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Contingent convertible lease modeling and credit risk management

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Abstract

The main objective of this study is to determine a lease agreement to finance an investment project and a solution for managing credit risk. This study investigates three types of contingent leases to reduce the costs associated with bankruptcy and compensate for the lessor's position. A leasing defaultable contract allows the lessor to obtain the rent that will be recovered if the lessee defaults. A leasing convertible contract can be automatically converted into shares when certain default conditions related to the cash flows generated by the firm are met. These conditions are triggered by the ratio of the firm's value and leasing payments. A Defaultable-Convertible-Leasing contract with a payback option grants the lessor the right but not the obligation to convert the remaining lease payments into stocks or to break up the contract and pick up the rented equipment when the firm reaches the default threshold. These contracts are motivated by contributing to the range of risk-management strategies by adding more flexibility to standard leasing contracts and contingent rents. Closed-form securities pricing solutions are set forward in a dynamic model for firms with existing assets and a growth option financed by shares and a contingent lease. Risk-neutral pricing theory and the backward induction method are used to determine the pricing of corporate securities. Numerical analysis shows that leasing convertible contracts and defaultable-convertible contracts with payback options impact the service value of the leased asset, maturity, and inefficiencies resulting from insolvency and asset substitution. An optimal conversion rate reduces inefficiencies, thus making the leasing convertible contract and defaultable-convertible-leasing contract with payback option a reliable solution to ensure business continuity and loss coverage of the leasers upon default.

Keywords: Contingent convertible lease, Growth option, Risk of default, Asset pricing, Stochastic process

JEL Classification: G12, G31, G32, G33

Introduction

Leasing plays an important role in the global economy. It is designed as a financial contract between two parts by which the owner of an asset, the lessor, sells the use of the asset to the lessee for a specified period without necessitating a transfer of ownership. It provides a separation of ownership from the user, with the lessee receiving the benefits of the use and the lessor receiving the lease payment flow, plus the residual value of the asset.

Pricing and credit risk management is one of the most popular research topics since the world economy has experienced deep crises affecting almost all industries. Nevertheless, financial engineering has not ceased proposing innovative alternative solutions to mitigate the adverse impacts of these economic crises. In this study, we deal with the problem of the valuation of leasing contracts by providing this type of contract, which is used mainly by small- and medium-sized companies, with more flexibility through the possibility of conversion in case of default.

Although the lease and rental agreements are similar, they are not the same. The lease agreement is specific in terms of detailing the responsibilities of both parties during the lease. It includes all necessary information to ensure that both parties are protected. In a classical lease agreement, the length of the lease and the amount of periodic rent are documented and cannot be changed. This ensures that the lessor cannot arbitrarily raise the rent, and the tenant cannot just leave the lease whenever they want without repercussions. The lease agreement is relevant for the specific time stated in the agreement and is then considered invalid. A rental agreement differs from a lease agreement in that it is not a long-term contract allowing the lessor to renegotiate the terms of the agreement when rental amounts can rise quickly. Both parties must enter a new lease agreement if tenants wish to keep the asset.

The issuance of contingent capital is initially reserved for banks and, to some extent, insurance companies in times of crisis. Nevertheless, there have been attempts to propose contingent capital and rentals as risk management solutions for companies. The Michelin Committed Long-Term Capital Solutions (CLOCS) Agreement may be an example of how companies can use contingent capital as a source of financial inefficiencies, as Culp (2002) reported. Song (2014) studied the effect of capitalizing operating leases and lease-contingent payments on accounting and financial indicators using a sample of US companies from 2001 to 2010. He finds that the capitalization of lease-contingent payments entails significant changes in the magnitude and relative rankings of total liabilities, debt ratios, current ratios, and ROA.

Contingent rentals are commonly based on price changes or inflation indices on the lessee's financial or operating performance and the lessee's use of the leased asset. Corporate risk management using contingent lease agreements can be demonstrated through contingent rentals based on the lessee's operating performance derived from the leased asset. Although lease agreements based on contingent rent are mainly used in estate leasing, this study extends this type of agreement to equipment leasing.

In this study, three models of asset lease agreements were developed to manage the credit risk of corporate finance lessees using a contingent claims approach. In particular, the defaultable-convertible-Leasing contract with a payback option (DCLCP) is an option-based valuation model. In these three models, it is assumed that firms have existing assets financed by equity and a growth option to increase existing assets, such as after investment expansion, earnings before interest, and taxes (EBIT) increase by a constant factor. Investment costs are financed by the issuance of equity and contingent leases. We developed pricing models for the leasing defaultable contract (LDC), leasing convertible contract (LCC), and DCLCP triggered by an event of default aimed at preserving the continuity of the business and preventing the lessee's default in times of financial difficulties. In the event of default, the LDC model allows the lessor to receive a fraction of the remaining lease value due to the default costs. The LCC model provides two possibilities. The first occurs when the tenant does not default until the maturity of the lease. In this case, the asset receives a periodic lease payment. However, the second occurs when the lessee cannot continue to pay the outstanding amount. In this case, the lessor agrees to convert this amount into shares and thus becomes a shareholder while alleviating the burden on the firm's liquidity.

The DCLCP, with the payback option model, grants the owner the choice to convert the remaining lease payments into equity or sell the leased asset on the market upon default to cover their position. There are also two possibilities to consider: the first is when there is no default, the lessee receives the benefits of using the asset, and the lessor receives the lease payment flow plus the residual value of the asset at the maturity of the lease. In this case, leasing is simply a mechanism for selling an asset for a predetermined period. Second, when the tenant defaults, the lessor has the right to choose the most relevant scenario that minimizes bankruptcy costs, either to trigger the conversion event or to obtain a set number of shares if the market value of the asset is less than the conversion value or to sell the leased asset in the market if the market value is higher. The difference between the LCC and DCLCP is the timing of the conversion and the existence of the payback option. The timing of the conversion in the LCC is determined when the lessee's stochastic cash flow falls to a lower conversion threshold; therefore, conversion occurs automatically and compulsorily. However, for the DCLCP, the conversion event is optional. This occurs when the lessor's financial situation falls to a certain level, and the benefits obtained from the conversion are greater than the market price of the leased asset.

This paper proposes to study contingent lease as a risk management instrument that extends and complements contingent rent agreements based on the operating activity of the firm or the use of the leased asset. The relationship between leasing and contingent capital may be explored from the risk management perspective between lessors and lessees. It may contribute to recent innovations in contingent capital and lease rentals, along with a discussion of their role in integrating corporate finance and risk management.

A numerical analysis was performed for the three types of contracts to examine and compare the sensitivities of different parameters. The results indicate that if the conversion rate is high enough, insolvency and asset substitution inefficiencies can be reduced, regardless of the financing policy, namely LCC or DCLCP equity. However, when the conversion rate is zero, the risk of inefficiencies increases. Moreover, it shows that the value of the company after the investment expansion first increases and then decreases with asset volatility. Since the conversion in the LCC occurs automatically at the moment of default, the numerical analysis reveals that the risk-shifting incentive under LCC equity financing is less than that under DCLCP equity for different conversion rates. For a conversion rate close to one, the risk-shifting incentive under the financing of DCLCP-equity if the lessor chooses to exercise the convertible lease option is less than abandoning it and selling the asset in the market.

The remainder of the paper is structured as follows. Second section presents the "literature review". Thrid section presents the "pricing methodology and contingent lease modelling". Fourth section examines the "numerical results and sensitivity analysis". Fifth section wraps up the closing part and exhibits some "concluding remarks".

Literature review

The contingent capital lease develops simple capitalization leasing contracts proposed by the Financial Accounting Standard Board (FASB) and the International Accounting Standard Board (IASB).

The lease clauses presented in this paper are intended to reduce credit risk. The LDC rental valuation model has been widely and extensively investigated in numerous research works. It is one of the most common rental structures in the world. The LDC elaborated in this paper is very similar to the valuation models of leases, which consider credit risk by entitling the lessor to receive a fraction of the rental value remaining under financial stress circumstances. Grenadier (1996) and Ambrose et al. (2018) addressed this type of contract. Moreover, a study on ex-post lease performance performed by Lease et al. (1990) found significant credit risk in typical leases. The leases in their sample underwent a failure rate of approximately 20% and a recovery rate of 38% over the asset's original cost or 64% of the present value of the remaining lease payments plus the estimated salvage value. Their analysis revealed that lease valuations should incorporate tenant credit risk because corporate bond values reflect default risk. These lease valuation models rely on the exogenous default thresholds. However, the default threshold of the LDC contract is determined endogenously, making the model analysis more descriptive and representative of the real world.

Many leases involve additional clauses to protect the lessor against tenant defaults. Most rental contracts in Western countries require a security deposit in an escrow account to cover the cost of a tenant default. In the Korean real estate lease market, rental payments can be converted into security deposits and vice versa, subject to a market conversion rate from zero to infinity (refer back to Park and Pyun (2020)). For example, a conversion rate of 10% implies that the tenant grants the landlord a refundable deposit of \$100 instead of \$10 in the annual rent. The LCC and DCLCP models are flexible enough to extend to various realistic lease structures. The LCC structure includes a practical and simple conversion mechanism that can be an alternative to absorb the losses of the lessee in the event of financial distress and, at the same time, cover the position of the lessor. This type of contract is similar to contingent convertible bonds. The leasing contract is a financing and investment alternative and substitution of debt, so the same reasoning can be applied to contingent rental contracts. In addition, the structure of this type of contract is similar to that of risk lease contracts. It aims to compensate for the loss of the remaining non-refundable rental payments by converting them into a predetermined number of shares. In addition, it serves to redress the financial situation of a company by quickly absorbing losses related to bankruptcy.

Most leases include clauses that allow the tenant (and sometimes the landlord) to change their maturity during the lease term. A cancellation option (also known as a "kick-out" clause) allows the tenant to end the lease before it expires. The DCLCP model extends the analysis of leases with cancellation options in Grenadier (2005) and McConnell and Schallheim (1983) by integrating the valuation of a call option into an equilibrium framework and using the American option methodology. McConnell and

Schallheim (1983) conducted a detailed call options analysis. The DCLCP model handles the valuation of asset call options under default risk conditions. The structure of the DCLCP model provides closed solutions for the equilibrium valuation of leases with purchasing options. Additionally, it incorporates a convertible lease option from which the contract can take an end before its maturity, that is, in a credit-risk environment.

Theoretical research in creating contingent claim leases has largely been conducted. McConnell and Schallheim (1983) created a contingent claim model to evaluate asset leases and illustrated its pertinence to a diversity of operating leases. The valuation of the contingent claim lease was analyzed according to a numerical framework based on interacting operating options. McConnell and Schallheim (1983) and Schallheim and McConnell (1985) developed a thorough analysis of various lease options and insurance contracts, following an option-pricing framework. Chiang et al. (1986) developed a contingent claims model to explore certain retail lease features. They used the contingent claim approach, which considers explicit risk without ad hoc risk adjustment. They assert that this approach has a significant advantage for such models through traditional discounted cash flow methods. Risk adjustments are made subjectively based on the discount rate. In addition, Chiang et al. (1986) asserted that the commercial lease percentage valuation model is elaborated in a manner that is comparable to Ingersoll's (1977) convertible bond valuation model, such that the excess rent is estimated by applying the Black and Scholes (1973) call option pricing methodology.

Leasing was previously regarded as a substitute for debt, especially for companies that are "too risky or unable to enter conventional debt markets, " as argued by Lease et al. (1990). The academic literature has devoted considerable attention to analyzing asset leases and determination of the rental rate. Theoretical modeling of the equilibrium determination of lease rates relies mainly on default-free leases. However, the risk of default is a key factor in the valuation of financing leases. Lewis and Schallheim (1992) developed an equilibrium leasing model by considering the risk of default. Krishnan and Moyer (1994) recorded that leasing reduces bankruptcy costs more than borrowing, which makes it an attractive financing option as the bankruptcy potential in a firm increases. Their model was developed in the context of minimizing the costs incurred by tenant default. A contingent leasing agreement is suggested as a means of financing an investment project and, in the meantime, as a solution to manage the risk of default by minimizing the costs associated with bankruptcy and compensating for the lessor's position. Sharpe and Nguyen (1995) assert that leasing may economize transaction costs for firms facing a high cost of external funds. Their results suggest that a low-rated firm should use more lease financing than a highly rated firm after controlling for firm size and other factors.

Similarly, Grenadier (1995) combined an option-pricing approach with a competitive industry equilibrium to provide a unified framework for valuing leases. Similarly, Grenadier (1996) reported that the default risk model is important for determining the equilibrium rental rates and lease clauses. It provides a unified model for the equilibrium determination of lease rates and credit spread on leases when payments are subject to default risk. Furthermore, it applies the results to several real-world leasing arrangements such as security deposits, embedded lease options, indexed leases, and lease credit insurance. Trigeorgis (1996) described a contingent claims analysis of operating lease options and suggested a numerical analysis method for valuing leases with some options. The author asserted that contingent claims are an adequate methodology for evaluating operating lease options and discussed a binomial numerical analysis of complex lease contracts. Stanton and Wallace (2009) studied a flexible model for valuing contingent claim leases and estimated it using a contract dataset comprising 711 leases from three types of properties in 11 different states. They analyzed the behavior of the net present value of various leasing contracts and proposed a new measure to compare different leases with various maturities. The same authors argue that there are remarkable pricing errors that cannot be explained using interest rates, lease expiration, or information about options embedded in contracts.

Several models have been developed to meet the requirements of the Financial Accounting Standards Board (FASB). The impact of capitalizing operational leases and lease contingent payments on reported accounting numbers and financial indicators were examined empirically by Song (2014), who found that capitalizing lease contingent payments would result in a substantial rise in average total liabilities, total assets, debt ratio, and current ratio. Kusano (2018) also considered the impact of constructively capitalizing operating leases on Japanese credit ratings. He used credit ratings as a proxy for debt cost to evaluate the risk relevance of operational leases and empirically studied the impact of accounting information reliability on the risk relevance of operating leases. Kusano's (2018) research reveals that credit ratings are linked to constructively capitalizing operational leases and claims that the accuracy of accounting data has a major impact on the risk relevance of operating leases.

The motivation for default risk in contingent convertible lease contracts is manifested by the exposure to the risk of non-refundable lease payments due to insufficient cash flow generated by using leased equipment in corporate finance companies. In a situation where the lessee firm is in default payment, the lessor traditionally recuperates the leased equipment, leases it to another company, or sells it on the market, leaving the firm to its own devices with consequences that can lead to business interruption. The idea behind convertible leasing is to provide more flexibility and collaboration between the lessee and the lessor by giving the latter the possibility to convert the non-refundable lease payments into shares after the event of default if they wish. Therefore, the lessor gives the company a chance to redress its financial situation, thus avoiding the risk of bankruptcy. The conversion mechanism remains a simple and practical technique that may absorb losses caused by the tenant's default and provides an alternative to the lessor of the leased asset to manage their risk exposition.

Recently, tenants' default risk evaluation has been investigated in several studies. Ambrose et al. (2018) addressed the problem of capital structure for firms issuing debt and leasing, considering counterparty risk. They introduced lessor and tenant default risk into the term structure of rental rates and showed that decisions related to the capital structure have an endogenous effect on the company. Moreover, Kouzmina and Okrepilov (2019) discussed risk management issues in leasing companies and projects. They adopted methods of risk management for leasing operations based on risk management processes, particularly in project management. They analyzed different risks associated with lessors of assets and addressed methodological approaches to leasing companies' risk management. The same authors developed an econometric model to predict the probability of the default of rental payments.

Park and Pyun (2020) built a rent-deposit option-based equilibrium model between the lessor of the asset and the tenant based on a stochastic process of the asset's use-value. Their model predicts that the deposit/rent equilibrium ratio increases with the owner's return on investment in real estate, whereas it decreases with the tenant's cost of capital. The study performed by Park and Pyun (2020) handles the rent-deposit equilibrium in Korean leases along the spectrum from rent-only to deposit-only contracts, considering the owner's rate of return on the property, real estate investment, the tenant's cost of capital, and their mutual default risks. They asserted that there are no deposit leases if tenants do not raise enough capital despite the high risk of tenant default. Their models predict that a tenant's cost of capital outweighs the risk of tenant default in Korean leases. Because a higher appreciation rate allows tenants to achieve a higher conversion rate through the deposit-to-rent conversion process, tenants also benefit from reduced total rental costs. Similarly, Qu et al. (2020) developed a debt cession financial leasing value model with an optimal economic pattern and risk assessment analysis to ameliorate both parties' pricing processes. They suggest that their model can help lessees extend their production scales, address the difficulty of funding large investment scales for firms, and recover the money involved in leased assets. Barykina and Chernykh (2021) discussed the risks of switching from financial to operating leasing, considering the interests of counterparties in leasing transactions and eliminating leasing risks.

Many bankruptcy prediction models have been proposed, especially for small and medium-sized enterprises (SMEs). Kou et al. (2021) proposed a two-stage multi-objective feature selection method that optimizes the number of features as well as the classification performance of the model. They found an optimal feature subset that optimized the model performance and feature subset size through multi-objective wrapper-based feature selection. They revealed that the feature importance assessment for the optimal subset features confirmed the importance of transactional data and variables determined in the payment network for predicting bankruptcy. Their models showed that the variables reflected in payments and transactional data determine the prediction of SME bankruptcy.

The speed of leasing modes in the peer-to-peer (P2P) platforms enables small and medium-sized enterprises (SMEs) to solve financing problems. Qu et al. (2020) established multi-period, continuous, and variable models of leased asset value assessment, taking rent, lease terms, and interest as independent variables to examine the transaction price of leased assets in a P2P platform. They proved that the price of leased assets is related to interest, rent per period, and the number of payments and rent changes when other factors are constant. Furthermore, they revealed that the business model of financial leasing in P2P platforms provides better macro-business direction and business micro-management guidance for the leasing industry, P2P platforms, and financial leasing companies. Wang et al. (2021) modeled credit scoring in P2P loans as a cost-sensitive multiclass classification problem. They propose a misclassification cost matrix for P2P credit scoring that considers the opportunity costs and actual losses associated with P2P loans. Equations and models are developed to calculate the cost of the misclassification cost matrix. Similarly, they analyzed the parameters in the proposed equations and models from the perspective of cost-sensitive classification and business operations. To validate the usefulness of the proposed misclassification cost matrix, Wang et al. (2021) used publicly available data from the Lending Club and revealed that cost-sensitive classifiers could reduce the total cost, which is essential for the survival and profitability of P2P platforms.

In China, the financialization process is facilitated by an under-regulated financial market (especially P2P financial organizations) and policy support for rental housing provision. In this context, Chen et al. (2022) proposed a timely exploration of the ongoing financialization process in China's rental housing industry based on the financial means used by low-capital "long-term apartment rental" (LAR) firms in this industry. Chen et al. (2022) examined tenants' financial risks and vulnerability embedded in the new financing strategies used by asset-light LAR companies using Shanghai as a case study. Their results add to the knowledge of the heterogeneity of financialization and the different impacts of financialization on housing rights and urban governance.

Devos et al. (2022) examined operating leases as an alternative source of financing for REITs (Real Estate Investment Trust) through lease information from 334 unique REITs from 1993 to 2018. They examined the determinants of the operating lease decision and found that REITs that have adopted operating leases tend to be larger and have more growth opportunities, as measured by Tobin's Q. However, they also have higher leverage, reporting lower funds from operations and higher risk. Overall, their models show that simple location contracts are used as an alternative source of financing by highly leveraged REITs and cannot rely much on their internal financing. Kou et al. (2022) generated an inventive problem-solving map of innovative carbon reduction strategies for transportation investment projects. They illustrated the causal relationships between innovative strategies for solar energy projects. The main contribution of Kou et al.'s (2022) study is the presentation of outstanding strategies to increase the efficiency of solar energy investment projects with a new hybrid decision-making methodology. Their results make a positive contribution in an attempt to solve the carbon emissions problem in the transportation industry.

Pricing methodology and contingent lease modeling

Methodology for analyzing leasing models

We consider three types of contingent leases that provide flexibility in dealing with default risk. The pricing problem of these leasing contracts is handled under a dynamic model for a firm with outstanding assets and growth options. We assume that the growth option's investment costs are financed by equity and a convertible lease. In each version of the model, we determine the lease contracts as claims contingent on the firm's cash flow. The evaluation models for the different types of contingent leasing contracts refer to Brennan and Kraus (1982), McConnell and Schallheim (1983), and Stanton and Wallace (2009). The underlying asset value $A(s)$ is determined by the present value of its future service flows,$s(t)$, which is provided by the diffusion process:

$$ds_t = \alpha_s s_t dt + \sigma_s s_t dz_s \tag{1}$$

where α_s is the service flow yield of the leased asset, σ_s is the volatility of the service flow, and z_s is the standard Wiener process.

According to Grenadier (1996), the expected value of the use of the asset at the maturity of contract$T$,$Y(s_0, T)$, is expressed as follows:

$$\begin{aligned} Y(s_0, T) &= E\left(\int_0^T e^{-rt} s_t dt\right) = E\left(\int_0^T e^{-rt} s_0 e^{\left(\alpha_s - \frac{1}{2}\sigma_s^2\right)t + \sigma_s z_s(t)} dt\right) \\ &= \frac{s_0}{r - \alpha_s}\left[1 - e^{-(r-\alpha_s)T}\right] \end{aligned} \tag{2}$$

where $r > 0$ represents the risk-free interest rate and $Y(s_0, T)$ are the explicit solutions of Eq. (1). (Please see the detailed derivation in the Appendix). Note that the service flow of the asset can appreciate or depreciate over time because of the sign of the expected asset service flow yield rate α_s is n

ot restrictive.

The rental time value V_T can be expressed by the following formula:

$$V_T = \int_0^T \exp(-rt) R_T dt = \frac{R_T(1 - \exp(-rT))}{r} \tag{3}$$

where R_T is the rental payment. At equilibrium, the rental value at the maturity of contractV_T, equals the value of the use of the asset at the maturity of contract $Y(s_0, T)$ which also corresponds to the asset lease value:

$$V_T = Y(s_0, T) \tag{4}$$

Using Eqs. (2) and (4), the rental payment R_T satisfies the following equilibrium equation:

$$R_T = \frac{r}{1 - e^{-rT}} Y(s_0, T) = \frac{r}{1 - e^{-rT}} \frac{s_0}{r - \alpha_s}\left[1 - e^{-(r-\alpha_s)T}\right] \tag{5}$$

Because the asset's value equals the asset's service flow value at equilibrium, the value of the underlying asset $A(s)$ is the perpetual value of $s(t)$, which is expressed as $A(s) = \lim_{T \to \infty} Y(s_0, T)$. The flow x is supposed to be generated by the company's ordinary activity, where there is a growth option θ and a conversion ratio β triggered by the default threshold. The firm should have assets in place and a growth option. At any time$t$, the assets in place generate profit before interest and taxes that correspond to the cash flows of the firm denoted by x and described by the following geometric Brownian motion:

$$dx_t = \alpha_x x_t dt + \sigma_x x_t dz_x \tag{6}$$

where x_t is the tenant state variable representing the cash flow of the firm, α_x is the expected risk-adjusted rate of return, σ_x is the rate of volatility, and z_x is a standard Brownian motion that is defined on a complete probability space (Ω, G, P) composed of a filtration $F \equiv \{F : t \geq 0\}$ describing the information flows available to an investor by satisfying the usual conditions.

Referring to Hackbarth and Mauer (2011) and Sundaresan et al. (2014), we infer that the firm has the right to exercise a growth option at any time to increase the scale of its operations by paying a fixed sunk cost noted, I. The growth option exercise implies an immediate increase in a firm's cash flow from x to $(1 + \theta)x$, where $\theta > 0$ is a constant representing the growth rate.

In this study, a firm's capital structure is composed of equity and contingent leases. According to Grenadier (1996), we assume that when a default event occurs, the asset owner rents the equipment for the remainder of the maturity to another financially more healthy company. Thus, for a leasing defaultable contract, owing to default, the lessor receives a portion of the remaining lease's value $(1-w)Y(s_0, T_D)$, where $0 \leq w \leq 1$ is the expected value of the loss due to default. However, for the LCC, the conversion takes place, and the shareholders must distribute to the owner of the leased asset a fractionβ, where $0 \leq \beta \leq 1$, of the residual cash flows under dividend form instead of paying R_T rental payments. The DCLCP allows the lessor to choose the exercise of the convertible lease option and to receive a fraction of shares or sell the leased asset on the market to minimize the cost of default. Fraction β represents the number of shares that can be held by LCC or DCLCP holders when conversion occurs. Thus, the conversion threshold must be determined exogenously. The default threshold of the LDC is determined endogenously by maximizing the value of shareholder equity. Owing to the homogeneity of time in our model, the conversion threshold of the convertible leasing and the default threshold of the LDC must be independent of time. To address the pricing of corporate securities, we apply a risk-neutral pricing approach and a backward induction method. The solution approach of the DCLCP is based on option-pricing analysis. In fact, the DCLCP contains two options: the payback option, which can be exercised at the maturity of the lease in case of default, and a convertible lease option that contains certain clauses that allow the lessor to determine the lease contract before its expiration. This option is known as a cancellation option in Grenadier (2005) or a Kick-out clause. In the numerical analysis, we referred to the parameters of Tan and Yang (2016) to determine the relevant variables of the proposed leases and examine the impact of LCC and DCLCP on the service value of the asset lease, maturity, and inefficiencies resulting from the problem of insolvency and asset substitution.

Leasing pricing and firm value

This section proposes a framework for the LDC and extends the work to an LCC and DCLCP. Each time we provide closed-form solutions for leasing and firm value, we examine the pricing of corporate securities using the backward induction method and risk-neutral pricing theory.

Leasing defaultable contract pricing (LDC)

We consider a contingent claim as a derivative instrument underlying the cash flow of a firm and $G(x)$ as its price, where x is a function of the current cash flow level. Generally, any claim should satisfy the partial differential equation (PDE), as mentioned by Goldstein et al. (2001). Indeed, if the lease has finite maturity, its value should be a function of the firm value and time. Hence, $G(x)$ must satisfy the partial differential equation (PDE), which includes a time derivative:

$$\frac{1}{2}\sigma_x^2 x^2 \frac{\partial^2 G(x)}{\partial x^2} + \alpha_x x \frac{\partial G(x)}{\partial x} + \frac{\partial G(x)}{\partial t} - rG(x) = 0 \tag{7}$$

In this study, we apply a perpetual growth approach that assumes a firm generates cash flows at a constant rate for perpetuity, as stated in Eq. (10). Thus, the firm continues to be financed by leases continuously over time, as if the lease has infinite maturity, so the leased asset generates perpetual cash flows until the firm goes bankrupt. From this perspective, the model becomes homogeneous over time. In other words, leasing contracts have indefinite maturity, and the firm is assumed to finance its growth options over its lifetime with similar replicated leases. This paper uses a time-homogeneous stochastic diffusion process that satisfies the Markov property in the modeling of contingent convertible leasing contracts. This implies that the contingent leasing contracts considered in this study are time-independent. The PDE then becomes an ordinary differential equation, which can be expressed as follows:

$$\frac{1}{2}\sigma_x^2 x^2 \frac{\partial^2 G(x)}{\partial x^2} + \alpha_x x \frac{\partial G(x)}{\partial x} - rG(x) + \xi = 0 \tag{8}$$

where ξ is a linear function of x generated by a contingent claim, expressed as $\xi = ax + b$, where a and b are constants. Note that ξ is always a linear function in the model to x for a stopping maturity $T_D = inf\{t \geq 0 : x_t \notin D\}$. The boundary D corresponds to the domain of cash flow limits. It determines the boundaries that must be satisfied by the contingent claim value. Using a standard approach, the general solution of Eq. (8) can be expressed as follows:

$$G(x) = \frac{ax}{r - \alpha_x} + \frac{b}{r} + B_1 x^{\gamma-} + B_2 x^{\gamma+} \tag{9}$$

where B_1 and B_2 are constants to be specified by the limit conditions determined again by the payoff of the claim, and $\gamma^{\pm} = \frac{-(\alpha_x - \frac{1}{2}\sigma_x^2) \pm \sqrt{(\alpha_x - \frac{1}{2}\sigma_x^2) + 2\sigma_x^2 r}}{\sigma_x^2}$ with γ^+ and γ^- represent the positive and negative solutions of the quadratic Equation $\frac{1}{2}\sigma_x^2\gamma^2 + (\alpha_x - \frac{1}{2}\sigma_x^2)\gamma - r = 0$.

The value of a fully financed firm with equity starting with the only investment project that generates $x(t)$ is denoted by:

$$V(x) = E\left[\int_t^{\infty} exp(-r(u-t))(1-\tau)x_u(1+\theta)du | x_t = x\right] = \frac{(1-\tau)(1+\theta)x}{r - \alpha_x} \tag{10}$$

where τ is the firm's effective tax rate. It is noteworthy that r must satisfy the condition where $r > \alpha_x$.

Leasing is used to finance the total assets needed for the entire project with a perpetual leasing payment, R_T. To determine the price of the LDC, we first denote the limit of the domain $D = (x_d, +\infty)$, $a = 0$ and $b = R_T$ with R_T representing the cash flow received by the lessor of the asset, and the default threshold is denoted by x_d. Second, we set two possible scenarios for the lessee to specify the limit conditions: if $x < x_d$, the firm is declared in default, and the lessor receives a portion of the remaining lease value $(1-w)Y(s_0, T_D)$. Hence, the first boundary condition of the leasing value is $L_d(x_d) = (1-w)Y(s_0, T_D)$. If $x > x_d$, then there is no default, and the lessor receives $\frac{R_T}{r}$. Therefore, the second boundary condition for the leasing value is $L_d(+\infty) = \frac{R_T}{r}$. Using the limit conditions set above, the value of a leasing defaultable contract is expressed as follows:

$$L_d = \frac{R_T}{r}\left[1 - \left(\frac{x}{x_d}\right)^{\gamma -}\right] + (1-w)Y(s_0, T_D)\left(\frac{x}{x_d}\right)^{\gamma -} \tag{11}$$

The first term of Eq. (11) represents the value of the leasing contract without default multiplied by one minus the stochastic discount factor in the case of default. The second term represents the amount of rent that will be recovered if a lessee defaults. Note that the value of a contingent lease that pays 1$ when cash flow x reaches the default level x_d for the first time is $\left(\frac{x}{x_d}\right)^{\gamma -}$.

Equity value pricing

If we assume that the capital structure of the company is composed of an LDC and equity, then the cash flow of equity is $(1-\tau)((1+\theta)x - R_T)$. For $x > x_d$, the equity value is determined using the following formula:

$$E_d(x) = (1-\tau)\left[\left(\frac{(1+\theta)x}{r-\alpha_x} - \frac{R_T}{r}\right) - \left(\frac{(1+\theta)x_d}{r-\alpha_x} - \frac{R_T}{r}\right)\left(\frac{x}{x_d}\right)^{\gamma -}\right] \tag{12}$$

As shareholders determine the default endogenously to maximize the market value of stocks, the equity value in Eq. (12) must satisfy the smooth-pasting condition at the default threshold:

$$\frac{\partial E_d(x)}{\partial x}|_{x=x_d} = 0 \tag{13}$$

Therefore, the optimal default threshold is equal to

$$x_d = \frac{\gamma^-}{\gamma^- - 1} \times \frac{r-\alpha_x}{r} \times \frac{R_T}{1+\theta} \tag{14}$$

Leasing-convertible pricing (LCC)

A leasing convertible is a rental contract where lease payments are automatically converted to a predetermined number of shares once a predefined trigger event occurs. This quantity represents the conversion rate held by the lessor of the leased asset denoted by β. Glasserman and Nouri (2012) emphasized that the conversion time can be obtained as follows:

$$T_c = inf\left\{t \geq 0 : \varphi V(x_c) \leq \frac{R_T}{r}\right\} \tag{15}$$

Equation (15) implies that the conversion of the lease into equity occurs once a fraction, denoted φ, of the shareholders' equity reaches the sum of the par values of the outstanding lease. Parameter ϕ[1] $\in$ (0,1) is specified as an index to measure the regulatory standard.

[1] Note that Basel III requires that the capital adequacy ratio must satisfy at least 8%. If the company's capital ratio is lower than the level $(1-\varphi)$, such as 8%, the contingent convertible debt must be converted to equity.

The conversion expression indicates that the firm value is greater than the value of the outstanding lease at the time of conversion. In the usual banking and insurance context, the conversion rule can retain the existing regulatory standard for the Tier 1 capital ratio if an appropriate parameter value φ is taken. Generally, a conversion threshold is required for financial regulations.

Accordingly, the smaller the parameter φ, the more stringent the regulatory standard. However, the conversion expression may be improved because the conversion threshold is independent of firm risk. Therefore, the larger the business risk, the higher the conversion threshold.

Departing from (15), the conversion threshold is determined by:

$$x_c = \frac{r - \alpha_x}{r} \times \frac{R_T}{1-\tau} \times \frac{1}{(1+\theta)\varphi} \tag{16}$$

To specify the convertible value of leasing, the following limit conditions must be imposed:

$CL^G(+\infty) = \frac{R_T}{r}$; in the case of no conversion, the lessor receives leasing payments for the entire period until maturity.

$CL^G(x_c) = \beta V(x_c)$; at the conversion moment, the lessor receives payoff $\beta V(x_c)$, where $V(x_c)$ is the firm value at time T_c.

Referring to Eq. (9) and the limit conditions above, the value of the leasing convertible is indicated in terms of

$$CL^G(x) = \frac{R_T}{r}\left[1 - \left(\frac{x}{x_c}\right)^{\gamma-}\right] + \beta V(x_c)\left(\frac{x}{x_c}\right)^{\gamma-} \tag{17}$$

Following Barucci and Del Viva (2012), the conversion rate is defined as follows:

$$\beta = min\left(\frac{\frac{R_T}{r}}{V(x_c)}, 1\right) \tag{18}$$

The conversion equation implies that at the time of conversion, leaseholders receive a portion of equity equal to its par value. In other words, if the equity value is less than the par value of the lease, the leaseholder will be the owner of the entire firm.

A conversion rate of $\beta = 1$ implies that the holders of the leased asset completely replace the former holders of shares at conversion. In this case, the owners of the asset suffer a loss because the face value of the lease is greater than the value of the equity $\frac{\frac{R_T}{r}}{V(x_c)} > 1$. In these circumstances, the completion of the conversion affects bankruptcy restructuring equivalently. The shareholders are kicked out of the firm, and the assets' owners hold the lease's residual value. For more accurate details of the conversion mechanism, refer to Barucci and Del Viva (2012) and Pennacchi (2010).

Note that after the conversion, the firm's capital structure is composed of equities and assets in the lease (which become the firm's property), as well as the lessor who becomes a shareholder.

Firm value pricing

A firm's capital structure is composed of equity and leasing convertible contracts. For $x \geq x_c$, the cash flow level is higher than the conversion level. In this case, the cash flow generated by the leased asset can be expressed as $\xi = (1-\tau)((1+\theta)x - R_T)$, with $a = (1-\tau)(1+\theta)$ and $b = -(1-\tau)R_T$. Let the domain limit $D = (x_c, +\infty)$ and the limit conditions be $E^G(x_c) = (1-\beta)V(x_c)$ represents the cash flow value when the conversion takes place, with $(1-\beta)$ corresponding to the loss rate endured by shareholders due to conversion.

$E^G(+\infty) = (1-\tau)\left(\frac{x}{r-\alpha_x} - \frac{R_T}{r}\right)$ represents the cash flow value at maturity; that is, there is no conversion.

The equity value before conversion of the leasing convertible is expressed in terms of:

$$E^G(x) = (1-\tau)\left[\left(\frac{(1+\theta)x}{r-\alpha_x} - \frac{R_T}{r}\right) - \left(\frac{(1+\theta)x_c}{r-\alpha_x} - \frac{R_T}{r}\right)\left(\frac{x}{x_c}\right)^{\gamma-}\right] + (1-\beta)V(x_c)\left(\frac{x}{x_c}\right)^{\gamma-} \tag{19}$$

The first term between brackets in Eq. (19) determines the investment cash flow value minus the leasing payment, R_T. Likewise, the second term stands for the same notion as the first term when the conversion takes place, and the term outside the brackets provides the value of equity after conversion. Note that $\beta V(x_c)$ represents the value of equity for the new shareholders and $(1-\beta)V(x_c)$ is the equity value for the old shareholders in the case of conversion.

The total value of the firm is the equity value plus the leasing convertible value and can be expressed as follows:

$$V^G(x) = V(x) + \frac{\tau R_T}{r}\left[1 - \left(\frac{x}{x_c}\right)^{\gamma-}\right] \tag{20}$$

The Equation for $V^G(x)$ is the sum of the value of the equity-financed firm and tax shields obtained by issuing an LCC before conversion minus lease payments after conversion.

Defaultable-convertible-leasing contract with payback option pricing (DCLCP)

The convertible lease option grants the lessor the right, but not the obligation, to convert the remaining lease payments into stocks or to break up the contract and pick up the rented equipment when the firm reaches the default threshold. In the case of no default, the firm acquires the option but not the obligation to pay back the equipment at the maturity of the contract at a fixed price, M_T. The payback option is exercised at maturity T when the asset market price A_T is higher than M_T.

Convertible lease option

The convertible lease option value upon default with an exercise price A_{T_D} is provided by

$$\begin{aligned} &payoff\,(C_{CL}) = \max\left(c, A_{T_D}\right) \\ &\left(c - A_{T_D}\right)^+ = \begin{cases} c - A_{T_D}, & if c \geq A_{T_D} \\ 0, & if c < A_{T_D} \end{cases} \end{aligned} \tag{21}$$

where c is the conversion value, and A_{T_D} is the market price of the leased asset upon default.

If $c > A_{T_D}$, the lessor will exercise the option and receive conversion rate β. Yet, if $c < A_{T_D}$, the lessor will not exercise the option and may choose to sell the rented asset at market price. The condition $c = A_{T_D}$ implies that the lessor can choose between converting non-refundable lease rents or selling the asset in the market. The decision rule related to the option exercise is triggered by two conditions: default event and option payoff. The CCL option C_{CL} is claimed only when the cash flow level x reaches the default level x_d. Let $1_{\{T_D \leq T\}}$ be the indicator function of an event when $x < x_d$. This function takes the value of 1 when an event occurs and becomes equal to zero in the opposite case. We can now express exercise price as a function of both variables c andA_{T_D}:

$$payoff(C_{CL}) = (c - A_{T_D})^+ = \begin{cases} c - A_{T_D}, ifc \geq A_{T_D} andx < x_d \\ 0, ifc < A_{T_D} \end{cases} \tag{22}$$

When the option payoff is zero, the lessor leaves the convertible lease option for the rented equipment and may choose to sell the asset if $x < x_d$ and $c < A_{T_D}$. In the case where $c = A_{T_D} andx < x_d$, the lessor may be indifferent. In fact, he would choose to either exercise or sell the equipment. Thus, we express the payoff of the convertible lease option at time T_D by the following formula:

$$F(s, x_d, T_D; R_T, T) = 1_{\{T_D \leq T\}} \left(\beta V(x_c) 1_{\left\{c > A_{T_D}\right\}} + A_{T_D} 1_{\left\{c < A_{T_D}\right\}} \right) \tag{23}$$

Note that the maturity of the option is the time when the firm is declared in default that is, at time T_D or at the maturity of the contract at time T in case of no default.

The exercise price of the option is the market price of rented equipment, which is determined by the following formula: $A_{T_D} = \frac{x_0 \alpha_s}{k}$, where α_s represents the return on service flows of the asset, x_0 stands for the current cash flow rate of the firm, and k corresponds to the weighted average cost of capital.

Capital structure composition and pricing

The firm's financial structure is composed of two asset classes: stocks and convertible-lease options. Convertible lease option pricing necessitates the following limit conditions: at the maturity of the option and in case of no default, payments are regularly carried out to the lessor, and the lessee will exercise the payback option. The renting rate and payback option are worth

$$CL_{\mathrm{DCLCP}}(T) = F(s, x, T; R_T, T) = max(A_T - M_T; 0) + R_T \tag{24}$$

However, in the case of default at time T_D, there are two scenarios to be tackled:

$$CL_{\mathrm{DCLCP}}(T_D) = F(s, x_d, T_D; R_T, T) = max\left(c, A_{T_D}\right) \tag{25}$$

If $c > A_{T_D}$, the conversion trigger event occurs, and the owner of the leased asset receives a predetermined number of shares, denoted as $\beta V(x_c)$. The limit condition of the conversion can be expressed in terms of:

$$CL_{\mathrm{DCLCP}}(T_D) = F(s, x_c, T_D, c; R_T, T) = \beta V(x_c) \tag{26}$$

where $V(x_c)$ represents the value of stocks at the time of conversion or the exercise time of the convertible lease option.

$$T_{CL} = inf\left\{t \geq 0 : c > A_{T_D}\right\} \tag{27}$$

If the lessor decides to exercise the convertible lease option, the time and the level of default will be the same as the time and the level of conversion for $c > A_{T_D}$ and is determined by Eq. (16). If $c < A_{T_D}$ The lessor leaves the convertible lease option and sells the rented asset at the market price.

The second limit condition can be indicated as follows:

$$CL_{\mathrm{DCLCP}}(T_D) = F\left(s, x_d, T_D, A_{T_D}; R_T, T\right) = A_{T_D} \tag{28}$$

According to the limit conditions and Eq. (9), the value of the convertible lease option can be expressed as follows:

$$\begin{aligned} CL_{DCLCP}(x) &= F(s, x, T; R_T, T)\left(1 - \left(\frac{x}{x_d}\right)^{\gamma-}\right) + F(s, x_d, T_D; R_T, T)\left(\frac{x}{x_d}\right)^{\gamma-} \\ &= \left(\frac{M_T}{(1+r)^T} + \frac{R_T}{r}\right)\left(1 - \left(\frac{x}{x_d}\right)^{\gamma-}\right) + \pi F\left(s, x_d, T_D, A_{T_D}; R_T, T\right)\left(\frac{x}{x_d}\right)^{\gamma-} \\ &\quad + (1-\pi)F(s, x_c, T_D, c; R_T, T)\left(\frac{x}{x_c}\right)^{\gamma-}, \pi \in \{0, 1\} \end{aligned} \tag{29}$$

The first term represents the value of the lease contract with a payback option at the maturity of the contract, the second term refers to the value of the lease at the time of default, and the third term determines the value of the lease if conversion occurs.

In case of conversion, $F(s, x_d, T_D; R_T, T) = F(s, x_c, T_D, c; R_T, T) = \beta V(x_c), \pi = 0$ and $F\left(s, x_d, T_D, A_{T_D}; R_T, T\right) = 0$.

In case of no conversion $F(s, x_d, T_D; R_T, T) = F\left(s, x_d, T_D, A_{T_D}; R_T, T\right) = A_{T_D}$, $\pi = 1$ and $F(s, x_c, T_D, c; R_T, T) = 0$.

After exercising the growth option, the cash flow of the firm at maturity is given by $\xi = (1-\tau)((1+\theta)x - R_T - max(A_T - M_T; 0))$. The following limiting conditions were used to determine the value of the stock:

At the maturity of the convertible lease option contract, the value of cash flow is

$$E_{\mathrm{DCLCP}}(T) = H(s, x, T; R_T, T) = (1-\tau)((1+\theta)x - max(A_T - M_T; 0) - R_T) \tag{30}$$

At the time of default T_D, two scenarios can be envisaged:

$$E_{\mathrm{DCLCP}}(T_D) = H(s, x_d, T_D; R_T, T) = max\left(c, A_{T_D}\right) \tag{31}$$

Ifc $> A_{T_D}$, there will be a conversion event, and the cash flow becomes:

$$E_{\mathrm{DCLCP}}(T_D) = H(s, x_c, T_D, c; R_T, T) = (1-\beta)V(x_c) \tag{32}$$

Table 1 Basic parameters

Reference	Tan and Yang (2016)	Koziol and Lawrenz (2012)	Glasserman and Nouri (2012)
Parameters	$r = 0.06$ $\tau = 0.15$ $\alpha_x = 0.01$ $\sigma_x = 0.25$ $w = 0.25$	$\beta = 0.4$	$1 - \varphi = 0.05$

where $(1 - \beta)$ determines the shareholder loss rate if the convertible lease option is exercised. If $A_{T_D} > c$, the lessor leaves the option and receives the market value of the rented asset, A_{T_D}.

The second limit condition is determined by:

$$E_{\text{DCLCP}}(T_D) = H\left(s, x_d, T_D, A_{T_D}; R_T, T\right) = V(x_d) - A_{T_D} \tag{33}$$

The stock value with the convertible lease option will be:

$$\begin{aligned}
E_{DCLCP}(x) &= H(s,x,T;R_T,T)\left(1 - \left(\frac{x}{x_d}\right)^{\gamma-}\right) \\
&\quad + H(s,x_d,T_D;R_T,T)\left(\frac{x}{x_d}\right)^{\gamma-} \\
&= (1-\tau)\left[\left(\frac{(1+\theta)x}{r-\alpha_x} - \frac{R_T}{r} - \frac{M_T}{(1+r)^T}\right)\right. \\
&\quad \left. - \left(\frac{(1+\theta)x_d}{r-\alpha_x} - \frac{R_T}{r} - \frac{M_T}{(1+r)^T}\right)\left(\frac{x}{x_d}\right)^{\gamma-}\right] \\
&\quad + \pi H(s,x_c,T_D,c;R_T,T)\left(\frac{x}{x_c}\right)^{\gamma-} \\
&\quad + (1-\pi)H\left(s,x_d,T_D,A_{T_D};R_T,T\right)\left(\frac{x}{x_d}\right)^{\gamma-}, \pi \in \{0,1\}
\end{aligned} \tag{34}$$

The first term of Eq. (34) represents the value of the stock, which is equal to the cash flow value at lease maturity minus the stock's value at time T_D. The second term represents shareholder loss in the event of a default. The lessor, in this case, has the right to exercise the option and receives $H(s, x_c, T_D, c; R_T, T)$ if $\pi = 1$, or sell the rented asset at the market price $H\left(s, x_d, T_D, A_{T_D}; R_T, T\right)$ if $\pi = 0$.

The total value of the firm is equal to the stock's value plus the convertible lease option.

$$\begin{aligned}
V_{DCLCP}(x) &= E_{DCLCP}(x) + CL_{DCLCP}(x) \\
&= (1-\tau)\left[\frac{(1+\theta)x}{r-\alpha_x} - \frac{(1+\theta)x_d}{r-\alpha_x}\left(\frac{x}{x_d}\right)^{\gamma-}\right] + \tau\left[\frac{R_T}{r} + \frac{M_T}{(1+r)^T}\right]\left[1 - \left(\frac{x}{x_d}\right)^{\gamma-}\right] \\
&\quad + \pi\left(F(s,x_c,T_D,c;R_T,T) + H(s,x_c,T_D,c;R_T,T)\right)\left(\frac{x}{x_c}\right)^{\gamma-} \\
&\quad + (1-\pi)\left(F\left(s,x_d,T_D,A_{T_D};R_T,T\right) + H\left(s,x_d,T_D,A_{T_D};R_T,T\right)\right)\left(\frac{x}{x_d}\right)^{\gamma-}, \pi \in \{0,1\}
\end{aligned} \tag{35}$$

Table 2 Landlord parameters

s_0	$A(s)$	A_{T_D}	A_T	M_T	V_{AM}
2	40	20	41	31	10

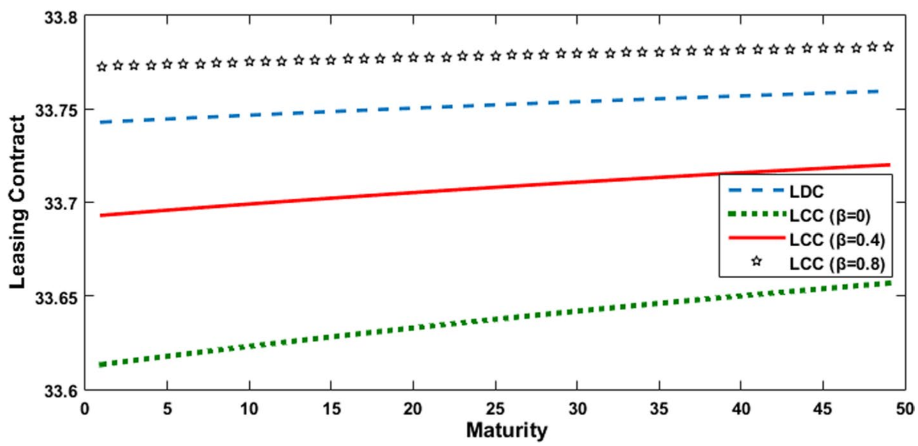

Fig. 1 Leasing contract versus Maturity under different conversion rates

Note that if the lessor exercises the option, then $\pi = 1$ and the functions $H(s, x_c, T_D, c; R_T, T)$ and $F(s, x_c, T_D, c; R_T, T)$ take their values, and the functions $H\left(s, x_d, T_D, A_{T_D}; R_T, T\right)$ as well and $F\left(s, x_d, T_D, A_{T_D}; R_T, T\right)$ are equal to zero. Otherwise, if the lessor abandons the option, then $\pi = 0$ and the functions $H\left(s, x_d, T_D, A_{T_D}; R_T, T\right)$ and $F\left(s, x_d, T_D, A_{T_D}; R_T, T\right)$ take their values, and the functions $H(s, x_c, T_D, c; R_T, T)$ and $F(s, x_c, T_D, c; R_T, T)$ become equal to zero.

Numerical results and sensitivity analysis

To clarify the impact of LCC and DCLCP on the service value of leased assets, maturity, and inefficiencies resulting from insolvency and asset substitution, we determine the parameter value by referring to Tan and Yang (2016), Koziol and Lawrenz (2012), and Glasserman and Nouri (2012). Thus, the values of the basic parameters in this study were selected as follows Table 1.

We note the risk-free interest rate $r = 0.06$, the bankruptcy cost $w = 0.25$, the drift rate $\alpha_x = 0.01$, the volatility $\sigma_x = 0.25$, the effective tax rate $\tau = 0.15$, the conversion ratio $\beta = 0.4$ and the capital adequacy ratio $1 - \varphi = 0.05$.

We incorporate suitable parameters and other factors to analyze the contingent lease models presented previously. We assume that the current cash flow is $x_0 = 200$ and the growth ratio of the cash flow (investment) is $\theta = 0.01$. The service value of the current asset is $s_0 = \alpha_s * x_0 = 2$, where the yield of the service flow $\alpha_s = 0.01$. Therefore, the asset value (at maturity) when T tends to infinity is determined by $A(s) = \frac{s_0}{r - \alpha_s} = 40$. To determine the exercise price of the buyback option, it is first assumed that the duration of the leased equipment is four years, and the duration of the DCLCP is $T = 3$ years. The amortization value of the equipment was equal to

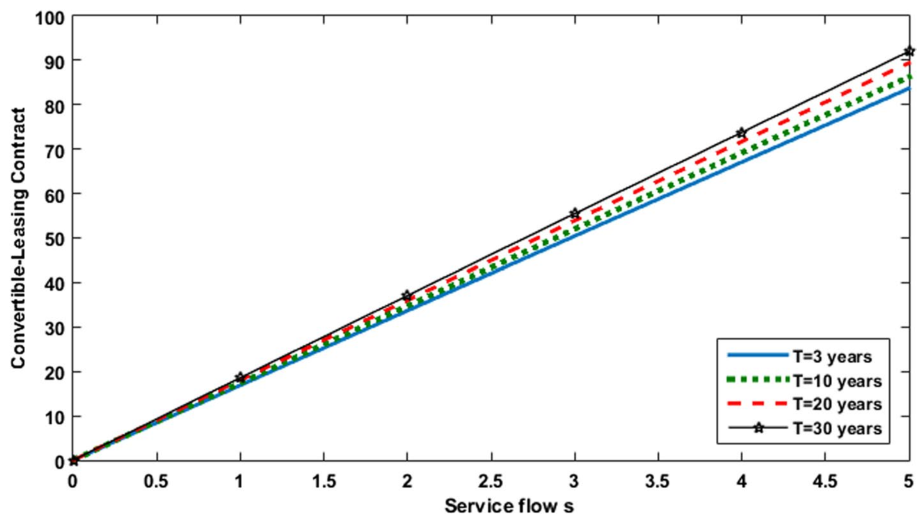

Fig. 2 The Convertible-Leasing Contract value versus initial service flow value under different maturities

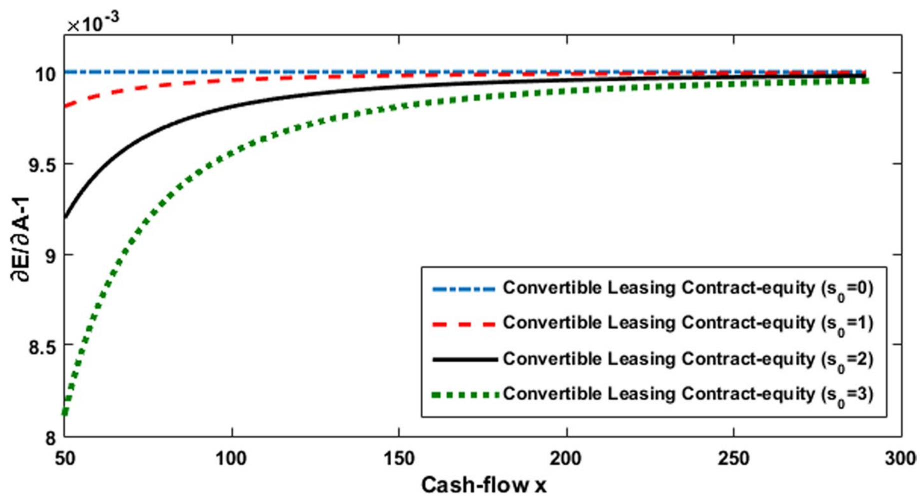

Fig. 3 The sensitivity of leverage ratio versus cash-flow under different service flow value

$V_{AM} = A(s) - \frac{A(s)}{4} * T = 10$. Thus, the Black–Scholes securities price valuation model is used to determine the future value of the assets leased at maturity T. Hence, we obtain the value $A_T = A(s)\exp(\alpha_s * T) = 41$. Finally, we calculate the exercise price of the leased asset as $M_T = A_T - V_{AM} = 31$. The exercise price of the convertible lease option is the value of the asset leased on the market at moment T_D obtained by the following formula: $A_{T_D} = \frac{x_0\alpha_s}{k} = 20$, where k refers to the cost of capital. We summarize the parameters of the lease contracts calculated in the following Table 2.

Departing from expressions (11) and (17), the values of the LDC and LCC are represented in Figure 1 for different values of the conversion rates β over time.

As portrayed in Fig. 1, we notice that the value of the LCC with a conversion rate ($\beta = 0.8$) is higher than that of the LDC. From this perspective, we deduce that the conversion rate significantly impacts the growth of leasing contract value. The closer

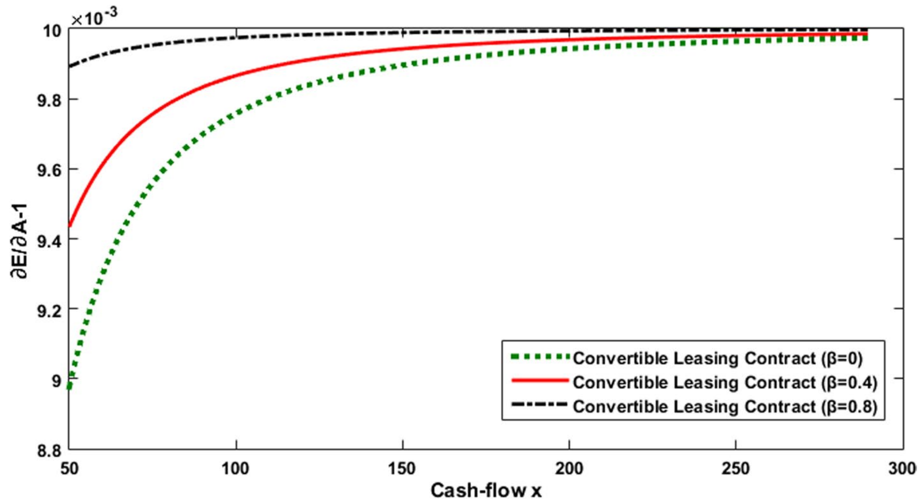

Fig. 4 The sensitivity of leverage ratio versus cash-flow under different Conversion Rates

the conversion rate is to one, the more the value of the lease increases over time. The position of the contract curves is reasonable because the convertible leasing contract with a high conversion rate offers more protection to its holder than the standard leasing contract in the case of a default.

According to Fig. 1, the LCC with a conversion rate of 0 increases sharply with the maturity period. However, the LCC with a conversion rate ($\beta = 0.8$) increases slightly with maturity. This implies that the higher the conversion rate, the less maturity of the lease affects the value of the convertible lease.

Based on Eq. (17), the evolution of LCC relative to cash flow for different maturities is shown in Fig. 2.

As shown in Fig. 2, the LCC value increases linearly according to the service flows of the leased assets. The higher the service flow of the asset, the more return it generates to the firm and the higher the value of the contract. In addition, the longer the maturity, the more important the LCC. However, when asset service flows are less than 2, the maturity of the contract does not affect LCC value. The difference between the slopes of the LCC under different maturities is very small for service flow values close to 0. This refers to the fact that with a very low service flow, the leased asset does not generate profits for the tenant, and therefore its value becomes very low regardless of the maturity of the lease. The impact of maturity on the evolution of LCC value emerges only if the lease maturity becomes very long.

According to Eq. (19), we represent the sensitivity of the lease-leverage ratio to cash flow under the different values of the service flow of the asset, as displayed in Fig. 3.

Figure 3 shows that the lower the service flow value of the asset is, the greater the sensitivity of equity to cash flows becomes. We infer that the lower the service flow value of the asset is, the greater the shareholder incentive to replenish equity becomes. The sensitivity of the lease-leverage ratio stabilizes slightly with the increase in the cash-flow value of the company for different service values of the asset $s_0 = 1$, $s_0 = 2$, and $s_0 = 3$. Hence, the higher the value of cash flows is, the more the company's leverage ratio stabilizes. For $s_0 = 0$, the lease-leverage ratio is almost stable for any cash-flow value. If

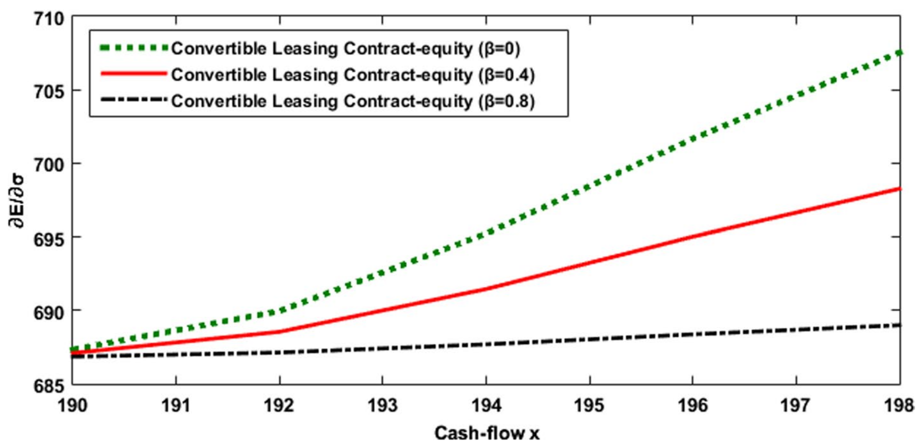

Fig. 5 Asset substitution versus cash-flow under different Conversion Rates and financing policies

$s_0 = 0$, the company does not introduce any means of financing in its capital structure. The company, in this case, considers self-financing to manage its business and investment project to raise capital. This entails a high and stable lease-leverage ratio compared to cash flows.

Based on Eq. (19), Fig. 4 plots the sensitivity of the lease-leverage ratio as a function of cash flow for a company that issues an LCC in its capital structure under different values of β with baseline parameter values. To measure a company's leverage, we calculate the net increment of equity value, $\partial E/\partial A - 1$ as suggested by Pennacchi et al. (2014). $\partial E/\partial A - 1$ is an index that calculates the sensitivity of the equity value to the asset value A. It indicates shareholders' capacity to cover all outstanding leasing. Thus, it can be viewed as a measure of a company's leverage ratio. A negative value signifies that shareholders' final value is lower than what they originally invested, which suggests a lease overhang distortion. The lower the ratio $\partial E/\partial A - 1$ is, the worse the leverage and the less the incentive from shareholders to replenish equity.

Figure 4 displays that after the exercise of the growth option, the conversion rate, β, has a significant effect on the sensitivity of the lease-leverage ratio. The higher the conversion rate $\beta = 0.8$ is, the higher the incentive for the shareholders to replenish equity.

The LCC has opposite effects on the value of equity, which can suffer from a significant decrease due to the dilution effect because, at the time of the conversion, a large part of the equity must be distributed to leaseholders. However, an increase in the share price may also indicate that shareholders will no longer pay leasing coupons after the conversion. The higher the conversion rate, the greater the dilution effect. If the conversion rate is sufficiently high, the effect of share dilution dominates the conversion effect. Consequently, shareholders will find it more profitable to issue new shares to delay or avoid conversion, which seems to be costlier for existing shareholders. From this perspective, we conclude that financing through an LCC reduces insolvency inefficiencies through higher cash flows. The reason lies in the fact that an increase in the company's cash flow through the injection of money from shareholders reduces the likelihood of bankruptcy.

Based on Eq. (19), Fig. 5 graphs the risk-shifting incentive after the exercise of the growth option under LCC equity financing and different conversion rates, β, with baseline parameter values. Following Pennacchi et al. (2014), we calculate the sensitivity of

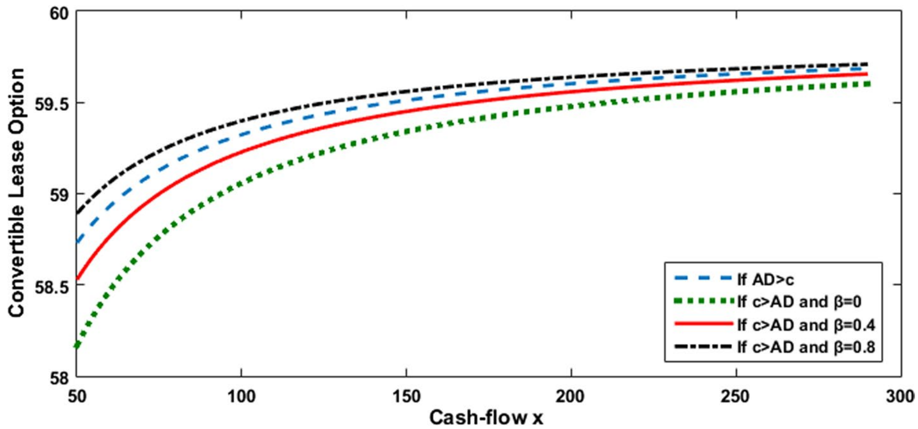

Fig. 6 Convertible Lease Option versus cash-flow under different Scenarios and Convertion Rates

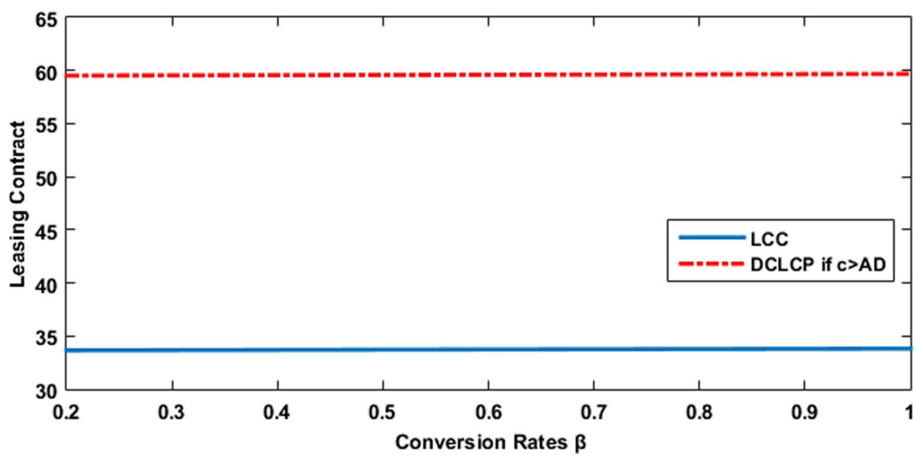

Fig. 7 Leasing contract versus Conversion Rates

the equity value to asset volatility, that is, $\partial E/\partial\sigma$, to measure the risk-shifting incentive. When shareholders increase an additional unit of risk, the value of the capital increase will be greater than zero, which leads to a distortion of asset substitution.

The $\partial E/\partial\sigma$ index refers to the sensitivity of the value of equity to asset volatility. If this value is greater than 0, then there is an increase in business risk, which leads to an increase in the value of equity; therefore, there is a risk-shifting incentive for shareholders. Certainly, the greater the value, the stronger the incentive.

Figure 5 highlights that after the growth option exercise, the risk-shifting incentive in LCC equity financing is very low for a higher conversion rate. This is assigned to the fact that, before conversion, shareholders can lose a larger fraction of equity if the conversion ratio is higher enough. Hence, the risk-shifting incentive under LCC-equity financing for a high conversion rate ($\beta = 0.8$) is less than that under LCC-equity financing with a low conversion rate $\beta = 0\, or\, \beta = 0.4$.

Figure 6 is elaborated according to the definition of a convertible lease option in (29).

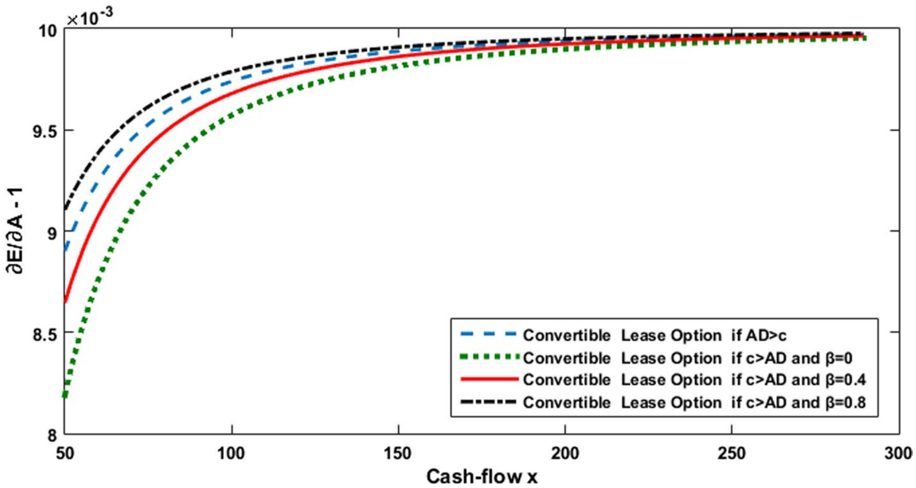

Fig. 8 The sensitivity of leverage ratio versus cash-flow under different Scenarios and Convertion Rates

This option offers the right to exercise the convertible lease option and receive several shares or sell leased assets on the market. Unlike LCC, DCLCP allows the lessor to choose the most efficient and least expensive scenario to cover his/her position at the time of default. Figure 6 reveals that the value of the DCLCP is higher when the lessor exercises the convertible lease option with a higher conversion rate ($\beta = 0.8$) as opposed to abandoning this option and therefore selling the leased asset on the market.

At this analysis level, we would assert that the conversion rate β has a significant effect on the value of the DCLCP. The higher the rate β, the higher the value of the convertible lease option. The option grants the holder more advantage in managing the tenant's risk of default.

At tenant default, if the leaseholder exercises the convertible lease option, the value of the DCLCP with a conversion rate ($\beta = 0.8$) is higher than the value of the contract in which the asset holder chooses to sell the leased asset in the market. It can then be deduced that the exercise of the convertible lease option with a conversion rate close to 1 covers the position of its holder and minimizes the losses of default.

Equations (17) and (29) are invested in examining the impact of the conversion rate β on the values of LCC and DCLCP in Fig. 7.

Figure 7 shows the evolution of LCC and DCLCP relative to the conversion rates, β. The evolution of LCC and DCLCP remains stable relative to the different conversion rates. As depicted in Fig. 7, the value of DCLCP was higher than that of LCC. The holder of the convertible lease option's assets has the choice to exercise the option at the time of default. The DCLCP covers more of the position of its holder than the LCC does. In the DCLCP, the tenant can pay back the leased asset at maturity if the exercise price of the underlying asset is lower than the market asset price. This option provides the holder with the right to repay the leased asset at a lower price.

Departing from Eq. (34), we present in Fig. 8 the sensitivity of the value of equity to cash flow for a company that issues a DCLCP for different values of β with baseline parameter values.

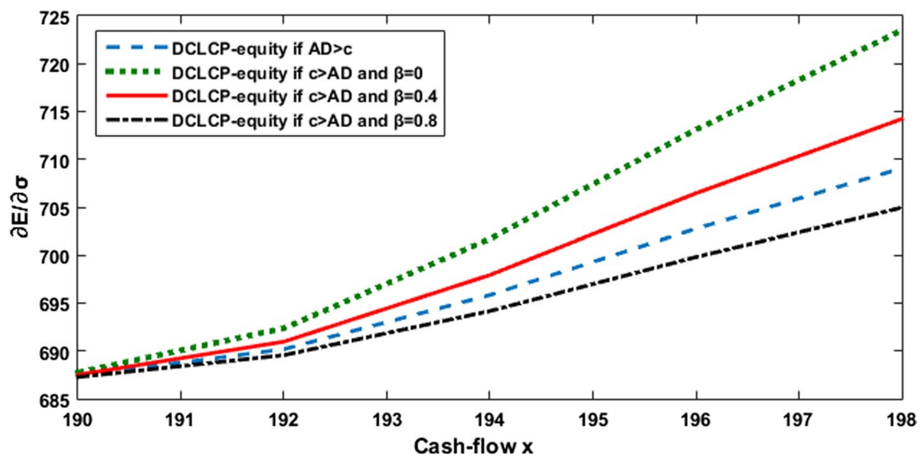

Fig. 9 The sensitivity of the value of equity to asset volatility versus cash-flow

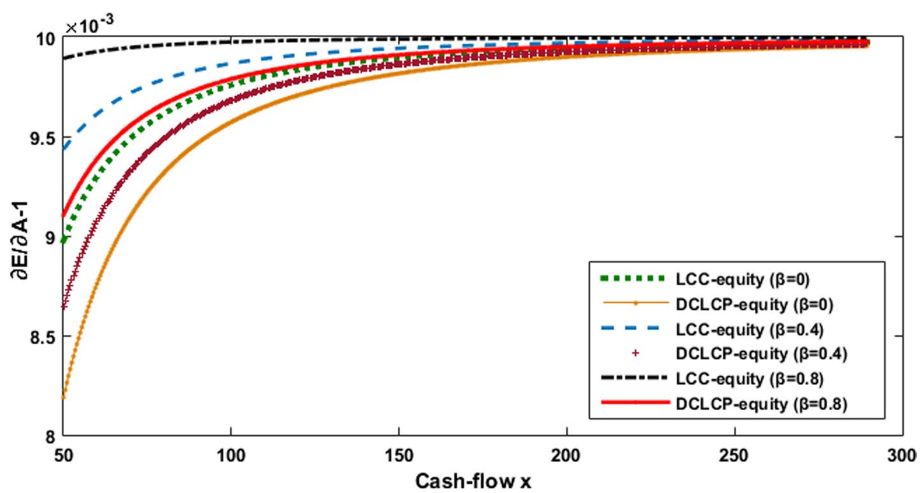

Fig. 10 Leverage sensitivity versus cash-flow under different Conversion Rates and financing policies

Figure 8 indicates that the conversion rate has a significant impact on the lease-leverage ratio, and a larger conversion rate can reduce inefficiencies from insolvency. Moreover, Fig. 8 shows that if the holder chooses to exercise the option at a significant rate ($\beta = 0.8$), then the lease-leverage ratio sensitivity of the DCLCP is higher than that of the same contract once the holder chooses to sell the leased asset at the market price at the time of default. This confirms that it is more advantageous for any company to exercise the convertible leasing option at the time of default rather than abandon it.

Regarding Eq. (34), Fig. 9 demonstrates the sensitivity of the value of equity to asset volatility against the cash flow x after investment under DCLCP-equity financing with different conversion rates, β, relative to baseline parameter values.

As shown in Fig. 9, the sensitivity of the risk-shifting incentive increases with the company's cash flow. The higher the level of cash flow, the higher the risk-shifting incentive.

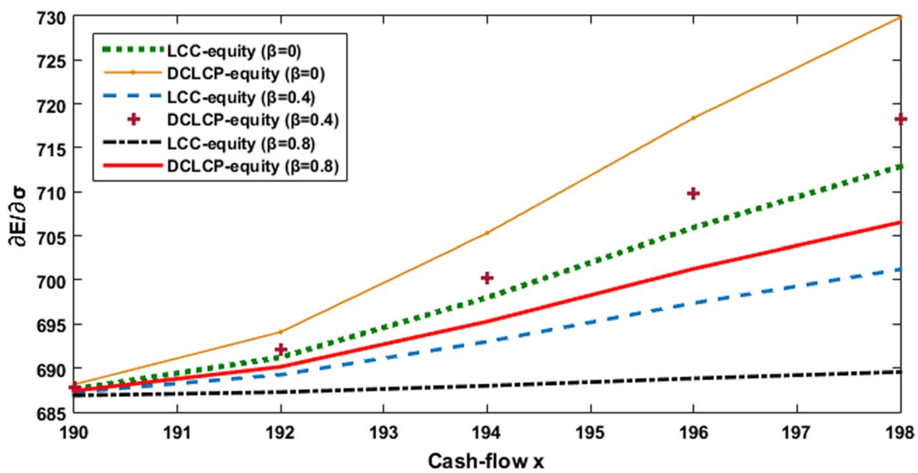

Fig. 11 Asset substitution versus cash-flow under different Conversion Rates and financing policies

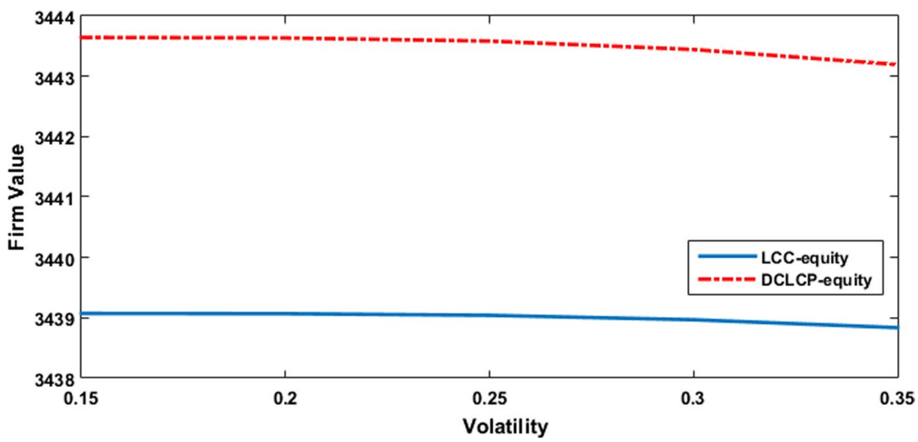

Fig. 12 Optimal firm value for different assets volatilities and financing policies

In addition, Fig. 9 shows that the lower the conversion rate, the higher the sensitivity of the risk-shifting incentive. For a conversion rate close to 1, such as $\beta = 0.8$, the likelihood of risk-shifting incentive under financing DCLCP-equity when the lessor exercises the convertible lease option becomes less than that when the lessor abandons it and sells the asset at the market. However, at the time of default, the event of conversion under financing of DCLCP equity does not occur automatically, which yields a very high risk to the company.

Resting on Eqs. (19) and (34), we plot in Fig. 10 the ratio of equity value to asset value A as a function of cash flows under different financing policies and values of β with baseline parameter values.

Figure 10 reports that the lease-leverage ratio sensitivity under the LCC-equity financing is higher than that under DCLCP-equity financing for all different values of conversion rates. This can be attributed to the fact that the LCC leads to better management of the financial situation of the firm at the time of default, which corresponds to the automatic conversion of non-refundable payments into a predetermined number of shares. However, in the DCLCP, the holder of the leased asset has the right to exercise or abandon the option at the time of the tenant's default to cover their position, which leads, in this case, to a huge loss to the company.

It is also worth noting that the conversion rate significantly affects the lease-leverage ratio. In fact, under LCC equity and DCLCP equity financing with sufficiently high conversion rates, the efficiency of insolvency can be reduced. This can be interpreted as the fact that if the conversion ratio is sufficiently high, shareholders will benefit from injecting new money to delay any conversion since conversion is more costly for existing shareholders.

Likewise, we invest the same Eqs. (19) and (34) to trace in Fig. 11 the sensitivity of risk-shifting incentives versus cash flows under different financing policies and the values of β with baseline parameter values.

Figure 11 represents the sensitivity of equity value to asset volatility against x after the exercise of the growth option under different financing policies and conversion rates, β. The sensitivity of the risk-shifting incentive relative to cash flows under LCC-equity financing is less than that under DCLCP equity financing for different conversion rates because the conversion in the LCC is carried out automatically at the moment of default of the company. The existence of a convertible lease option in the DCLCP grants its holder the right, and not the obligation, to exercise the conversion, which leads to a higher risk-shifting incentive, as outlined in Fig. 11.

Figure 12 illustrates the impact of asset volatility on the value of the firm for different asset volatilities, σ, and funding policies based on the values of the basic parameters. We use Eqs. (20) and (35) to plot Fig. 12.

The value of the firm decreases with asset volatility regardless of the financing policy. After the investment expansion, DCLCP equity financing leads to a maximum firm value, which is not the case for LCC equity financing. This result indicates that introducing a DCLCP into the company's capital structure generates more profitability for the company's shareholders than an LCC. This finding is accounted for by more risk in the DCLCP than in the LCC.

Conclusion

We explore three contingent leases as risk management instruments that extend and complement contingent rent agreements based on the firm's operating activity. Leasing defaultable contracts allows the lessor to receive a fraction of the lease value due to default costs if the lessee defaults. Convertible leasing is a financial instrument that allows a lessor to convert non-refundable lease payments into shares when a trigger occurs in the event of financial distress. Therefore, it can reduce the financial situation of a company and avoid the risk of bankruptcy. The conversion mechanism of the contingent lease is a practical technique that may absorb losses caused by the tenant's default and provide an alternative to the lessor of the leased asset to manage their risk exposure. The Defaultable-Convertible-Leasing Contract with Payback option grants the lessor the right to exercise the convertible lease option or to sell the rented equipment on the market when the firm reaches the default threshold. The main reason for introducing these contracts into corporate finance was to expose the risk of non-refundable lease payments due to the insufficient cash flow generated by using leased equipment.

In this study, we consider a dynamic model for a company with existing assets and a growth option and assume that investment costs are financed by equity and a contingent lease to increase existing assets. We provide closed-ended solutions for contingent lease prices and equity by setting the growth option after expansion investment as part of different financing policies. Next, we investigate the effect of the inclusion of contingent leases on the capital structure of corporate finance firms. We examine how the contingent lease affects the service value of the asset for different types of leases, financial stability, the inefficiencies arising from risk-shifting and insolvency, and the sensitivity of the company's value to changes in asset volatility.

The numerical analysis indicated that regardless of the financing policy, namely LCC or DCLCP equity, asset substitution, and insolvency inefficiencies after the exercise of the growth option are reduced for a conversion rate close to 1. LCC and DCLCP can eliminate the inefficiencies resulting from the agency's conflict of interest. The value of the company after investment expansion first increases and then decreases with asset volatility. The DCLCP is more advantageous to the lessor of the leased asset than the LCC because it better covers their position at the time of the tenant's default. If the conversion ratio of the LCC is sufficiently high, the LCC offers more protection to its holder than the LDC does. The results reveal that the value of the LCC increases linearly with the service flow of the leased asset and the maturity of the lease. The impact of maturity on LCC value appears only for long-term lease maturity. In particular, the lower the asset's service flow value, the greater the leverage ratio sensitivity and the greater the shareholder incentive to replenish equity.

In this study, it is assumed that the value of the leased asset follows a geometric Brownian motion, which is considered more flexible and adapted to the modeling of leasing, as in Grenadier (1996), because the service flows of the leased asset can depreciate or appreciate over time. However, modeling the dynamics of a firm's value with jump processes is likely to represent better the uncertainty associated with the conversion or default event, as pointed out by Pleger (2012). Moreover, the use of jump processes reduces the conflict of interest problem, as mentioned by Duffie and Lando (2001), between lessees and lessors of the leased asset, due to the accounting of sudden movements that can instantly lead to the bankruptcy of the firm. From this point of view, modeling the default using a jump-diffusion process that considers the timing and size of the jumps could provide a significant extension of our work.

Appendix

Derivative of Eq. (2): Using the properties of log-normal variables in Eq. (1), we obtain

$$s_t = s_0 e^{\left(\alpha_s - \frac{1}{2}\sigma_s^2\right)t + \sigma_s z_s(t)}$$

The present value of future service flow is:

$$E\left(\int_0^T e^{-rt} s_0 e^{\left(\alpha_s-\frac{1}{2}\sigma_s^2\right)t+\sigma_s z_s(t)} dt\right) = s_0 E\left(\int_0^T e^{-rt} e^{\left(\alpha_s-\frac{1}{2}\sigma_s^2\right)t+\sigma_s z_s(t)} dt\right)$$

Then, if we apply the Fubini's Theorem (refer back to Arnold (1977)), then

$$\begin{aligned} E\left(\int_0^T e^{-rt} e^{\left(\alpha_s-\frac{1}{2}\sigma_s^2\right)t+\sigma_s z_s(t)} dt\right) &= \int_\Omega \int_0^T e^{-rt} e^{\left(\alpha_s-\frac{1}{2}\sigma_s^2\right)t+\sigma_s z_s(t)} dtdP \\ &= \int_0^T e^{\left(\alpha_s-\frac{1}{2}\sigma_s^2-r\right)t} \int_\Omega e^{\sigma_s z_s(t)} dPdt \\ &= \int_0^T e^{\left(\alpha_s-\frac{1}{2}\sigma_s^2-r\right)t} E\left(e^{\sigma_s z_s(t)}\right) dt \end{aligned}$$

where $E\left(e^{\sigma_s z_s(t)}\right)$ defines the moment generating function of the Brownian motion $z_s(t)$ and Ω refers to the sample space of the random variable $z_s(t)$. Hence, $E\left(e^{\sigma_s z_s(t)}\right) = e^{\left(\frac{t\sigma_s^2}{2}\right)}$. Consequently, $E\left(\int_0^T e^{-rt} s_t dt\right) = \frac{s_0}{\alpha_s - r}\left(e^{(\alpha_s-r)T} - 1\right) = Y(s_0, T)$.

Abbreviations

LDC	Leasing defaultable contract
LCC	Leasing convertible contract
DCLCP	Defaultable-convertible-leasing contract with a payback option

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Author contributions
OT participated in the design of the study and performed the numerical analysis and wrote the manuscript. FA conceived the study, participated in its design and coordination, draft the manuscript, and checked the numerical analysis. All authors have read and approved of the final manuscript.

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Availability of data and materials
The datasets used and/or analyzed during the current study are available in the manuscript. We refer to Tan and Yang (2016) to determine the baseline parameter values. Matlab numerical analysis procedure are available upon request.

Declarations

Competing interests
The authors declare that they have no competing interests.

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