of NFTs can be separated into intangible flows  $v(W)$  and capital gains that stem from the growth rate of  $v(W)$ . However,  $v(W)$  belongs solely to the holder of an NFT and cannot be sold unless the buyer's  $v(W)$  is higher. As flows are cumulative during the holding period after the NFT is purchased, the holder accumulates past  $v(W)$  and expects  $v(W)$  in the future. Under the Markovian property and in line with standard financial analysis, past  $v(W)$  has no bearing on agents' decision making.

A terminal boundary condition is set at time  $T$ , which is yet to be specified, and  $q$  is the quantity of NFT,  $q = [0,1]$ . Agent 2 is assumed to maximize a pecuniary function such that

$$\text{Max}_{[q=0,1]} [\max\{E'F(P_t), Ev(W_t)\}, 0] \text{ yields } F^*(P_t, v(W_t)) \quad (2)$$

where  $F^*(\cdot)$  is the value of the NFT.

Subsequently, three different models that assume variable dependency<sup>7</sup> are proposed and analyzed.

1.  $dv(W_t) = f(dP_t, L)$
2.  $dv(W_t) = g(dF_t, L)$
3.  $dv(W_t)$  and  $dF_t$  are jointly bivariate normally distributed.

Several intertwined factors imply that our economy differs slightly from the standard economic framework. First, the supply of the underlying asset is fixed, and its consumption does not reduce supply; that is, the notion of scarcity does not apply. The NFT supply is given by  $q = [0,1]$ . Subpopulation  $s$  consists of multiple type 2 agents with heterogeneous beliefs about  $v(W)$ . Thus, setting the value of an NFT may not be dependent on a market-clearing condition but rather on the non-contradictory notion that the highest-value user sets the market price.

We make the critical yet realistic assumption that the overall economy-wide value of  $v(W_t)$  across all NFTs may not exceed the overall values of all underlying assets. Then, the first and second partial derivatives are

$$0 \leq \frac{\partial v(W_t)}{\partial P_t} \leq 1 \text{ or } \frac{\partial^2 v(W_t)}{\partial P_t^2} \leq 0, \quad (3)$$

It is possible to justify both cases of the second derivative, that is, being negative or positive. However, for our purpose, whether  $v(W_t)$  is increasing at a decreasing/increasing rate, it asymptotically tends to  $P_t$ , that is,  $\lim_{t \rightarrow T} (v(W_t)) = P_t$ .

Agent 3 is a speculator and an expected gain maximizer who believes that agent 2 exists. Thus, agent 3 creates an NFT if and only if

<sup>7</sup> We assume dependency between the variable, i.e., an NFT is a derivative asset. One can assert that the value of an NFT is completely independent of the values of the related variables, but in this case, a model would solely rely on the assumptions of the underlying parameters and therefore would lack robustness.



$$PV[EF(P_t) - c] > 0 \quad (4)$$

where  $c$  denotes the exogenously given cost of setting up a wallet and installing the software necessary for participating on the platform, the cost of the likelihood of legal action, and the costs of the non-universality of the NFT. Note that  $c$  is uncorrelated with  $F$  (see Krishnamurthy and Vissing-Jorgensen 2012 and earlier studies for further discussion). The existence of agent 3 or agent 1, who acts like agent 3, is a necessary condition for our economy to be viable.

Formally, (4) implies

$$\text{if } \begin{cases} E' \int_{t=0}^T e^{-\mu t} dF_t = E \int_{t=0}^T e^{-rt} dF_t - c \geq 0, \text{ then } q = 1 \\ E' \int_{t=0}^T e^{-\mu t} dF_t = E \int_{t=0}^T e^{-rt} dF_t - c < 0, \text{ then } q = 0 \end{cases} \quad (5)$$

In summary, agent 3 is an intermediary between agent 1, the producer, and agent 2, who is the end buyer of the NFT. The creator of the NFT can be either the producer (agent 1) or the speculator (agent 3). This point is elaborated in section "Goal of this study".

#### Case 1: $dv(W_t) = f(dP_t, L)$

We follow Merton's continuous-time framework to account for the fact that the underlying asset is continuously tradable or that there exist tradable substitute securities that mimic the payoffs of the underlying asset. The value-generating process underlying asset  $P(t)$  is a geometric Brownian motion (GBM), that is,

$$dP = P(\mu dt + \sigma dZ), \quad (6)$$

where  $dZ$  is the standard Brownian motion under the physical measure and  $\mu$  and  $\sigma$  are constant parameters that reflect the drift and standard deviation, respectively.

We describe the methodology and theoretical justifications from a discipline other than finance. Merton was the first scholar to suggest a continuous-time framework for asset pricing. This framework enables market participants to form a self-financed (i.e., zero net outlay) portfolio. By continuously updating the weight of each asset in the portfolio, the net outcome becomes riskless. If such a portfolio is formed, the rate of return must be the riskless rate of return (e.g., the rate of interest in government bonds or the rate of return for a market-orthogonal<sup>8</sup> asset). The portfolio contains  $n-1$  assets whose values are given exogenously and one asset (generally a derivative of such an option) whose value is sought. As the overall portfolio yields the riskless rate of return, one can derive the implied value of the  $n$ 'th asset whose value is sought. This partial equilibrium methodology is labeled as risk-neutral valuation.

Following Duffie (1994) and Campbell (2017), the stochastic discount factor (SDF) with an instantaneous correlation between  $\gamma$  and  $P$  is as follows:

$$dk = k(-rdt - \gamma dZ^k), \quad (7)$$

<sup>8</sup> Orthogonal means an asset whose rates of return have zero correlation with the market returns.



While

$$dZ^k = dZ'^k + \gamma dt$$

Thus, under the risk-neutral measure,

$$dP_t = P_t((\mu - \rho\gamma\sigma)dt + \sigma dZ) = P_t(\mu^* dt + \sigma dZ) \quad (8)$$

and

$$dv(W_t) = \int_t^T \frac{\partial f}{\partial P}(P)P(t)qdt$$

Let  $f(\cdot)$  be a specific<sup>9</sup> function given  $P_t$  such that if  $q = I$ ,

$$dv(W_t) = \int_t^T \frac{\partial v(W)}{\partial P}(P_t)dt = A + \alpha dP(t) + \delta dP^2(t) \quad (9)$$

where  $\alpha$  is a positive scalar and  $\delta \in L(\cdot)$  reflects a “status” preference parameter of the highest-value user among all type 2 agents in subgroup  $s$ . Recall that agent 2 evaluates  $F^*$  as in (2).

On the basis of Ito’s lemma and our assumptions,

$$dF_t = \begin{cases} \left[ \left( \frac{\partial F}{\partial t} + \frac{\partial F}{\partial P} P_t \mu^* + .5 \frac{\partial^2 F}{\partial P^2} P_t^2 \sigma^2 \right) dt + \frac{\partial F}{\partial P} P_t \sigma dZ \right] & \text{if } F_t > v(W_t) \\ A + \alpha dP(t) - \delta dP(t)^2 & \text{if } F_t = v(W_t) \end{cases} \quad (10)$$

where

$$\mu^F = \begin{cases} \left[ \left( \frac{\partial F}{\partial P} P_t \mu^* / F_t + .5 \frac{\partial^2 F}{\partial P^2} P_t^2 \sigma^2 / F_t \right) \right] & \text{if } F_t > v(W_t) \\ A + \alpha dP(t) - \delta dP(t)^2 \mu^* & \text{if } F_t = v(W_t) \end{cases} \quad (11)$$

To preserve continuity at the first point in time when  $F_t = v(W_t)$ , we set

$$\left( \frac{\partial F}{\partial P} P_t \mu^* / F_t + .5 \frac{\partial^2 F}{\partial P^2} P_t^2 \sigma^2 / F_t \right) = (A + \alpha dP(t) - \delta dP(t)^2) \mu^*$$

Moreover,

$$\sigma^F = \begin{cases} \frac{\partial F}{\partial P} P_t \sigma / F_t & \text{if } F_t > v(W_t) \\ \frac{\partial v(W)}{\partial P} P_t \sigma / v(W) & \text{if } F_t = v(W_t) \end{cases} \quad (12)$$

Under the self-financing paradigm and adding a non-stochastic cash flow distributed from the underlying asset,  $P_t$ , the following are the final partial differential equations (PDEs) that govern the behavior of  $F$  and  $v(W)$ :

<sup>9</sup> This specific functional form describes the evolution of  $v(W)$  to account for a maximum  $v(W)$  at a finite point in time. Alternatively, the case where  $dv(W)$  is independently stochastic is less informative as it yields a trivial PDE. Implementing any specific relationship may be more informative.

$$\begin{cases} \frac{\partial F}{\partial t} + .5 \frac{\partial^2 F}{\partial P^2} P_t^2 \sigma^2 - rF_t + (r - \vartheta) P_t \frac{\partial F}{\partial P} = 0 & \text{if } F_t > v(W_t) \\ \frac{\partial v(W)}{\partial t} + (r(v(W) - (\alpha - 2\delta P_t)) + P_t^2 \sigma^2 \delta^3 = 0 & \text{if } F_t = v(W_t) \end{cases} \quad (13)$$

The second line in (13) is derived as follows: Under the self-financing assumption,

$$dv(W_t) = \varphi_1(t) \alpha dP_t + \varphi_2(t) rB_t \quad (14)$$

where  $B_t$  is a riskless bond whose rate of interest is  $r$ .  $\varphi_i$  denotes the continuously changing weights that ensure equality (14) at any point in time.

$$dv(W_t) = \int_t^T \frac{\partial v(W_t)}{\partial P_t} dP_t = A + \alpha dP(t) - \delta dP^2(t) \quad (15)$$

As  $(dZ)^2 = dt$ , following Ito's lemma,

$$v(W_t) = \left[ \frac{\partial v(W_t)}{\partial t} + \alpha \frac{\partial v(W)}{\partial P} dP_t + .5 \frac{\partial^2 v(W)}{\partial P^2} (\sigma \delta)^2 \right] dt + \frac{\partial v(W)}{\partial P} \sigma dZ \quad (16)$$

Therefore,

$$\varphi_1(t) \alpha dP_t + \varphi_2(t) rB_t = \left[ \frac{\partial v(W_t)}{\partial t} + \alpha \frac{\partial v(W)}{\partial P} dP_t + .5 \frac{\partial^2 v(W)}{\partial P^2} (P_t^2) (\sigma \delta)^2 \right] dt + \frac{\partial v(W)}{\partial P} \sigma dZ \quad (17)$$

as  $\varphi_1 = \frac{\partial v(W)}{\partial P} = \alpha - 2\delta P_t$ , and under the self-financing requirement,

$B_t = \frac{v(W) - \varphi_1 \alpha P_t}{\varphi_2}$ , then  $\varphi_2(t) rB_t dt = (r(v(W) - (\alpha - 2\delta P_t))) dt$ . Substituting this into Eq. (17) yields  $(r(v(W) - (\alpha - 2\delta P_t)) + P_t^2 \sigma^2 \delta^3$ .

It is important to incorporate a tax parameter into the model as realized NFTs' gains are taxed at rate  $\tau$ , that is, the capital gain tax rate (long or short term), whereas neither accumulated past benefits,  $dv(W)$ , nor future benefits are taxable. This follows from Proposition 1 if  $Ev(W_t) > EF(P_t)$  and the NFT is never going to be sold. Thus, agent 2, while deciding on an action (buy, sell, hold), performs the following optimization:

$$\text{Max}_{[q=0,1]} [\max\{(EF(P_t)(1 - \tau), Ev(W_t)), 0\}] \quad (18)$$

Then, her decision to buy depends on her opportunity costs as of time  $t$ :

$$\text{if } \begin{cases} \text{Max}[\text{Max}\{(1 - \tau)(E'F_t e^{-rT} - F_t), Ev(W_t)\}] \geq \int_t^T r(1 - \tau) F_t^* dt & \text{Buy} \\ \text{Otherwise} & \text{Avoid} \end{cases} \quad (19)$$

This condition is necessary for timing a purchase, but it not a sufficient one as agent 2 may choose NFTs that have the largest differences in (19) given her budget constraint.

If  $\max\{(1 - \tau)(E'F_t e^{-rT} - F_t), Ev(W_t)\} = (1 - \tau)(E'F_t e^{-rT} - F_t)$ , agent 2 acts like agent 3, but uniquely for agent type 2,  $\max\{(1 - \tau)(E'F_t e^{-rT} - F_t), Ev(W_t)\} = Ev(W_t)$ .

If agent 2 buys an NFT, then

$$\text{if } \begin{cases} \text{Max}[(1 - \tau)(EF_t e^{-rT} - F_t), Ev(W_t) - v(W_t)] > (1 - \tau)(EF_t e^{-rT} - F_t) dt, & \text{then Hold} \\ \text{Otherwise} & \text{Sell} \end{cases} \quad (20)$$

At the point of the optimal sale date, agent 2, as defined above, has no reason to continue to hold, whereas the buyer who needs to pay  $F_t = \text{Max} \left[ (1 - \tau) \left( EF_t e^{-rt^*} - F_t \right), Ev(W_t) - v(W_t) \right]$ , if such a buyer exists, must have a higher valuation  $Ev(W_t)$  of the NFT. This leads us to a trivial proposition; however, one is needed for the next section.

**Proposition 1** *If agent 2's preferences are constant and subpopulation  $s$  is fixed in size, agent 2, who is the highest-value user, will never sell an already held NFT.*

*Proof* At time  $t$ , agent 2 holds an NFT if

$$\text{Max} \left[ (1 - \tau) \left( EF_t e^{-rt^*} - F_t \right), Ev(W_t) - v(W_t) \right] > (1 - \tau)(F_t - F)e^{-rt} dt. \quad (21)$$

Being the highest-value user (constant preferences are assumed), her valuation  $Ev(W_t)$  is the highest; therefore, her selling price  $F_t^*$  is high enough that no one else belonging to fixed subpopulation  $s$  will be willing to pay the price. Moreover, at any subsequent point in time, as beliefs about  $P_t$  are shared by all members and are embedded in the holder's  $Ev(W_t)$ , the above inequality remains indefinitely. This is true even if  $P_t$  does not follow a martingale. Moreover, at time  $T$ ,  $P_t = 0$ . (The assumption of fixed subpopulation  $s$  is relaxed in section "Goal of this study".)  $\square$

No closed-form solution to (13) is available because of the existence of a free boundary condition. In Appendix A, we present a numerical method to solve the PDE subject to the estimation of the parameters.

**Case 2:**  $v(W_t) = g(dF_t, L)$

What if the movements of  $v(W_t)$  are affected by the value of the NFT, which, in turn, is governed by the behavior of the underlying asset? In this case,  $v(W_t)$  and the NFT value are set recursively. The underlying asset price,  $P_t$ , is still governed by the same process as in case 1.

$$dP_t = P_t((\mu - \rho\gamma\sigma)dt + \sigma dZ) = P_t(\mu^*dt + \sigma dZ)$$

First, by Ito's lemma,

$$dF_t = F_t \left[ \left( \frac{\partial F}{\partial t} + \frac{\partial F}{\partial P} P_t \mu^* + .5 \frac{\partial^2 F}{\partial P^2} P_t^2 \sigma^2 \right) dt + \frac{\partial F}{\partial P} P_t \sigma dZ \right] = F_t \mu^F dt + F_t \sigma^F dZ \quad (22)$$

Then, once again,

$$dv(W_t) = \left[ \left( \frac{\partial v(W)}{\partial t} + \frac{\partial v(W)}{\partial F} F_t \mu^F + .5 \frac{\partial^2 v(W)}{\partial F^2} F_t^2 \sigma^{F2} \right) dt + \frac{\partial v(W)}{\partial F} F_t \sigma^F dZ \right] \quad (23)$$

and  $dv(W_t) = \varphi_1(t)dF_t + \varphi_2(t)rB_t$  by the self-financing requirement, that is,

$$\begin{aligned}
& \left[ \left( \frac{\partial v(W)}{\partial t} + \frac{\partial v(W)}{\partial F} F_t \mu^F + .5 \frac{\partial^2 v(W)}{\partial F^2} F_t^2 \sigma^{F^2} \right) dt + \frac{\partial v(W)}{\partial F} F \sigma^F dZ \right] \\
& = \varphi_1(t) dF_t + \varphi_2(t) r B_t \\
& = \varphi_1(t) F_t \mu^F dt + \varphi_1(t) F_t \sigma^F dZ + \varphi_2(t) r B_t
\end{aligned} \tag{24}$$

In this case,  $\varphi_1(t) = \frac{\partial v(W)}{\partial F}$ , and thus,

$$\left( \frac{\partial v(W)}{\partial t} + 5 \frac{\partial^2 v(W)}{\partial F^2} F_t^2 \sigma^{F^2} \right) = \varphi_2(t) r B_t = \frac{\varphi_2(t) r [v(W) - \varphi_1 F_t]}{\varphi_2(t)} = r v(W) - r \frac{\partial v(W)}{\partial F} F_t \tag{25}$$

Inserting  $\sigma^{F^2}$  from (12) into (25) yields

$$\frac{\partial v(W)}{\partial t} + 5 \frac{\partial^2 v(W)}{\partial F^2} \left[ F_t P_t \sigma \frac{\partial F}{\partial P} \right]^2 + r \frac{\partial v(W)}{\partial F} F_t - r v(W_t) = 0 \tag{26}$$

Meanwhile, the PDE for  $F_t$  is the standard one, that is,

$$\frac{\partial F}{\partial t} + 5 \frac{\partial^2 F(W)}{\partial P^2} [P_t \sigma]^2 + r \frac{\partial F}{\partial P} P_t - r P_t = 0 \tag{27}$$

The PDE for  $v(W)$  relies on an additional assumption: Let  $g(F_t)$  be a specific functional form that describes the evolution of  $v(W)$  to account for a low  $v(W)$  at the beginning of the holding period, becoming explosive subsequently, and decaying at the end. For example,

$$dv(W_t) = \int_t^T \frac{\partial g}{\partial F}(F_t) dF = \alpha \left( dF_t t - \frac{t}{(\delta)^{dF_t}} \right) \tag{28}$$

Then, the PDE for  $v(W)$  is

$$\frac{\partial v(W)}{\partial t} - .5(F_t + 1)t\delta^{-F_t-2} \left[ F_t P_t \sigma \left( \alpha F_t - \delta^{-F_t-1} \right) \right]^2 + r \left( \alpha F_t - \delta^{-F_t-1} \right) F_t - r v(W_t) = 0 \tag{29}$$

In summary, if and once  $F_t = \text{MAX}(Ev(W_t), E'F_t) = Ev(W_t)$ ,  $F_t$  and  $dv(W_t)$  are recursively set, and  $v(W_t)$  may or may not converge to a finite number depending on parameter  $\delta$ .

$$\begin{cases} \frac{\partial F}{\partial t} + .5 \frac{\partial^2 F}{\partial P^2} P_t^2 \sigma^2 - rF + (r - \vartheta) P \frac{\partial F}{\partial P} = 0 & \text{if } F_t > v(W_t) \\ \frac{\partial v(W)}{\partial t} - .5(F_t + 1)t\delta^{-F_t-2} \left[ F_t P_t \sigma \left( \alpha F_t - \delta^{-F_t-1} \right) \right]^2 + r \left( \alpha F_t - \delta^{-F_t-1} \right) F_t - r v(W_t) = 0 & \text{if } F_t \leq v(W_t) \end{cases} \tag{30}$$

Note that the distributed cash flow component  $\vartheta$  does not appear in the second line above as the NFT does not have any distributable cash flows.

The optimal stopping rules (buy, sell vs. hold) for agent 2 remain the same as in case 1.

### Case 3: $F_t$ and $v(W)$ are jointly bivariate normally distributed

What if the co-movements of  $v(W_t)$  and  $F_t$ , each of which is normally distributed, are known to be bivariate normally distributed with a positive correlation? In other words, both variables are price-related, but each has a stochastic component that depends on

the underlying price. If this is the case, we may be able to derive a closed-form solution for the value of an NFT as the value of an option on a maximum of two assets.

Let  $v(W_t)$  and  $F_t$  be governed by a GBM derived by Ito's lemma from

$$dP = P((\mu - \rho\gamma\sigma)dt + \sigma dZ) = P(\mu^* dt + \sigma dZ)$$

That is,

$$dF_t = \left( \frac{\partial F}{\partial P} P_t \mu^* + .5 \frac{\partial^2 F}{\partial P^2} P_t^2 \sigma^2 \right) dt + \frac{\partial F}{\partial P} P_t \sigma dZ = \mu^F dt + \sigma^F dZ$$

and

$$dv(W) = \left( \frac{\partial v_w}{\partial P} P_t \mu^* + .5 \frac{\partial^2 v_w}{\partial P^2} P_t^2 \sigma^2 \right) dt + \frac{\partial v_w}{\partial P} P_t \sigma dZ = dv_t = \mu^W dt + \sigma^W dZ \quad (31)$$

Following the self-financing strategy, the final PDE is

$$\begin{aligned} \frac{\partial F^*}{\partial t} - rF^* - (r - \theta)F \frac{\partial F^*}{\partial F} + (r - \theta)v(W) \frac{\partial F}{\partial v(W)} \\ - .5 \left( F^2 \sigma_F^2 \frac{\partial^2 F^*}{\partial F^2} + v(W)^2 \sigma_W^2 \frac{\partial^2 F}{\partial v(W)^2} - 2v(W)F \rho_{FW} \sigma_F \sigma_W \frac{\partial^2 F}{\partial F \partial v(W)} \right) = 0 \end{aligned} \quad (32)$$

$F_t^* = \text{Max}[\max\{F_t, v(W_t)\}; TH, T], 0]$ , and  $(TH)$  is a threshold condition representing the carry (opportunity) costs. In other words,  $F_t^*$  is an American-type option on the maximum of  $F_t$  and  $v(W_t)$ , where at the first point in time and at any point in time subsequently, when  $F_t \leq v(W_t)$ ,  $F_t$  may be rationally exchanged for  $v(W_t)$ , incurring the “exercise” price that is equal to the after-tax carry costs.

Agent 2 may sell an NFT if the expected gain in  $\max\{\max[F_t^*, v(W_t)], 0\}$  is equal to or less than the carry (opportunity) costs; that is,

$$\text{if } \left\{ \begin{aligned} & \left[ \frac{\text{MAX}(E_t F_{t+dt}, E_t v(W_{t+dt}))}{\text{MAX}(F_t, v(W_t))} - 1 \right] > \frac{((1-\tau_r)F_t^* - \tau_g(F_t^* - F_0))rdt}{\text{MAX}(F_t, v(W_t))}, \text{ then Hold} \\ & , \text{ otherwise, Sell} \end{aligned} \right.$$

where the term on the left-hand side is the after-tax, risk-adjusted, expected rate of return if agent 2 continues to hold. The term on the right-hand side is the after-tax opportunity rate of return, embedded in the valuation of  $F_t^*$ , minus the rate of return on the deferral of already accumulated tax liabilities.  $\tau_g$  is the capital gain tax rate, and  $\tau_r$  is the ordinary tax rate. Thus,  $TH = ((1 - \tau_r)F_t^* - \tau_g(F_t^* - F_0))rdt = ((1 - \tau_r - \tau_g)F_t^* + \tau_g F_0)rdt$  for any interval  $dt$ . If taxes are disregarded, claiming that  $F_t^*$  reflects after-tax returns, it becomes a simpler option to exchange  $F_t$  for  $v(W_t)$  (see Margrabe 1978).

To derive one form of a closed-form solution (see Appendix B), we set the boundary condition  $F(T) = 0$ . We can derive a closed-form solution when  $T \mapsto \infty$  (see Martyr 2017). For any other boundary condition, we may need to use bivariate finite-difference methodology (see Appendix A).

### NFTs and IPRs: an extension

What if agent 1, the producer, creates an NFT of her own asset? Essentially, the creation of NFTs may be beneficial because they enable producers to securitize their assets as the NFT market is really a securities market. Producers may choose to protect the exclusivity of the use of their asset, that is, to protect potential cash flows using any restricting mechanism or one that is governed by an IPR. Alternatively, the producers may refrain from any protection (i.e., the asset is likely to become a public good) and may instead create related NFTs. Under both alternatives, the producers are exposed to the risk of loss in some states of nature irrespective of whether or not they use a protection mechanism. Thus, the question needs to be phrased differently: Under what conditions are IPRs rationally replaceable by NFTs?

The applicable intuitive condition is that IPRs are replaceable by NFTs when the present value of the expected net proceeds generated by the protected underlying asset less the costs of production,  $C$ , is smaller than the expected value of the NFT less the costs of creation,  $c$ ; that is, when

$$PV\{EP - C\} < PV\{F - c\}. \quad (33)$$

Unit-wise, the NFT has  $q = [0, 1]$  while the underlying asset,  $P_t$ , is measured by the value of the entire production as a single number irrespective of the quantity produced. We assume a finite horizon to allow for an expiration date of the protection or for the fact that the underlying asset is perishable, may become obsolete or out of fashion, is economically depreciable, etc. However, we analyze assets for which consumption does not reduce supply. We continue to assume that the value of the underlying asset is governed by a GBM, although one may argue that the distribution of  $F$  is skewed relative to the log-normal distribution. However, we believe that the qualitative conclusion is not altered by the choice of the underlying process. The market for NFTs is assumed to be efficient in the sense that the ex-ante NPV of NFTs is zero.

We relax the assumption of a fixed subpopulation  $s$  of type 2 agents. Utilizing the valuation models in section "[Literature review](#)", the following conclusions can be drawn.

**Proposition 2** *If the use of an underlying asset whose value is  $P_t$  is protected or governed by an IPR, then the value of the related NFT can never exceed  $P_t$ . If the underlying asset is unprotected (i.e., a public good), a sufficient (but not necessary) condition for the value of the NFT to exceed  $P_t$  is when  $\frac{\partial v_W}{\partial P} s' > 1$  for the case where  $v(W)$  increases in  $P_t$  at an increasing rate and  $\frac{\partial v_W}{\partial P} s' - .5 \frac{\partial^2 v_W}{\partial P^2} P_t^2 \sigma^2 > 1$  for the case where  $v(W)$  is increasing in  $P_t$  at a decreasing rate. If these conditions are met, the producer will be better off creating an NFT than relying on an IPR.*

**Proof** (see Appendix C).

### Implications

Empirical observations of the behavior of NFT markets show very volatile prices. One may wonder whether there is a rational pricing mechanism that can serve as a benchmark for pricing NFTs. The models derived in the previous section establish valuation

tools under a strong assumption of market participants' rationality. A common assertion in finance states that even if one can question the rationality of the participants, the NFT markets will eventually settle into some sort of rational equilibrium, as assumed in the models above. In any event, rational pricing can serve as a benchmark for NFT prices.

To derive the actual values of NFTs using the models presented in this study, one has to empirically estimate the parameters underlying the assumed processes. This task, however, has already been performed in the empirical literature cited in the Literature Review section.

Further implications of the results enable us to analyze the likelihood that NFTs would serve as a substitute for IPRs. Given the models presented herein, we can set the financial condition in which NFTs would ever serve as substitutes for IPRs.

### Diversification by a bundle of NFTs

A single NFT possesses idiosyncratic risk, which is reflected in our assumption about parameter  $\sigma_F$ . It makes sense to consider the diversification of this type of risk by holding multiple NFTs. However, several issues are associated with one's ability to diversify away an idiosyncratic risk. First, under a continuous-time framework, we can set constant weights for each asset in a portfolio, that is,  $w_i = q_i F_{ti}$ , by continuously adjusting the quantity of each asset to the changes in value, that is,  $dq = dF_t \frac{\partial q}{\partial F}$ . However, by definition, the quantity of each NFT is  $q=1$ ; therefore, the holder is unable to optimize the desired level of diversifiable risk. More importantly, the notion of a correlation among NFTs may not be clear as the value of an NFT may stem from one's desire to obtain  $v(W)$ , which may be uncorrelated with someone else's  $v(W)$  for a different NFT. This condition implies that the correlation with the underlying asset,  $P$ , can be justified only under model 2 (see case 2 above) and that under models 1 and 3, the set of correlations between NFTs, given the set of correlations between the underlying assets, could have a life of their own. Finally, there may be an issue with the supply of NFTs as NFTs are exclusively held.

Thus, under the assumption that NFTs are uncorrelated, the diversification effect on the risk of a portfolio of NFTs at any point in time  $t$  is given by

$$\overline{d\sigma_F^2} = \left( \frac{dF_{t1}}{\sum_1^n F_{tn}} \right)^2 \sigma_{F1}^2 + \left( \frac{dF_{t2}}{\sum_1^n F_{tn}} \right)^2 \sigma_{F2}^2 + \left( \frac{dF_{t3}}{\sum_1^n F_{tn}} \right)^2 \sigma_{F3}^2 + \dots \quad (34)$$

### Short-term NFTs and privileged information

An NFT economy ultimately allows for novel business models and improved market efficiency through the increased transparency of privileged information possessed by insiders. Before the tokenization era, Prof. Mann (1966), the former director of the SEC, was of the opinion that allowing insiders to take advantage of privileged information may expedite the disclosure of new information to the public. However, in any country worldwide with a centralized security exchange, stringent regulations against the exploitation of privileged information are implemented. The enforcement of these regulations is far from being efficient as it is a costly process and is subject to numerous lengthy court actions. Criminal actions against insiders will not curb their



appetite for gains in lieu of owned privileged information. Allowing insiders to create NFTs may be an efficient mechanism to serve both insiders and the public. NFTs will serve as a signal indicating an undisclosed piece of information and will be short-lived until the actual disclosure is made. Why would anyone buy this type of NFT? They would do so for the same reasons that anyone would buy other types of NFTs: status, power, and fame, or the belief that an individual would seek to buy NFTs for these reasons.

## Conclusion

Irrespective of the type of model that enables the evaluation of NFTs, a value-generating mechanism exists while status, clout, and fame are always sought by human nature and can be translated into pecuniary values. That is to say, market participants are willing to pay hard cash for “higher social ranking.” A status-free assumption regarding financial arenas may miss a critically powerful engine. NFT marketplaces may embed these factors.

This study provides a theoretical framework for pricing NFTs. The main limitation of the various versions of the model is its ability to empirically estimate the underlying parameters, such as the standard deviation. However, as more continuous data become available in the future, this issue will be less critical. Further research, aided by additional data, will be able to determine which of the three versions of the model depicted herein is more applicable.

In addition to the issue of pricing NFTs, this study analyzes whether NFTs can potentially substitute IPRs. Currently, IPRs are an acceptable mechanism that encourages producers to create new products. Therefore, they enhance social welfare by encouraging the creation and distribution of products. The market for NFTs enables producers to be rewarded for their products without the need to rely on the protection provided by IPRs under certain conditions while avoiding costly intermediaries. Obviously, some new legal definitions must be proposed to reduce the involvement of legal systems (see legal citations in Frye 2021).

This study establishes the financial condition for NFTs to potentially be a rational substitute for IPRs. However, this issue must be investigated further by law scholars, who may raise legal issues that are not discussed in this paper. It is our assessment that if NFTs are a more efficient and less costly mechanism to protect the rights of producers, then a legal solution to potential legal issues will be found.

Potential extensions of the above can take several directions. (i) Empirical estimations of the underlying parameters can be conducted to determine whether the findings impose constraints on the conclusion of the model. More importantly, empirical estimations of the parameters may provide a conclusion regarding which version of the model is more relevant. (ii) A behavioral factor (other than the ones mentioned herein) that may allow irrational pricing may be added. (iii) Legal constraints that may change the conclusion regarding the likelihood that NFTs may substitute IPRs may be added.

## Appendix A

This appendix explains the method to solve the PDE and get an explicit solution using a numerical methodology.

Solving the following PDEs,

$$\begin{aligned} \frac{\partial F}{\partial t} + .5 \frac{\partial^2 F}{\partial P^2} P_t^2 \sigma^2 - rF_t + (r - \vartheta) P_t \frac{\partial F}{\partial P} &= 0 \quad \text{if } F_t > v(W_t) \\ \frac{\partial v(W)}{\partial t} + r(v(W) - (\alpha - 2\delta P_t)) + P_t^2 \sigma^2 \delta^3 &= 0 \quad \text{if } F_t = v(W_t) \end{aligned} \quad (35)$$

by using the Backward Finite Difference method (similar to Courtadon 1982). This is done by Dividing a two-dimensional space, price,  $P$ , and time into a two-dimensional discrete coordinates  $i, i = 0, N$  and  $j, j = 1, M$  respectively, where  $u_{ij}$  represents the value at these coordinates.  $h$  and  $k$  denote the differences respectively. Then,

$$\begin{aligned} \frac{\partial F}{\partial t} &\cong (u_{ij+1} - u_{ij})/k \\ \frac{\partial F}{\partial P} &\cong (u_{i+1j} - u_{i-1j} + u_{i+1j+1} - u_{i-1j+1})/4h \\ \frac{\partial^2 F}{\partial P^2} &\cong (u_{i+1j+1} - 2u_{ij+1} + u_{i-1j+1} + u_{i+1j} - 2u_{ij} + u_{i-1j})/2h^2 \end{aligned}$$

Implementing these definitions into the first line in A1, and given the parameters,  $r, \theta, \sigma, a, \delta, k$  and  $h$ , yields  $jk - 1$  partially overlapping equations in a diagonal  $jk$  lines matrix,

$$\begin{bmatrix} U_{j+1} & 0 & 0 \\ 0 & U_j & 0 \\ 0 & 0 & U_{j-1} \end{bmatrix}$$

where, at  $U_j$ , current  $j$  value is a function of the next value  $j - 1$ , (going backward) i.e.,

$$\begin{aligned} &\left(\frac{rki}{4}\right)u_{i-1j+1} + \left(1 + \sigma^2 i^2 k\right)u_{ij+1} - \left(\frac{rki + \sigma^2 i^2 k}{4}\right)u_{i+1j+1} \\ &= \left(1 - \sigma^2 i^2 k\right)u_{ij} - \left(\frac{rki}{4}\right)u_{i-1j} + \left(\frac{rki + \sigma^2 i^2 k}{4}\right)u_{i+1j} \end{aligned} \quad (36)$$

Then we create similar two-dimensional space,  $v(W)$ , and time into a two-dimensional coordinates  $i$  and  $j$  respectively, where  $v_{ij}$  represents the value at these coordinates.  $h$  and  $k$  denote the differences respectively

$$\frac{\partial v(W)}{\partial t} \cong (v_{ij+1} - v_{ij})/k \quad (37)$$

Implementing (37) into the second line in (35) yields,

$$v_{ij+1} = (1 + kr)v_{ij} - (a - 2k\delta u_{ij}) + k\sigma^2 \delta^3 (u_{ij}^2) \quad (38)$$

We form  $jk-1$  equations that can be solved for the value of an NFT at time  $t$  ( $j=M$ ) and an arbitrary  $P$  ( $=i'h$ ) by Gauss elimination, subject to the following conditions:

$u_{i0} = v_{i0} = 0$ , and  $u_{0j} = v_{0j} = 0$ , and the highest  $i$  coordinates,  $N, u_{Nj} - u_{n-1j} = h$ ,

and most importantly, at any  $i$  and  $j$ ,  $u_{ij+1}/v_{ij+1} = \max[u_{ij}, v_{ij}]$  as  $v_{ij+1}$  is known by (38).

## Appendix B

This appendix describes the explicit solution to the case where the two variables are binomially distributed.

Given a fixed date  $T$ , and a threshold,  $TH$ , the PDE ,

$$\begin{aligned} & \frac{\partial F^*}{\partial t} - rF^* - (r - \theta)F \frac{\partial F^*}{\partial F} + (r - \theta)v(W) \frac{\partial F}{\partial v(W)} \\ & - .5 \left( F^2 \sigma_F^2 \frac{\partial^2 F^*}{\partial F^2} + v(W)^2 \sigma_W^2 \frac{\partial^2 F}{\partial v(W)^2} - 2v(W)F \rho_{WF} \sigma_F \sigma_W \frac{\partial^2 F}{\partial F \partial v(W)} \right) = 0 \end{aligned} \quad (39)$$

would have the following explicit solution,

$$F^*(T) = C_1(F_t, TH_t, T) + C_2(v(W_t), TH_t, T) - Mn(F_t, v(W_t), TH_t, T) \quad (40)$$

where  $C_1$  and  $C_2$  are the standard European Black–Scholes options, and

$$\begin{aligned} Mn(.) = v(W_t)N_2 & \left\{ \frac{\ln\left(\frac{v(W)}{TH}\right) + rT + .5\sigma_W^2 T}{\sigma_W \sqrt{T}}, \frac{\ln\left(\frac{F}{v(W)}\right) - .5(\sigma_W^2 + \sigma_F^2 - 2\rho_{WF}\sigma_W\sigma_F)\sqrt{T}}{\sqrt{T(\sigma_W^2 + \sigma_F^2 - 2\rho_{WF}\sigma_W\sigma_F)}}, \frac{(\rho_{WF}\sigma_F - \sigma_W)}{\sqrt{\sigma_W^2 + \sigma_F^2 - 2\rho_{WF}\sigma_W\sigma_F}} \right\} \\ & + F_t N_2 \left\{ \frac{\ln\left(\frac{F}{TH}\right) + rT + .5\sigma_F^2 T}{\sigma_F \sqrt{T}}, \frac{\ln\left(\frac{v(W)}{F}\right) - .5(\sigma_W^2 + \sigma_F^2 - 2\rho_{WF}\sigma_W\sigma_F)\sqrt{T}}{\sqrt{T(\sigma_W^2 + \sigma_F^2 - 2\rho_{WF}\sigma_W\sigma_F)}}, \frac{(\rho_{WF}\sigma_W - \sigma_F)}{\sqrt{\sigma_W^2 + \sigma_F^2 - 2\rho_{WF}\sigma_W\sigma_F}} \right\} \\ & - TH_t e^{-rT} N_2 \left\{ \frac{\ln\left(\frac{v(W)}{TH}\right) + rT + .5\sigma_W^2 T}{\sigma_W \sqrt{T}}, \frac{\ln\left(\frac{F}{TH}\right) + rT + .5\sigma_F^2 T}{\sigma_F \sqrt{T}}, \rho_{WF} \right\} \end{aligned} \quad (41)$$

$N_2$  is the bivariate CSND, and  $\sigma_F$  is given in (12) and similarly  $\sigma_W$ .  $\rho_{WF}$  is the correlation coefficient. Inserting the parameters and the initial condition for  $v(W)$  into (41) requires further assumptions about the empirical estimations of these parameters.

## Appendix C

**Proof of proposition 2** Relaxing the assumption of  $s$  being a fixed subgroup of agents type 2, let  $s'$  be the normalized size of this subgroup, i.e.,  $s' = s/s_p$  where  $s_p$  is the size of the subgroup when the underlying asset is protected or governed by IPR. By definition, if the underlying asset becomes a public good the exposure and therefore the consumption of the underlying asset is greater and  $s'$  is equal or greater than unity. In more general terms, let  $s'$  represents a deterministic cardinal ranking  $v(W)$ . Let  $Ed(P_t)$  represents the expected change in the producer's payoff if protected.

Then, from the producer's point of view (agent 1), the difference in the payoff, if protected, or creates an NFT is,

$$\begin{aligned}
E'd(P_t) - \text{Max}[F_t, \text{Edv}(W_t)] &= P\mu^* \left( 1 - s' \left[ \frac{\partial v_w}{\partial P} - .5 \frac{\partial^2 v_w}{\partial P^2} P_t^2 \sigma^2 \right] \right) dt \\
&\quad + P_t \sigma E \left\{ \left( 1 - \frac{\partial v_w}{\partial P} s' \right) \right\} dZ \\
&= P\mu^* \left( 1 - s' \left[ \frac{\partial v_w}{\partial P} - .5 \frac{\partial^2 v_w}{\partial P^2} P_t^2 \sigma^2 \right] \right)
\end{aligned} \tag{42}$$

Under our assumption (3), and if  $\frac{\partial v_w}{\partial P} s' > 1$  and  $\frac{\partial^2 v_w}{\partial P^2} > 0$  (42) is always negative, and if  $\frac{\partial v_w}{\partial P} s' > 1$  but  $\frac{\partial^2 v_w}{\partial P^2} < 0$  then, if  $\frac{\partial v_w}{\partial P} s' - .5 \frac{\partial^2 v_w}{\partial P^2} P_t^2 \sigma^2 > 1$  (42) is negative. When (42), which represents the producer's payoff is negative, i.e. when  $\frac{\partial v_w}{\partial P} s' > 1$  or  $\frac{\partial v_w}{\partial P} s' - .5 \frac{\partial^2 v_w}{\partial P^2} P_t^2 \sigma^2 > 1$ , the producer is better off creating an NFT. QED

### Abbreviations

NFT	Non-fungible token
IPR	Intellectual property rights
GBM	Geometric Brownian motion
LHS	Left hand side
RHS	Right hand side

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### Declarations

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