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A theory of very short-time price change: security price drivers in times of high-frequency trading

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Abstract

Academic research has identified several factors that affect price movements; however, the scenario changes abruptly in the case of very short time price changes (VSTPC). This topic is not specifically examined in the existing literature; nonetheless, the behavior of the market microstructure is quite different at the subsecond scale. Indeed, below a certain psychological time threshold, most factors typically influencing price changes cease to apply. This paper analyzes several parameters considered to affect price changes and identifies four of them as potentially influencing VSTPC. These factors are previous volatility, scarce liquidity, high quantity exchanged, and stop-loss (SL) orders (seldom mentioned in the literature). These four parameters are examined by means of a mathematical model, audit trail data analysis, Granger-causality testing, and agent-based model. The results of these four techniques converge to suggest a nonlinear combination of previous volatility, liquidity, and SL orders as the main causes of excess volatility. However, contrary to mainstream literature on trading time above a certain psychological threshold, the volumes exchanged are not integral agents for VSTPC. Currently, financial markets face many ultrafast orders, yet a coherent theory of price change at time scales incomprehensible by humans and only manageable by computers is still lacking. The theory presented in this paper attempts to fill this gap. The outcome of such a theory is important for purposes of market stability, crisis avoidance, investment planning, risk management, and high-frequency trading.

Keywords: High-frequency trading, Subsecond time scale, Volatility, Liquidity, Exchange volume, Stop-loss orders

JEL Classification: G01, G12, G14, G17

Introduction

The debate regarding the factors affecting the price of securities has persisted for a long time among academics and practitioners. Although the presence of arbitrage opportunities as well as noise traders are theoretically accepted feats, a commonly accepted theory among academics and practitioners is that efficient markets are populated by well-informed, rational investors. Yet, arbitrage, which aligns asset prices to actual values and corrects the damage caused by market anomalies, is regarded as a factor mitigating such damage. As long as arbitrage opportunities circulate among investors, it is impossible to generate abnormal profits consistently. On the contrary, should one agent display a permanent advantage over all other participants, the game would no longer be fair, and markets would cease to be efficient. This phenomenon has been taking place over the last two decades as a result of the introduction of high-frequency trading (HFT). Fortunately, for market fairness, it is never just one player who enjoys high-frequency (HF) capabilities. Although they are a minority, several trading firms access exchange servers at subsecond speed. This phenomenon ensures a clear advantage over traders that operate at "human" speed. However, competition is still possible; HFT is simply a different way of trading. Slower traders exploit their superior capabilities in market analysis and hope to achieve long-term profitability, whereas fast traders act upon other parameters to win a short-term game.

Gaps identified in the current literature

The main reason to propose a theory of Very Short-Time Price Changes (VSTPC) is the lack of a comprehensive and coherent theory to explain their characteristics. Numerous hypotheses and vast knowledge about price changes are scattered sparsely among several papers, books, and conference proceedings, but they do not constitute a unified theory. According to the findings of this research, some well-established price change factors do not carry much weight when observed at a Very Short Time (VST) interval, whereas other important factors are often overlooked. Academic research has produced a large body of literature about price changes, yet it fails to differentiate generic price changes from the much more peculiar field of VSTPC or systematize the currently available research into a coherent and comprehensive theory. Indeed, as will be discussed below, most widely mentioned factors are irrelevant at VST. This theory attempts to demonstrate that very often, only four factors affect price changes at VST—volatility, liquidity, volume, and stop-loss (SL) orders. The main hypothesis states that the nonlinear combination of otherwise tolerable factors may lead to a nonlinear outcome, potentially causing financial instability, extreme events, and crises.

Drivers of very short time price changes (VSTPC)

Price changes of financial securities are usually considered a symptom of market volatility that are usually computed as the standard deviation of the price over a certain period. Although this currently holds true, the trading environment created by algorithms differs from the traditional one in many respects (Kou et al. 2021a): the ultrafast environment precipitated by HFT has changed the rules of the game. Speed has always been a competitive advantage on the trading floor. With each technological innovation, the "same old movie" has been played faster and faster until something broke the metaphor. Computer-based trading displays "the potential to lead to a qualitatively different and more obviously nonlinear financial system" (Foresight 2012, p. 73). Recent studies have started raising doubts about market efficiency: abnormal price patterns have begun appearing at higher frequencies, and at higher frequencies, the more numerous are the anomalies. Johnson and Zhao (2012) focused on extreme events causing price changes greater than 0.8%, which were observed for at least 10 up or down ticks with no opposite movements in between and lasting no longer that 1500 ms over the 2006–2011 period. Their study found more than 18,520 qualifying events, that is, more than 10 anomalous events per trading day. But the most revealing finding relates to the distribution of such events over a certain duration: plotting the number of events against the event duration depicts an exponentially down-sloping curve. The shorter the time window is, the more extreme are the events. Nothing similar can be observed at "human" frequencies. This certainly does not look like "the same old movie" played faster: it is a brand-new movie. Summarizing a conspicuous set of studies, Foresight (2012, p. 85) concluded that "an important speed limit has been breached" and at subsecond level, a phase transition has occurred. One of those studies by Cartlidge and Cliff stated that "[a]t subsecond timescales, below the robot transition, the robot-only market exhibits 'fractures'—ultra-fast swings in price akin to mini flash crashes" (2012, p. 3). It is then imperative to understand what happens at such timescales and, more importantly, why.

The purpose of this study is to identify the main drivers of market price changes in the recently established VST environment and verify whether they differentiate themselves from traditional drivers.

Generic drivers of stock price changes

The first 250 academic articles sorted in order of relevance appearing in a Google Scholar search performed in June 2018 for "stock price change" yielded the following nondisjointed list of items: business and macroeconomic news, fundamental analysis, corporate, accounting and tax policies, market regulations, correlation between securities on the same venue, alternative venues dealing with the same securities, technical analysis, arbitrage, the random walk model (RWM), previous volatility, liquidity, volumes exchanged, bid–ask book order imbalances, transaction costs, the total number of investors on the market, market manipulation, speculation, strategies of large investors, investors' feelings, margin calls, and noise trading. Most of the Googled papers consider the factors that affect how investors evaluate stock value as exogenously driven. Instead, the VST impact on prices, as this research investigates, is driven by endogenous factors, irrespective of any external intervention. It must be said that ultrafast agents occasionally exploit exogenous factors thanks to their superior speed, yet it is endogenous parameters that lead to frequent subsecond swings, establishing themselves as the main features of algorithmic trading. All the factors found by the Google Scholar search potentially affect price changes, but only a few of them are relevant at VST. Business news, macroeconomic news, and fundamental analysis of company-related news are traditional price movers. However, as far as the impact of automated, ultrafast news trading on a VSTPC is concerned, news arrival is certainly too rare an event to be significant for theory. Corporate, accounting, and tax policies seldom change and therefore do not impact VSTPC. Similarly, changes to market regulations occur infrequently and therefore do not meet the requirements of a VSTPC parameter. Market inefficiencies do occur, but they are rare events if compared to the frequency of subsecond trading. Arbitrage and margin calls are certainly factors affecting price changes at VST but given their quantitative impact over the total number of transactions, they can be safely assumed to have a negligible impact on the theory. Indeed, their occurrence is only occasional. It is true that under some exceptional conditions, arbitrage and margin calls might impact price but, on average, this occurs rather seldomly and inconsistently on the subsecond scale.

Volume exchanged

The volume exchanged is nearly unanimously considered a factor of price changes. Market shocks are often accompanied by peaks in transactions. Stickel and Verrecchia (1994) mentioned the Wall Street wisdom that "volume is the fuel for stock prices." Their hypothesis is that an increase in trading volume is due to a greater probability of transactions being conducted by informed rather than uninformed traders. The consequence is that volume-driven price changes reflect the "real" price and are unlikely to be reversed in the future. On the contrary, large price variations supported by weak volumes tend to regress to the mean over the long term. Gündüz and Hatemi (2005) tested Granger-causality in emerging markets and found inconclusive evidence in the Czech Republic but unidirectional or bidirectional relationships in the other markets surveyed. Therefore, as a first attempt, volume exchanged will be considered a candidate driver of VSTPC.

Volatility

Volatility is at the very core of price changes. The two terms could be deemed to be one and the same, but a subtle distinction is worth making. A price change usually represents the difference between the initial and final price over a certain time period. Alternatively, as in the simulation presented later, a price change could be computed as the difference between the maximum and minimum price over the same period. The difference between the maximum and minimum prices (MAX–MIN price) is an interesting parameter as it indicates the greatest price change displayed by an asset over a certain period of time, usually days or, in case of HF data, hours, or minutes. This could be called "global volatility." However, there is another nuance of volatility that could be called "local volatility," that is, the price change over a VST. This will certainly stay within the MAX–MIN range, yet it has an impact on price and returns at a VST. The question that this study will attempt to answer is whether preexisting volatility may lead to further volatility, that is, if at a VST, price changes are affected by volatility in the previous seconds or fractions of seconds. A high level of volatility makes markets nervous, and several studies have adopted the autoregressive conditional heteroskedasticity (ARCH) and generalized autoregressive conditional heteroskedasticity techniques to detect whether volatility comes in bursts, creating clustering effects (Kou et al. 2014; Li et al., forthcoming) in which volatility at the beginning of the burst influences volatility throughout. Previous volatility is a candidate parameter for studying how prices change over a VST.

Liquidity

Liquidity also affects market prices, both at VST and otherwise. Indeed, liquidity is a sought-after feature of markets as it can smooth out excess volatility. On the contrary, if liquidity is thin, even a low volume of transactions has the potential to cause a sharp increase in volatility, with unforeseen consequences. Liquidity is therefore a candidate parameter for a VSTPC theory.

Stop-loss (SL) orders

Although the 250 papers used as the main references for identifying the drivers of price changes mention a wide range of possible causes of price changes, none of them deals with the SL mechanism. Instead, the impact of SL orders on price changes is both direct and intuitive. The SL mechanism is a "mechanical" driver of prices; risk and uncertainty are the conceptual price drivers related to it. If a price drops (increases) at or below (above) a certain level and if, corresponding to that level, there are SL orders, they will automatically be converted into market orders by the exchange server before adding any new limit order to the books or executing new market orders. If the number of SL orders is great or liquidity is low or both, the execution of SL orders turned into market orders is likely to consume the liquidity available at that price level, potentially changing the price even more.

Conclusions regarding the drivers of stock price changes

This analysis prompts us to conclude that among the several factors potentially affecting VSTPC, those most directly involved in the price formation process can be limited to the following four parameters: substantial volatility in the very recent past, scarce liquidity, a high volume of trades, and many SL orders. No other market parameters are assumed to directly take part in the process. These will be considered the pillars of the theory and discussed throughout the remainder of this article.

The remainder of this paper is organized as follows. "Literature review" section reviews the literature on the four parameters assumed to influence VSTPC; "Methodology" section describes the methodologies used to verify the influence of such parameters; "Theoretical foundations of very short time price changes (VSTPC)" section proposes a mathematical model; "Results of the data analysis" section analyzes audit-trail data at VST; "Results of the granger-causality test" section demonstrates the results of Granger-causality tests applied to real data; and "Results of the agent-based model (ABM) experiment" section presents the results of a computer simulation using an agent-based model (ABM). "Discussion" section discusses the results, and "Conclusion" section concludes.

Literature review

The existing literature on the drivers of market prices is abundant. Discovering what could generate the mythical *alpha* is the ultimate dream of traders, speculators, arbitrageurs, and academics alike. Yet, as stated earlier, most existing literature focuses on slow markets, so missing to investigate the drivers of HFT that, according to several authors, account for 50% or more of all transactions: Of course, a copious literature exists on this timely topic, but a comprehensive theory of price drivers at the subsecond scale has not yet been established. This paper attempts to fill this gap by incrementally building on previous studies while leaving aside those factors that display little impact at VST and interpreting the remaining drivers in the light of their capability to affect ultrafast price swings.

Volatility (V)

Market price volatility is nearly unanimously considered a measure of risk (Wen et al. 2019; Kou et al. 2021b), and it is often calculated as the standard deviation of the price over the time horizon under analysis. Many studies focus on the impact that volatility at time t has on volatility at time $t+1$, finding a strong relationship. Some of the most widely used models relate today's price to yesterday's. The standard RWM can be described as follows:

$$\mathrm{P_t} = \mathrm{P_{t-1}} + \varepsilon_\mathrm{t} \tag{1}$$

In this model, the price on day *t* is equal to the previous day's price, with the addition of a random disturbance, ε_t. The basis of the efficient market hypothesis is that the disturbance ε_t is totally random, and there is therefore no hope to gain an abnormal return by speculating on the stock market. This is a questionable statement, otherwise several thousand investors and speculators around the world would not attempt to make a living doing so. Whatever the truth, Eq. (1) is developed as follows (Gujarati and Porter 2009):

$$\mathrm{P_1} = \mathrm{P_0} + \varepsilon_1;\ \ \mathrm{P_2} = \mathrm{P_1} + \varepsilon_2 = \mathrm{P_0} + \varepsilon_1 + \varepsilon_2 \tag{2}$$

and therefore,

$$\mathrm{P_t} = \mathrm{P_0} + \sum \varepsilon_\mathrm{t} \tag{3}$$

$$\mathrm{E(P_t)} = \mathrm{E}\left(\mathrm{P_0} + \sum \varepsilon_\mathrm{t}\right) = \mathrm{P_0} \tag{4}$$

$$\mathrm{Var(P_t)} = \mathrm{Var}\left(\mathrm{P_0} + \sum \varepsilon_\mathrm{t}\right) = \mathrm{t}\sigma^2 \tag{5}$$

Equation (4) shows that in case of the RWM, the expected price over time is equal to the initial price, but its variance increases indefinitely as *t* grows (Eq. 5). Since all disturbances occurring at the generic time *t* are stored in price values for higher values of *t* via the summation element, the RWM is said to have an infinite memory: it is a nonstationary stochastic process. However, at VST, things are no longer so clear-cut, and the threshold seems to again be the barrier of the psychological time below which the human brain cannot grasp physical events. The math is similar, as in the RWM presented earlier, there is no reference to the length of time *t*, and therefore, months or microseconds do not affect the mathematical treatment of the equations. Yet, when the formulas are filled with real-world data, they show different behaviors according to the data frequency (Johnson and Zhao 2012). Zigrand et al. (2012) failed to find direct evidence of a positive impact of HFT on volatility, and inconclusive results were also claimed by Zervoudakis et al. (2012) and Brogaard (2010). Myers and Gerig (2014) studied the consequences of low-latency activity on a market, and their findings displayed a reduction in volatility. Similarly, Hasbrouck and Saar (2013) noticed that HFT tends to reduce short-term volatility. On the opposite side, Abrol et al. (2016) admitted that positive feedback loops driven by HFT activity can exacerbate price shocks and may increase systemic risk when certain events occur at a speed below human reaction time. Zhang (2010) positively correlated HFT with volatility, and research by Aldridge and Krawciw (2015) also correlated aggressive HFT strategies with volatility.

Liquidity (L)

After the May 6, 2010 so-called "Flash Crash," practitioners swiftly found an ideal culprit in HFT for charges of excessive liquidity consumption and withdrawal from liquidity provision under severe stress conditions. Yet, at the academic level, Groth (2011) found weak or no evidence of HFT withdrawing liquidity during periods of high volatility, and Myers and Gerig (2014) determined that higher liquidity was provided when HFT activity flourished. The study also reported more probabilities of transactions, which the authors interpret as a further indication of abundant liquidity directly linked to HFT activity. Benos and Sagade (2016) and Hagströmer and Nordén (2013) concluded that, in general, HF traders supply more liquidity than they consume. Jarnecic and Snape (2014) even found that HFT is capable of resolving temporary liquidity imbalances. Yet not all scholars share the same view of HFT on market parameters. Baron et al. (2012) investigated the profitability and HFT and found significantly higher earnings when trading aggressively and thus consuming liquidity than when quoting passive orders and, in so doing, providing liquidity. The motivation therefore seems strong for HF traders to absorb liquidity rather than provide it. Cvitanić and Kirilenko (2010) developed a mathematical model showing that HF traders tend to withdraw from providing liquidity during critical times. Buchanan (2015) took a balanced stance by stating the advantages and disadvantages of HFT: on one hand, the research highlights the bad practice of fleeting liquidity (i.e., liquidity that suddenly disappears after a transaction takes place), while on the other hand, it recognizes the deeper markets that HFT generates. Overall, the literature does not seem to agree regarding the effects of HFT on the supply of liquidity.

Volume or quantity[1] (Q)

The price–quantity relationship is important, according to Karpoff (1987), for four reasons: (1) better understanding market structure; (2) it supports drawing inferences about informational content; (3) insights about the empirical distribution of speculative prices; and (4) price variability affects the quantities traded in futures markets. Among the many papers he researched, Ying found that “(1) A small volume is usually accompanied by a fall in price. (2) A large volume is usually accompanied by a rise in price. (3) A large increase in volume is usually accompanied by either a large rise in price or a large fall in price.” (1966, p. 676). Karpoff (1987) found that most articles written between 1964 and 1987 supported a positive correlation between the absolute value of a price change and quantities exchanged. Overall, it seems that in times certainly not affected by HFT, quantities exchanged showed an influence over price changes. Godfrey et al. (1964) identified SL orders, together with buy-above-market orders, as causes of the correlation between quantities exchanged and the squared value of the daily open minus the close price. Tauchen and Pitts (1983) assume that the correlation between Q and $|\Delta p|$ increases with the variance of the daily rate of information flow. All the papers mentioned so far in this section attribute great importance to information. However, at a VST, information is not the main price change driver because the arrival of information is a rare event compared to their total number over the timespan of the observed period (whatever it is). At a VST, the only price drivers are those that impact the price directly (e.g., liquidity and SL orders) or those that depend on the trader but information about

[1] Since the initial “V,” introduced in Volatility (V)” section, refers to “Volatility,” to avoid confusion in the following, traded volumes will be referred to as “traded quantities” or “Q” with the same meaning of “volume” in this context.

which can be found directly in the market books, as is the case for volatility and quantities exchanged. Overall, Q is a potential VST price driver worth investigating further.

Stop-loss orders (S)

Despite having been criticized for presenting the Findings Regarding the Market Events of May 6, 2010 ("Final Report" CFTC-SEC 2010b) 6 months after the Flash Crash, it only took 12 days for the two Commissions to complete the Preliminary Report (CFTC-SEC 2010a) on the events of May 6, 2010. Being "preliminary," it did not reach definitive conclusions but highlighted several possible causes that, in some cases, further research at governmental and academic levels confirmed. One of the parameters identified by the Preliminary Findings Regarding the Market Events of May 6, 2010 ("Preliminary Report" CFTC-SEC 2010a) as a possible driver of the Flash Crash was the impact of SL orders. Some academic studies (Angel 2011; Leland 2011; Zigrand 2011 and other papers contributing to Foresight 2012) seriously considered the matter, although without in-depth quantitative analysis. As the Preliminary Report (CFTC-SEC 2010a) pointed out, "[a]n additional hypothesis as to why some securities suffered more severe declines than the broader market on May 6 is that they were particularly affected by stop-loss market orders" (p. 5) and "during times of extreme market volatility, the use of market orders when stop-loss levels are triggered could result in executions at aberrant prices if all other liquidity has already been exhausted" (p. A-12). The time seems right for investigating this issue in more depth. Unfortunately, the major stock exchanges make commercial information about SL orders unavailable; therefore, direct audit-trail data analysis seems impossible. Yet, there are two other ways to approach a solution, namely, an indirect data analysis and a computer-based simulation. Neither is as reliable as direct data analysis but, lacking the required data, they provide an acceptable approximation, as will be explained later.

Why a theory on VSTPC is important: threats to financial stability

VSTPC have a substantial effect on various aspects of the financial markets. Aside from allowing HF traders to accrue great and perhaps abnormal profits, financial authorities, regulators, and exchange executives worry about the threats that HFT may pose to financial stability. In the era of subsecond trading when things happen below the threshold of people's perception, financial stability is the most sought-after parameter. When stocks were exchanged under an oak, prices were written in chalk on a blackboard, and open-outcry floor-based trading was the norm, a well-trained eye could control all major aspects of the market. This is no longer the case. Much more is happening behind the scenes, either in dark pools or on the subsecond scale, such that no one possesses global oversight. During her testimony on severe market disruption before the governmental subcommittee in charge of investigating the Flash Crash, Mary Shapiro, the chair of the Securities and Exchange Commission (SEC), framed it most clearly (Shapiro 2010): trading technology has progressed far too and too fast for regulatory authorities to keep apace. If markets are not under the control of the surveillance authorities, any deviation from market stability may easily develop into a major crisis. Various countries have developed financial cultures that are heavily influenced by their history, so the concept of financial stability is defined differently by central banks around the world. However, without entering the debate about the subtleties of financial stability, a few parameters can be agreed upon as constituting the core of the matter.

Market efficiency

Although the theory states that future market prices cannot be known in advance, many market participants spend a lot of time attempting to falsify them: the RWM is a mantra only in academia; nobody really believes in it on the trading floor, otherwise, being a trading professional would be pointless. This is the "trader's paradox"—traders seek financial stability to carry out their strategies, but the very existence of a trading strategy implies that future prices can be foreseen—something that would deny market efficiency and therefore financial stability. The matter has been exacerbated by today's ultrafast trading.

Quick price discovery

The price must be right, as dealing at the wrong price might be risky. Yet, in recent times, trading at the right speed has been more important than knowing the right price. The process of price formation must be quick; noise is a disturbance. Yet, trading on noise could be profitable, even though instability could ensue (Burton and Sunit 2013).

Reasonable volatility

The financial market would not exist without volatility. Nobody would buy an asset knowing that he or she could only resell it later at the exact same price, yet financial regulators worldwide have the goal of preventing excessive volatility, as this is a major threat to financial stability. Anderson et al. stated that "although episodes of heightened volatility and short-term illiquidity are not necessarily threats to financial stability, they could become so if they were to persist, amplify or spill over" (2015, p. 4). A moderate level of volatility is what policymakers, exchanges, and nonspeculating investors seek.

Sufficient liquidity

Unlike volatility, liquidity is never too abundant a parameter: it indicates the ease of asset-to-cash conversion. Since investors consistently look for liquid assets, liquid markets attract clients—and their fees. In recent years, some newly created exchanges have proposed fee structures that are specifically designed to attract HF traders, which often play the role of liquidity suppliers. Deep markets are capable of mitigating price shocks, so depth is therefore a market stabilizer. Yet, ultrafast trading also has the capability of enabling ultrafast liquidity consumption, which may have unforeseen consequences on market stability.

Nonlinear sensitivities

Zigrand et al. (2012) identified three mechanisms that may generate instability: "[N]onlinear sensitivities to change (where small changes can have very big effects), incomplete information (where some agents in the market have more, or more accurate, knowledge than others), and internal "endogenous" risks based on feedback loops within the system" (p. 8). According to the authors, such mechanisms are particularly effective "when financial markets involve significant proportions of CBT" (p. 8), where CBT stands for "computer-based trading," a prerequisite of HFT.

Methodology

This research will proceed as follows: it will (1) establish a theoretical foundation, (2) conduct audit-trail data analysis, (3) perform Granger-causality tests, and (4) perform an ABM simulation.

Theoretical foundations of very short time price changes (VSTPC)

The four parameters identified as the pillars of the theory interact with each other either directly or indirectly. In the model, all changes in a variable and its consequences on the other variables will be discussed. It is therefore possible to gauge the direct or indirect relationship between cause and effect according to the following scheme.

(a) Direct relationship: A change in the denominator causes a change in the nominator. For example, an increase in the traded quantity causes liquidity to shrink, as the former directly acts on the latter.
(b) Indirect relationship: An increase in the number of SL orders executed augments the quantities exchanged and, because of (a) above, higher quantities traded diminish liquidity.
(c) Behavioral relationship: Cause and effect is mediated by the will or preferences of investors. When volatility is high, liquidity suppliers usually refrain from quoting limit orders because of the risk of being picked off by more informed investors. That diminishes the liquidity supplied. However, not all investors may share the same opinion; some may behave differently.
(d) The most indirect relationship occurs when cause and effect is mediated through more than one behavioral connection.

Data analysis

Data analysis must first define its object of study. A decision was made to restrict the simulation to one of the most significant markets—one that can display situations in which very large volatility spikes happened at VSTs. As the Flash Crash was characterized by the volatility spikes required to examine this phenomenon, the analysis of data from May 6, 2010 prompted the authors to select the E-Mini Standard & Poor's (S&P) 500 futures June 2010 contracts at the Chicago Mercantile Exchange (CME) as its object of study. On that day, many U.S. exchanges displayed erratic behavior, and, according to the Commodities and Futures Trading Commission (CFTC) and the SEC (CFTC-SEC 2010a, b), the CME was where it all began. Several authoritative studies (Kirilenko et al. 2017; Menkveld and Yueshen 2018) have also assigned the CME a central role in the crisis. From a practical viewpoint, the CME seems to be an appropriate choice as the object for this research as E-Mini S&P 500 futures are only traded on that market, which limits possible interactions with other venues. Moreover, the maximum price change at the CME on that day was around 9%, a significant drop, but nothing as extreme as what happened to other securities traded on other markets. Lastly, this study restricts the analysis to the 3.5 min between 18:42:00 GMT (the beginning of the sharpest drop) and 18:45:28.115 GMT (when the 5-s stop logic at the CME started) on May 6, 2010. Using a longer period may divert the focus from VST features and using a shorter one might overlook significant events.

Granger-causality tests

The correlation between variables only observes the link between data but does not accomplish the main task required by a theory, which is establishing a causal relationship between a phenomenon and its effect(s), which is the ultimate way to acquire knowledge about an event. Although statistics provides some causal models, their link to "causation" in the common sense of the word is doubtful, and the authors themselves insist on qualifying causation so that no one can mistake a statistical causal relationship for the one referenced in common language. To avoid confusion, Diebold (2007) prefers to call it "predictive causality" and Edward Leamer (cited in Gujarati and Porter 2009, p. 653) leans toward the term "precedence over causality." The same authors recognize that "the question of causality is deeply philosophical with all kinds of controversies. At one extreme are people who believe that "everything causes everything" and at the other are people who deny any existence of causation whatsoever. This text will not delve into such a philosophical debate. The definition of a Granger test states that if a regression including one more independent variables provides a better explanation of the dependent variable than the regression that does not include it, then the added variable "Granger-causes" the dependent one (Granger 1969). Granger tests will be run between (lower) volatility in an earlier period versus (higher) volatility in a later period, and then, one at a time, between liquidity, quantities exchanged, and SL orders versus volatility. This will make it possible to verify whether the independent variables Granger-cause the dependent one (i.e., volatility). Once again, the financial market analyzed is the CME on the date and time described in the Data Analysis section. The test regresses volatility during the first half of the period under observation (18:42:00.001 to 18:43:44.057) onto the second half of the period (18:43:44.058 to 18:45:28.114), whereas liquidity, exchanged quantities, and SL orders will be regressed over the whole period. All times are given in Universal Time Coordinated or Greenwich Mean Time (GMT).

Agent-based modeling: a simulation

Simulations are characterized by several desirable features. Computer science has developed rapidly over the last 50 years, and algorithms are known to be rigorous mathematical abstractions (Knuth 1985). This experiment has been divided in two scenarios: a slow market with no HFT activity (i.e., 0%) and a rapid market with increasing percentages of HFT (i.e., 33%, 50%, 67%, 75%, and 90% of all activity). These simulations are run under eight different conditions: (1) a random walk (RW); (2) a trend (i.e., high volatility); (3) a RW with a quantity (QTY) effect; (4) a trend with a QTY effect; (5) a RW with an SL effect; (6) a trend with an SL effect; (7) a RW with both a QTY and an SL effect; and (8) a trend with both a QTY and an SL effect.

Theoretical foundations of very short time price changes (VSTPC)

The equations for liquidity, quantity, SL orders, and volatility at VSTs are described below.

Liquidity [L = L (Q, S, V, t)]

Liquidity is mostly relevant at the "top of book," that is, at lowest ask and highest bid prices. However, in the case of stressed markets, liquidity at higher ask and lower bid levels may also be relevant.

(a) A high quantity exchanged drives liquidity down, as each trade will reduce liquidity to a greater extent than in the case of a low quantity exchanged, as follows:

$$\frac{\partial L}{\partial Q} < 0 \tag{6}$$

(b) Volatility is a diminishing factor for liquidity, as a high level of volatility tends to scare investors and drive them away from quoting limit orders (which increase liquidity). It must be said that, both in general and at VSTs, traders welcome a certain degree of volatility as it will facilitate their ability to close a position profitably. However, given the tiny profits with which HF traders are content, a low level of volatility will suffice. When volatility grows substantially, risk-averse liquidity suppliers step back, shrinking liquidity, regardless of trading speed.

$$\frac{\partial L}{\partial V} < 0 \tag{7}$$

(c) SL orders, which are converted into market orders when the price reaches the trigger level, will increase the quantity traded (see "Quantity exchanged [Q = Q (L, S, V, t)]" section) and therefore consume liquidity.

$$\frac{\partial L}{\partial S} = \frac{\partial L}{\partial Q} \cdot \frac{\partial Q}{\partial S} < 0 \tag{8}$$

Liquidity displays an inverse relationship with the three other market parameters.

Quantity exchanged [Q = Q (L, S, V, t)]

(a) SL orders directly increase the quantity traded since when the price reaches the trigger level, they would automatically be transformed into market orders that, on execution, increase the quantity traded.

$$\frac{\partial Q}{\partial S} > 0 \tag{9}$$

(b) The quantity exchanged has a direct relationship with liquidity, as well as a behavioral one: a low level of liquidity discourages trading high quantities, as the trading price would penalize large orders, and a high level of liquidity would precipitate the opposite effect.

$$\frac{\partial Q}{\partial L} > 0 \tag{10}$$

(c) The matter is less clear-cut regarding the implications of volatility on Q. Unidirectional volatility, which is regarded as risky, would discourage large trading quantities, whereas bidirectional volatility would encourage a high level of trading, as market orders are more likely to be profitable, especially at VSTs. But, again, as in the case of liquidity, a high level of volatility, even though bidirectional, might reduce both limit and market orders, as turbulent markets tend to repel investors. Again, the relationship is mainly behavioral.

$$\frac{\partial Q}{\partial V} = \frac{\partial Q}{\partial L} \cdot \frac{\partial L}{\partial V} < 0 \tag{11}$$

Stop-loss orders [S = S (L, Q, V, t)]

(a) When volatility is high, the trigger price of SL orders is more likely to get hit. It can be argued that volatile markets tend to scare investors, whereas SL is a safe, risk-averse measure, and this may support the thesis of the behavioral relationship between the two market parameters. However, there is a difference between executed SL orders (those considered in Eq. 12) and quoted SL orders, which certainly depend on behavioral factors but are not considered by any of the equations that describe VSTPC.

$$\frac{\partial S}{\partial V} > 0 \tag{12}$$

The relationship between SL orders and volatility is direct.

(b) The relationship between liquidity and SL orders is indirect, in as much as a low level of liquidity leads to greater volatility (as will be seen in "Volatility [V = V (L, Q, S, t)]" section), which, in turn, triggers outstanding SL orders.

$$\frac{\partial S}{\partial L} = \frac{\partial S}{\partial V} \cdot \frac{\partial V}{\partial L} < 0 \tag{13}$$

(c) Similarly, exchanging high quantities consumes liquidity, and the scenario described by Eq. (13) repeats. These are two examples of triangular relationships, so it can be concluded that SL orders depend directly only on volatility; the other two dependencies are indirect.

$$\frac{\partial S}{\partial Q} = \frac{\partial S}{\partial V} \cdot \frac{\partial V}{\partial L} \cdot \frac{\partial L}{\partial Q} > 0 \tag{14}$$

Table 1 Categorization of the cause-effect relationship for the four VSTPC parameters

Direct	Indirect	Behavioural	Indirect behavioural
(1) $\frac{\partial L}{\partial Q} < 0$	(5) $\frac{\partial L}{\partial S} = \frac{\partial L}{\partial Q}\frac{\partial Q}{\partial S} < 0$	(10) $\frac{\partial L}{\partial V} < 0$	
(2) $\frac{\partial Q}{\partial S} > 0$		(11) $\frac{\partial Q}{\partial L} > 0$	(12) $\frac{\partial Q}{\partial V} = \frac{\partial Q}{\partial L}\frac{\partial L}{\partial V} < 0$
(3) $\frac{\partial S}{\partial V} > 0$	(6) $\frac{\partial S}{\partial L} = \frac{\partial S}{\partial V}\frac{\partial V}{\partial L} < 0$ (7) $\frac{\partial S}{\partial Q} = \frac{\partial S}{\partial V}\frac{\partial V}{\partial L}\frac{\partial L}{\partial Q} > 0$		
(4) $\frac{\partial V}{\partial L} < 0$	(8) $\frac{\partial V}{\partial Q} = \frac{\partial V}{\partial L}\frac{\partial L}{\partial Q} > 0$ (9) $\frac{\partial V}{\partial S} = \frac{\partial V}{\partial L}\frac{\partial L}{\partial Q}\frac{\partial Q}{\partial S} > 0$		

Volatility [V = V (L, Q, S, t)]

Volatility is often calculated as the standard deviation of price. Yet, the change of a variable over a small change in another variable (whether it is time or otherwise) leads to the concept of the first derivative. Since the focus of algorithmic trading rests with the price change over a short time, it makes sense to consider the first derivative of the price over time as the most appropriate definition of price volatility. In the following, the terms "volatility" and "price movement" will be used interchangeably.

(a) Volatility depends inversely on liquidity, as a lower level of liquidity is more likely to move the price.

$$\frac{\partial V}{\partial L} < 0 \tag{15}$$

(b) Volatility depends indirectly on the quantity exchanged as it tends to change the price by consuming liquidity (Eqs. (6) and (15)).

$$\frac{\partial V}{\partial Q} = \frac{\partial V}{\partial L} \cdot \frac{\partial L}{\partial Q} > 0 \tag{16}$$

(c) Many outstanding SL orders that are converted into aggressive orders when the price reaches the trigger level precipitates greater traded quantities, which, in turn, affects existing liquidity, the reduction of which may potentially move the price.

$$\frac{\partial V}{\partial S} = \frac{\partial V}{\partial L} \cdot \frac{\partial L}{\partial Q} \cdot \frac{\partial Q}{\partial S} > 0 \tag{17}$$

Cause–effect relationship

Table 1 summarizes the equations above and categorizes them according to the four degrees of directness/indirectness defined above (i.e., direct, indirect, behavioral, indirect behavioral).

From Table 1, it appears that only relationships (1) to (4) are direct, all others are either a combination of direct relationships (indirect relationships), depend on human judgment, tastes, preferences, or a risk-averse attitude (behavioral), or represent a combination of behavioral relationships (indirect behavioral). This means that relationships (1) to (4) suffice to explain "mechanical" market dynamics that are not

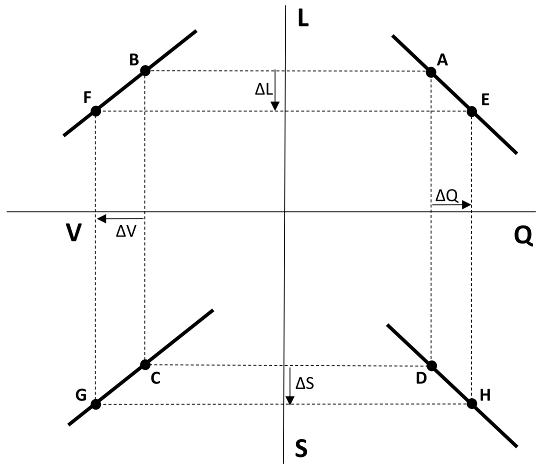

Fig. 1 Reaching a new equilibrium in case of change in one market parameter

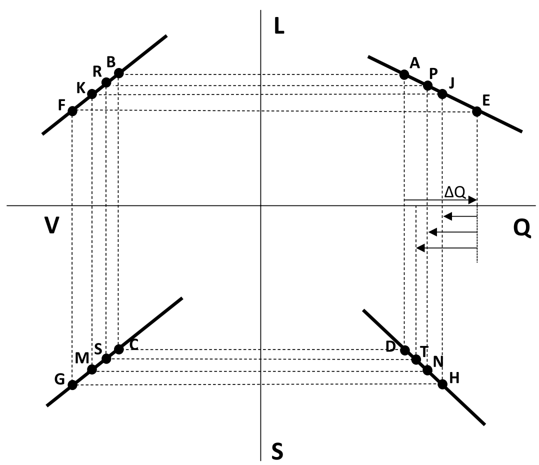

Fig. 2 Converging to the old equilibrium in case of rotation of the Q-L line

mediated by other factors. Figure 1 depicts an equilibrium status, as represented by Points A, B, C, and D.

Should any one of the four parameters change, a new equilibrium status will be reached. If, for example, investors decide to increase the amount of their market orders, represented by ΔQ, this change will cause liquidity to diminish (Point E), with the consequence of making price jumps more frequent and increasing volatility by ΔV (Point F). More price jumps will trigger more SL executions (under the *ceteris paribus* assumption, quoted SL orders remain constant), ΔS (Point G), and the system will settle at a new equilibrium quantity of trades at Point H. A change in the value of a parameter causes a shift from one equilibrium to another in the case of an equal change in both variables, which is depicted by unitary elasticity. However, unitary elasticity is a rather rare occurrence that cannot be taken for granted. An endogenous change (a change in the value of the variable on either axis) will result in a shift of the point along the line, but an exogenous change will display a different behavior. Investors can act on market parameters in several ways. Large orders can be split into

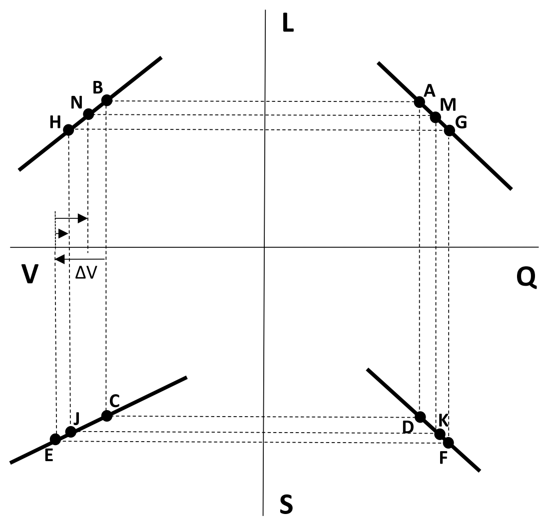

Fig. 3 Converging to old equilibrium in case of clockwise rotation of the V-S line

several smaller ones with the purpose of impacting liquidity and therefore market price less. This results in a smaller negative value in the slope of the Q–L line (Fig. 2).

The rationale behind the line rotation is that at the same level of quantity traded, liquidity will suffer less, as more time will be given to market-makers to provide it. If, from the equilibrium status A–B–C–D, the mood of investors shifts and they decide to increase the size of their market orders by ΔQ, new dynamics develop by lowering liquidity (Point E) and increasing volatility (Point F), which raises SL order execution (Point G). However, the new level of quantity traded (Point H) is lower than the original value plus ΔQ. Thus, liquidity increases again to Point J, volatility decreases to K, as does SL executions (Point M). The cycle starts again, going through Points N, P, R, S, and T until eventually the original equilibrium (A–B–C–D) is reached again. This scheme works in both directions with opposite results. It does not make much sense to investigate the opposite Q–L scenario as it would imply that several investors joined their market orders with the purpose of negatively impacting liquidity, which is clearly unrealistic. A more realistic double-sided scenario materializes when considering SL order quoting. Optimistic investors may decide not to protect their trades with SL orders as they believe that even adverse price movements will only be temporary and that in the longer term (albeit, still at the subsecond scale), their trading will be profitable. This scenario may be represented by a V–S line with a lower slope, as in Fig. 3.

The clockwise rotation of the V–S line implies that an increase in volatility, ΔV, would result in a lower number of SL executions (Point E) and therefore in a smaller traded quantity (Point F). This results in lower liquidity (Point G), yet it is higher than in the case of unitary V–S elasticity. A new cycle then begins, going through Points H, J, K, M, and N, getting closer and closer to the original A–B–C–D cycle.

Quite a different scenario is depicted in the case of pessimistic investors that experience more uncertainty and therefore decide to protect their trades by quoting more SL orders (Fig. 4).

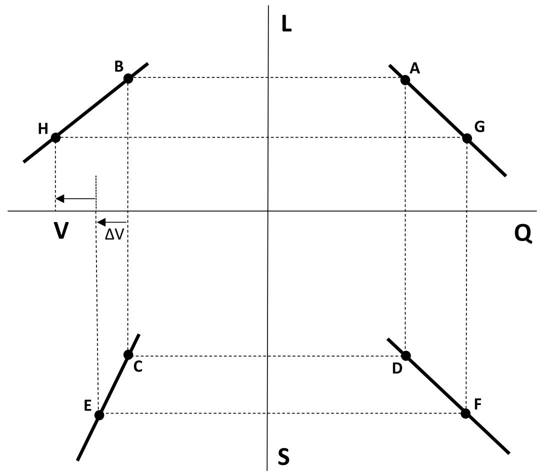

Fig. 4 Diverging parameters in case of anti-clockwise rotation of the V-S line

Table 2 ARCH(p) model detecting volatility clustering on the day/time of audit trail data

Lags	$(X_{t-1})^2$	$(X_{t-2})^2$	$(X_{t-3})^2$	$(X_{t-4})^2$	$(X_{t-5})^2$
1	18.869***	17.745***	14.511***	14.569***	14.468***
2		17.489***	14.116***	13.862***	13.082***
3			91.218***	90.914***	90.631***
4				− 1.989**	− 2.180**
5					2.503**

Results for lags = 1 to 3 display volatility clustering (at 1%, indicated as ***) whereas at higher lags ARCH yields oscillating results (sign changes) and significance only at 5% (indicated as **). Since the number of observations is 208,114, critical t-values are: 2.576 (at 1%) and 1.96 (at 5%)

An initial increase in volatility, ΔV, as in the previous case, leads to even higher SL order execution (Point E) and equally higher quantities traded (Point F). The resulting lower liquidity (Point G) would cause volatility to increase even more (Point H), generating an outward-spiraling sequence.

Results of the data analysis

Audit-trail data show that volatility did occur in bursts. Table 2 displays the results of an ARCH(p) model with lags 1 to 5; volatility bursts appear when regressing the model up to three lags.

Some studies (CFTC-SEC 2010a and a few papers in the Foresight 2012 set) mentioned SL orders in the context of the severe liquidity shortage and the surge in abnormal volatility experienced on May 6, 2010. Direct SL observation is not possible as the required level of detail is not publicly available. However, it is possible to use a proxy to estimate the impact of SL orders on extreme volatility. A suitable proxy is the length of a "run" —a run is an uninterrupted series of homogeneous trades—and its length is the number of trades occurring within that run.[2] To evaluate the likelihood of the role played by SL orders in trade runs, it is appropriate to compare two runs of

[2] For a more detailed description see Virgilio (2020).

Table 3 Run rates

Date	03-May	04-May	05-May	06-May	07-May
Runs	11,399	9293	8843	12824	6656
Runs/sec	1.7	2.6	3.0	33.1	3.1

Table 4 Comparison of liquidity

Date	03-May	04-May	05-May	06-May	07-May
Contracts	17,433	15,303	12,359	310	2659
USD value	1,045,293,463	893,156,475	715,793,213	16,638,063	147,680,563

Table 5 Large price movements within a run on May 6, 2010

Delta price	4.50	4.00	3.50	3.25	2.75	2.50	2.25	1.75	1.75	1.50	1.25
Occurrences	1	1	1	2	1	2	3	3	6	9	9

length 8 that occurred on 5/5/2010 on the E-Mini S&P 500 futures contracts with an expiration date of June 2010 on the CME. The first run started at 18:45:51.319 GMT and lasted 75 ms, whereas the second run started and terminated at 18:27:27.115. Although, as a matter of principle, it is not possible to rule out investor's orders in both cases, it seems unlikely to categorize eight trades occurring within the same millisecond as exogenous. In the same way, it seems unreasonable to identify an endogenous mechanism, such as a sequence of SL order executions, that is spread over a 75-ms period. Therefore, the assumption is that runs spanning a short amount of time were SL-driven and those distributed over comparably longer periods experienced external intervention. In any case, even long runs do not necessarily imply an increase in volatility if all or most of the quantities traded are absorbed by existing liquidity.

Number of trade runs

As seen above, long trade runs over a VST provide an indication of SL orders being executed. Table 3 shows the number of runs executed over the same number of events (580,864 at around the same time) on five consecutive days between May 3 and May 7, 2010.

The day affected by the highest volatility (May 6) not only experienced the most runs, but also the greatest run/second rate. This confirms the close relationship between SL orders and volatility, especially when liquidity is scarce, and it also suggests performing further tests to verify the cause–effect relationship between SL orders and abnormal volatility (this will be the subject of the next section). Table 4 displays a comparison of liquidity (at the same time) on different days in the same week.

In all tables shown in this chapter, May 7 was not a business-as-usual day as investors were understandably still shocked by the previous day's events.

Table 6 Price changes within runs

Date	03-May	04-May	05-May	06-May	07-May
Runs showing price change	6	27	26	476	62

Max price drop within a run

It is interesting to analyze the price differences within a run. As shown in Table 5, on the most illiquid day (May 6), the maximum price drop was 4.5 index points (one occurrence) for a value of 225 US dollars (USD) (1 E-Mini index point = 50 USD), followed by other large price movements. On the remaining days of the same week, the maximum price drop was 0.25 (on May 3, 4, and 5) or 0.5 index points (on May 7).

This suggests that the combination of scarce liquidity and SL orders might unchain volatility spikes.

Number of runs showing price jumps

A further confirmation of the above is provided by Table 6, which displays the number of runs that experienced a price change. Again, the day displaying the highest illiquidity and most SL activity experienced, by far, the most price movements within a run.

This analysis shows, once again, that substantial SL activity and scarce liquidity are strongly correlated to volatility, in this case, as described by the number of price changes that occurred within runs.

Discussion

The analysis in this section is affected by a lack of detailed data and therefore approximation was unavoidable to estimate SL activity. Considering this, the results indicate that the events experienced during the time characterized by abnormal volatility are related to SL orders and decreasing liquidity. Going one step further, it is possible to argue that volatility spikes can be caused by a combination of three apparently innocuous factors: falling prices, scarce liquidity, and heavy SL activity. Each of these is a relatively common occurrence in the market without necessarily giving rise to a memorably volatile day, and two such conditions can appear simultaneously without leading to a critical situation. But when all three conditions appear simultaneously, the chances seem better that a volatility crisis will materialize. To verify the existence of a cause–effect relationship, it is necessary to perform a formal causal test. This is the topic of the next section.

Results of the granger-causality test

This section investigates Granger-causality between the four variables identified and excess volatility. As in most time series, all the variables used for these tests are nonstationary; therefore, first differences have been used. The first differences for all four datasets displayed stationary behavior.

Test the Granger-causality of volatility on extreme volatility

Volatility has been computed using the top-of-book price differences reported at 1-ms (msec) intervals. The hypothesis is that between 18:42:00.000 and 18:45:28.114, volatility in the first half of the period (18:42:00.000 through 18:43:44.057) somehow influenced

Table 7 Granger-causality test on volatility at 10^{-3} s

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Volatility at 1/1000 s				
1	1.14118	0.28541	0.00166	CANNOT reject at 25%
2	1.36304	0.25589	0.00294	CANNOT reject at 25%
3	1.29382	0.27456	0.00761	CANNOT reject at 25%
4	0.97760	0.41828	0.00759	CANNOT reject at 25%
5	0.79484	0.55314	0.00837	CANNOT reject at 25%
6	1.60233	0.14189	0.00871	CANNOT reject at 25%
7	1.44409	0.18252	0.00925	CANNOT reject at 25%
8	5.88033	0.00000	0.00985	REJECT at 1%

Table 8 Granger-causality test on volatility at 10^{-2} s

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Volatility at 1/100 s				
1	3.01818	0.08237	0.02048	CANNOT reject the null hypothesis at 5%
2	4.72813	0.00886	0.02682	REJECT the null hypothesis at 1%
3	4.37457	0.00439	0.03174	REJECT the null hypothesis at 1%
4	3.45788	0.00788	0.03307	REJECT the null hypothesis at 1%

volatility in the second half (18:43:44.058 through 18:45:28.114). This means that all traders, both slow and fast, noticed that volatility was high for 1 min and 44 s, and this caused them to become nervous during the remaining 1 min and 44 secs before the execution of stop logic. This is intuitive and confirmed by common experience: the price of the E-mini dropped 14.50 index points (725 USD per contract, or more than 1.3%, in less than 2 min) in the first period, and this allegedly led to a further drop of nearly 4% (43.50 index points, or 2175 USD per contract) in the second period. Although it may not be surprising that a drop over a short time leads to a larger drop in a subsequent short period, the figures demonstrate a dramatic event unfolding. According to the Granger test, the hypothesis of Granger causality at the millisecond level with 8-lag cannot be rejected (Table 7). Correspondingly, at a hundredth of a second, Granger causality yields the same result as at 2-lag (when the first four lags are displayed in Table 8). Both tests show Granger causality between a previous period's volatility and the later period, as per the equations below.

$$V_t = \sum_{i=1}^{t/2} \alpha_i V_{t-i} + u_t; \quad V_t = \sum_{i=1}^{t/2} \alpha_i V_{t-i} + \sum_{j=t/2+1}^{t-1} \beta_j V_{t-j} + u_t \tag{18}$$

Test the Granger causality of a liquidity shortage on extreme volatility

Yet, further investigation showed that volatility during the previous minutes was not the only cause of the extreme volatility experienced later. Scarce liquidity was another

Table 9 Granger-causality test on liquidity-to-volatility at 3 liquidity levels

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Liquidity → volatility (3 levels)				
1	2139.38867	0.00000	0.01215	REJECT the null hypothesis at 5%
2	1203.54114	0.00000	0.01472	REJECT the null hypothesis at 1%
3	813.52325	0.00000	0.01900	REJECT the null hypothesis at 1%
4	610.98523	0.00000	0.01901	REJECT the null hypothesis at 1%

Table 10 Granger-causality test on liquidity-to-volatility at 5 liquidity levels

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Liquidity → volatility (5 levels)				
1	2321.79102	0.00000	0.01300	REJECT the null hypothesis at 5%
2	1221.69885	0.00000	0.01489	REJECT the null hypothesis at 1%
3	834.84369	0.00000	0.01930	REJECT the null hypothesis at 1%
4	626.72125	0.00000	0.01931	REJECT the null hypothesis at 1%

Table 11 Granger-causality test on liquidity-to-volatility at 7 liquidity levels

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Liquidity → volatility (7 levels)				
1	2266.32373	0.00000	0.01274	REJECT the null hypothesis at 5%
2	1214.91748	0.00000	0.01482	REJECT the null hypothesis at 1%
3	826.32660	0.00000	0.01918	REJECT the null hypothesis at 1%
4	620.29791	0.00000	0.01919	REJECT the null hypothesis at 1%

Table12 Granger-causality test on liquidity-to-volatility at 10 liquidity levels

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Liquidity → volatility (10 levels)				
1	1886.33667	0.00000	0.01096	REJECT the null hypothesis at 5%
2	1044.57202	0.00000	0.01323	REJECT the null hypothesis at 1%
3	710.26221	0.00000	0.01755	REJECT the null hypothesis at 1%
4	533.07294	0.00000	0.01756	REJECT the null hypothesis at 1%

strong candidate to share responsibility for the event. Therefore, another Granger test was run on the liquidity values between 18:42:00.000 and 18:45:28.114, considering liquidity at the top 3, top 5, top 7, and top 10 levels of the bid book. The results are reported in Tables 9, 10, 11 and 12, respectively. The results show that liquidity did Granger-cause a peak in volatility. The equations are as follows:

Table 13 Granger-causality test on volatility-to-liquidity at 3 liquidity levels

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Volatility → liquidity (3 levels)				
1	4.75698	0.02918	0.00584	REJECT the null hypothesis at 5%
2	6.59386	0.00137	0.00686	REJECT the null hypothesis at 1%
3	6.73184	0.00015	0.00762	REJECT the null hypothesis at 1%
4	6.95852	0.00001	0.00823	REJECT the null hypothesis at 1%

Table 14 Granger-causality test on volatility-to-liquidity at 5 liquidity levels

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Volatility → liquidity (5 levels)				
1	18.41980	0.00002	0.004444	REJECT the null hypothesis at 1%
2	31.42196	0.00000	0.00600	REJECT the null hypothesis at 1%
3	27.73387	0.00000	0.674	REJECT the null hypothesis at 1%
4	23.12213	0.00000	0.007466	REJECT the null hypothesis at 1%

Table 15 Granger-causality test on volatility-to-liquidity at 7 liquidity levels

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Volatility → liquidity (7 levels)				
1	8.74758	0.00310	0.00679	REJECT the null hypothesis at 1%
2	21.10105	0.00000	0.00954	REJECT the null hypothesis at 1%
3	17.99542	0.00000	0.00975	REJECT the null hypothesis at 1%
4	15.44412	0.00000	0.01032	REJECT the null hypothesis at 1%

Table 16 Granger-causality test on volatility-to-liquidity at 10 liquidity levels

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Volatility → liquidity (10 levels)				
1	5.66481	0.01731	0.00661	REJECT the null hypothesis at 5%
2	7.06459	0.00086	0.00737	REJECT the null hypothesis at 1%
3	5.91824	0.00049	0.00755	REJECT the null hypothesis at 1%
4	6.17625	0.00006	0.00823	REJECT the null hypothesis at 1%

$$V_t = \sum_{i=1}^{t-1} \alpha_i V_{t-i} + u_t; \quad V_t = \sum_{i=1}^{t-1} \alpha_i V_{t-i} + \sum_{j=1}^{t-1} \beta_j L_{t-j} + u_t \tag{19}$$

and

$$L_t = \sum_{i=1}^{t-1} \alpha_i L_{t-i} + u_t; \quad L_t = \sum_{i=1}^{t-1} \alpha_i L_{t-i} + \sum_{j=1}^{t-1} \beta_j V_{t-j} + u_t \tag{20}$$

Table 17 Granger-causality test on quantity exchanged-to-volatility

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Quantity → volatility				
1	21.06322	0.00000	0.01453	REJECT the null hypothesis at 1%
2	12.93685	0.00000	0.01871	REJECT the null hypothesis at 1%
3	10.66973	0.00000	0.01989	REJECT the null hypothesis at 1%
4	9.04931	0.00000	0.02146	REJECT the null hypothesis at 1%

Although the tests provide evidence that liquidity Granger-caused the increase in volatility, this is not the end of the story.

The same test should be performed in the other direction to verify whether there exists a bidirectional Granger causation effect. Should that be the case, the possibility of a third factor causing both effects examined would be a serious possibility. Granger tests claim success only in the case of one-directional Granger causation. Tables 13, 14, 15 and 16 show the results of Granger tests on the volatility-to-liquidity regression for 3, 5, 7, and 10 liquidity levels, respectively.

The volatility-to-liquidity tests also suggest Granger causality. However, a deeper analysis should dispel all doubts about the actual directionality of the Granger causation: this is provided by the *F*-value statistics. The comparison between the two sets of tests is unambiguous: *F*-values for the liquidity-to-volatility tests range from 2139 (at three levels of liquidity) to 2322 (at five liquidity levels), to 2266 (at seven levels), to 1886 (at 10 levels). The strongest Granger causation occurs at five liquidity levels. The opposite case is much less clear-cut. It still displays Granger causation, but the *F*-values range from 4.76 (at three liquidity levels), to 18.42 (at five levels), to 8.75 (at seven levels), to 5.66 (at 10 levels). Again, the strongest result corresponds to five liquidity levels, somehow confirming the result of the previous case. The two sets of *F*-values are scarcely comparable; the ratio between the *F*-values at the same liquidity level always yields results with two orders of magnitude: 450 at three liquidity levels, 126 at five levels, 259 at seven, and 333 at 10. It can therefore be stated that liquidity does Granger-cause volatility.

Test the Granger causality of quantity exchanged on extreme volatility

The quantity exchanged is another potential driver of VSTPC. A Granger test has been run on quantity and volatility to verify if any type of Granger causality exists using the following equations.

$$V_t = \sum_{i=1}^{t-1} \alpha_i V_{t-i} + u_t; \quad V_t = \sum_{i=1}^{t-1} \alpha_i V_{t-i} + \sum_{j=1}^{t-1} \beta_j Q_{t-j} + u_t \tag{21}$$

$$Q_t = \sum_{i=1}^{t-1} \alpha_i Q_{t-i} + u_t; \quad Q_t = \sum_{i=1}^{t-1} \alpha_i Q_{t-i} + \sum_{j=1}^{t-1} \beta_j V_{t-j} + u_t \tag{22}$$

The results of testing whether quantity exchanged Granger-caused volatility in the period between 18:42:00.000 and 18:45:28.114 for lags 1 through 4 are shown in Table 17.

Table 18 Granger-causality test on volatility-to-quantity exchanged

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Quantity $\rightarrow$ *volatility*				
1	14.83718	0.00012	0.01453	REJECT the null hypothesis at 1%
2	9.34719	0.00009	0.01871	REJECT the null hypothesis at 1%
3	7.35734	0.00006	0.01989	REJECT the null hypothesis at 1%
4	5.53375	0.02154	0.02146	REJECT the null hypothesis at 1%

Again, running the same test in the opposite direction provides a useful indicator against the robustness of the result. Table 18 displays the outcome of such tests.

In this case, contrary to testing previous volatility or liquidity, the result is not definitive. Quantity exchanged seems to Granger-cause volatility and the volatility-to-quantity relationship also seems true. Whereas in the liquidity case, the *F*-values were always two orders of magnitude greater than the liquidity-to-volatility causal direction, in the quantity exchanged case, the ratio is just 1.4. This does not suggest as strongly as in the previous case a unidirectional Granger causation between quantity exchanged and volatility. Therefore, based on this test, it cannot be stated that quantity exchanged Granger-causes volatility, nor the other way around. A third factor is the likely cause of the quantity-to-volatility relationship.

Test the granger causality of trade runs on extreme volatility

Another factor that may have significantly contributed to the volatility spike on the day under observation is the triggering of numerous SL orders. This is a delicate issue, as commercially available data provided by the main exchanges do not report postings of SL orders or the trading orders generated by SL triggering. Therefore, the existence of SL orders can only be deduced by other indicators. As in the Data Analysis section, the indicator used in this research is the length of a "run." As seen earlier, a "run" is an uninterrupted sequence of trades, all in the same direction (i.e., all buy, or all sell). A run could simply be the outcome of many human- or computer-generated aggressive orders all arriving at the exchange within a VST span. Alternatively, it could be the consequence of several SL orders being triggered because the price had reached an established level, coupled with insufficient liquidity to absorb the market orders generated by the SL mechanism. Lacking information in the audit-trail data about SL orders, it can only be inferred heuristically whether a run was SL-generated, and the only available discriminating criterion is the run length—the number of trades within a run. This is by no means a precise discriminating criterion, and there is no "critical" number above which it can be surely stated that the SL mechanism was triggered; therefore, the Granger test takes into consideration several different run lengths, ranging from $5+$ to $10+$. It uses the following equations:

$$V_t = \sum_{i=1}^{t-1} \alpha_i V_{t-i} + u_t; \quad V_t = \sum_{i=1}^{t-1} \alpha_i V_{t-i} + \sum_{j=1}^{t-1} \beta_j S_{t-j} + u_t \tag{23}$$

$$S_t = \sum_{i=1}^{t-1} \alpha_i S_{t-i} + u_t; \quad S_t = \sum_{i=1}^{t-1} \alpha_i S_{t-i} + \sum_{j=1}^{t-1} \beta_j V_{t-j} + u_t \tag{24}$$

Tables 19, 20, 21, 22, 23 and 24 show results of Granger tests on SL orders to volatility for different run lengths.

In this case, the outcome is unambiguous, as the volatility-to-SL orders parameter results in no Granger causality, as shown in Tables 25, 26, 27, 28, 29 and 30.

Table 19 Granger-test on stop-loss orders-to-volatility at run length > 10

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Stop-loss assumed at run length > 10				
1	14.44016	0.00017	0.21697	REJECT the null hypothesis at 1%
2	7.94687	0.00042	0.26988	REJECT the null hypothesis at 1%
3	5.45041	0.00112	0.28749	REJECT the null hypothesis at 1%
4	2.76547	0.02736	0.32958	REJECT the null hypothesis at 1%

Table 20 Granger-test on stop-loss orders-to-volatility at run length > 9

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Stop-loss assumed at run length > 9				
1	15.84370	0.00008	0.22649	REJECT the null hypothesis at 1%
2	7.94347	0.00041	0.27563	REJECT the null hypothesis at 1%
3	6.42414	0.00029	0.30054	REJECT the null hypothesis at 1%
4	3.41960	0.00912	0.33316	REJECT the null hypothesis at 1%

Table 21 Granger-test on stop-loss orders-to-volatility at run length > 8

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Stop-loss assumed at run length > 8				
1	14.38607	0.00017	0.21663	REJECT the null hypothesis at 1%
2	7.88797	0.00043	0.27396	REJECT the null hypothesis at 1%
3	6.53369	0.00025	0.30157	REJECT the null hypothesis at 1%
4	3.34332	0.01030	0.33745	REJECT the null hypothesis at 1%

Table 22 Granger-test on stop-loss orders-to-volatility at run length > 7

Lag	F-value	*p*-value	Adj.R^2	Null hypothesis of NO Granger causation
Stop-loss assumed at run length > 7				
1	13.00856	0.00034	0.17530	REJECT the null hypothesis at 1%
2	7.48942	0.00062	0.22547	REJECT the null hypothesis at 1%
3	5.54560	0.00094	0.24020	REJECT the null hypothesis at 1%
4	3.33697	0.01031	0.24406	REJECT the null hypothesis at 1%

Table 23 Granger-test on stop-loss orders-to-volatility at run length > 6

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Stop-loss assumed at run length > 6				
1	13.46431	0.00026	0.10511	REJECT the null hypothesis at 1%
2	6.63166	0.00141	0.18795	REJECT the null hypothesis at 1%
3	4.79999	0.00259	0.19730	REJECT the null hypothesis at 1%
4	4.53995	0.00128	0.25952	REJECT the null hypothesis at 1%

Table 24 Granger-test on stop-loss orders-to-volatility at run length > 5

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Stop-loss assumed at run length > 5				
1	15.40207	0.00010	0.10890	REJECT the null hypothesis at 1%
2	8.20498	0.00030	0.17590	REJECT the null hypothesis at 1%
3	5.74544	0.00069	0.17985	REJECT the null hypothesis at 1%
4	4.83712	0.00074	0.21412	REJECT the null hypothesis at 1%

Table 26 Granger-test on stop-loss orders-to-volatility at run length > 9

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Stop-loss assumed at run length > 9				
1	0.37895	0.53850	0.00000	CANNOT reject the null hypothesis at 25%
2	0.43564	0.64714	0.00000	CANNOT reject the null hypothesis at 25%
3	0.42503	0.73516	0.00000	CANNOT reject the null hypothesis at 25%
4	0.28310	0.88891	0.00000	CANNOT reject the null hypothesis at 25%

Table 27 Granger-test on stop-loss orders-to-volatility at run length > 8

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Stop-loss assumed at run length > 8				
1	0.55911	0.45500	0.00000	CANNOT reject the null hypothesis at 25%
2	0.51377	0.59858	0.00000	CANNOT reject the null hypothesis at 25%
3	0.57382	0.63247	0.00000	CANNOT reject the null hypothesis at 25%
4	0.37309	0.82785	0.00000	CANNOT reject the null hypothesis at 25%

Although Tables 19, 20, 21, 22, 23 and 24 and Tables 25, 26, 27, 28, 29 and 30 only display results for the first four lags, Granger tests have been executed for up to 32 lags with consistent outcomes.

Table 25 Granger-test on stop-loss orders-to-volatility at run length > 10

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Stop-loss assumed at run length > 10				
1	0.00266	0.95886	0.00000	CANNOT reject the null hypothesis at 25%
2	0.31532	0.72975	0.00000	CANNOT reject the null hypothesis at 25%
3	0.20240	0.89471	0.00000	CANNOT reject the null hypothesis at 25%
4	0.22671	0.92339	0.00000	CANNOT reject the null hypothesis at 25%

Table 28 Granger-test on volatility-to-stop-loss orders at run length > 7

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Grangercausation
Stop-loss assumed at run length > 7				
1	0.60693	0.43629	0.00000	CANNOT reject the null hypothesis at 25%
2	0.45862	0.63241	0.00000	CANNOT reject the null hypothesis at 25%
3	0.33869	0.79737	0.00000	CANNOT reject the null hypothesis at 25%
4	0.29109	0.88378	0.00000	CANNOT reject the null hypothesis at 25%

Table 29 Granger-test on volatility-to-stop-loss orders at run length > 6

Lag	F-value	*p*-value	Adj. R^2	Null hypothesis of NO Granger causation
Stop-loss assumed at run length > 6				
1	0.00833	0.92729	0.00000	CANNOT reject the null hypothesis at 25%
2	0.11251	0.89361	0.00000	CANNOT reject the null hypothesis at 25%
3	0.06385	0.97892	0.00000	CANNOT reject the null hypothesis at 25%
4	0.15505	0.96071	0.00000	CANNOT reject the null hypothesis at 25%

Table 30 Granger-test on volatility-to-stop-loss orders at run length > 5

Lag	F-value	*p*-value	Adj. R^2	NULL HYPOTHESIS of NO Granger causation
Stop-loss assumed at run length > 5				
1	0.00168	0.96727	0.00000	CANNOT reject the null hypothesis at 25%
2	0.28135	0.75484	0.00000	CANNOT reject the null hypothesis at 25%
3	0.38169	0.76623	0.00000	CANNOT reject the null hypothesis at 25%
4	0.29139	0.88245	0.00000	CANNOT reject the null hypothesis at 25%

Conclusions on the granger-causality tests

Although Granger testing is subject to criticism (as is any econometric tool), it is still one of the best indicators of causality that econometrics provides. Clive Granger was awarded the Nobel Prize in Economics Sciences in 2003. The analyses carried out in the previous sections prove beyond any doubt the existence of Granger causality between

Table 31 Volatility expressed as Delta Price (MAX – MIN) and Standard Deviation of price at different ratio of HF traders' participation, and different scenarios

		0%	33%	50%	67%	75%	90%
RANDOM WALK (RW)	Delta price	0.63	1.18	1.23	1.15	1.02	0.82
	Std. dev	0.20	0.25	0.28	0.29	0.27	0.23
TREND	Delta price	0.93	1.84	2.99	2.41	2.03	1.32
	Std. dev	0.28	0.56	0.89	0.72	0.61	0.39
RW + QUANTITY (QTY)	Delta price	0.7	1.62	1.86	1.84	1.69	1.61
	Std. dev	0.21	0.36	0.42	0.44	0.39	0.38
TREND + QTY	Delta price	0.96	3.04	3.88	3.71	3.63	2.99
	Std. dev	0.29	0.96	1.13	1.09	1.06	0.87
RW + STOP-LOSS (SL)	Delta price	4.39	4.41	4.21	4.03	3.99	4.37
	Std. dev	0.88	1.06	1.03	0.97	0.94	0.94
TREND + SL	Delta price	4.41	4.55	5.22	5.09	4.71	4.64
	Std. dev	1.08	1.29	1.32	1.30	1.22	1.20
RW + QTY + SL	Delta price	4.38	4.48	4.79	4.51	4.37	4.35
	Std.dev	0.91	1.06	1.01	1.04	1.01	0.96
TREND + QTY + SL	Delta price	4.44	4.89	5.70	6.26	6.24	6.2
	Std. dev	1.13	1.34	1.38	1.59	1.59	1.52

previous volatility and excess volatility within a relatively short period and between SL orders and excess volatility. Furthermore, quite certainly, there is a Granger-causal relationship between scarce liquidity and excess volatility, whereas there is no evidence of such a relationship between a high quantity of securities exchanged and excess volatility.

Results of the agent-based model (ABM) experiment

The next tool used to verify the impact of previous volatility, scarce liquidity, a high quantity exchanged, and SL orders on excess volatility is an ABM simulation. As was expected from previous simulation-based research, the percentage of HFT participation influences volatility since a moderate level of HFT activity propels volatility, whereas when most market agents are homogeneous (i.e., when they are all slow or all fast), volatility tends to behave according to the RWM and no unexpected excessive volatility appears. Volatility varies according to other parameters, but it usually reaches its maximum level when HFT activity compared to all market activity is in the 33% to 75% range. In the simulation, volatility is defined in two ways: maximum price less minimum price over the observation period and as the standard deviation of the price over the same period. The results are shown in Table 31.

The simulation displays results generally compliant with the other methodologies presented earlier. The only exception is the scenario "SL BASE," where the Delta Price rises as HFT participation increases, reaches a maximum corresponding to 50%, as in several other scenarios, and then declines for higher percentages of HFT participation. The exception is the peak at 90% participation (4.37 versus 3.99 at 75%), which does not appear in the other cases. As expected, the trend case shows higher volatility than the base case because of the definitive direction that the algorithm imposed to aggressive orders (either upward or downward). This confirms that preexisting volatility increases volatility after a VST interval. Moreover, when the market experiences a rise in the quantity exchanged, volatility is augmented, as it is when investors make use of SL orders to protect their limit orders. Again, as expected, the combination of these factors accumulates as volatility increases. Small discrepancies are expected because of the heavy use of random number generation in ABM. Yet, the combination of one or more factors does not result in an exponential peak in volatility. This should not surprise us. Not all butterfly's wings flapping in Beijing cause a hurricane in the Caribbean, therefore it is normal to expect that in most cases, causes do accumulate linearly even though they may occasionally accumulate nonlinearly, sometimes with disastrous consequences. The unpredictability of nonlinear behavior is the greatest risk factor. Indeed, as was expected, in Table 31, volatility grows as the quantity effect is added to the simulation (both in the base and trend cases), and it grows even more when SL orders are included in the system, and it grows even further when both effects are combined. The ABM simulation confirms that excess volatility is a possible consequence of the four drivers identified above.

Discussion

VSTPC have been evaluated using a mathematical model and three practical techniques: audit-trail data, Granger testing, and ABM simulation. The Granger tests provide strong evidence of previous volatility Granger-causing higher volatility within a VST period. Granger testing also displays the Granger causality of SL orders or liquidity to high volatility. All previous results match common sense and are in accordance with the mathematical model. Volatility causes concern among investors and is likely to cause higher volatility either directly or through liquidity reduction due to uncertainty (a behavioral effect). The quantities exchanged do not show Granger causality to high volatility equally clearly. In a similar fashion, the audit-trail data analysis matches the results of the Granger tests as far as volatility-to-volatility, liquidity-to-volatility, and SL-orders-to-volatility are concerned. The ABM simulation also demonstrates a definitive impact of SL orders on volatility, whereas the influence of quantity exchanged does not seem as clear and fails to exhibit definitive results.

Overall, the analyses performed in the previous sections strongly suggest a combination of previous volatility, scarce liquidity, and SL orders to be prerequisites of high volatility at VST, where the impact of large quantities, although reasonable, is not supported by the evidence.

The results presented in the previous section, although very suggestive, cannot be taken as conclusive. Each of the approaches used can be criticized from either a theoretical or practical perspective. The data analysis restricted its horizon to one market, a limited period, and on a few days only. Granger causality is not, neither does it pretend to be, causality in the common sense of the word. ABM simulation is an artificial construction that forces the real world into a model that is simpler than the reality. Yet, all the techniques used point in the same direction: excess volatility appears when markets show previous volatility, scarce liquidity, and a considerable number of SL orders or a combination thereof. The opposite is obviously not true: volatility does not necessarily appear if and only if the three drivers are present. There is no mechanical cause–effect relationship. Trading is not mechanical since it depends on human behavior or, slightly differently but perhaps not so much, on human-programmed algorithms. Some level of unpredictability is unavoidable. The main issue about the approaches used in this research concerns the possibility of VSTPC being caused by something other than the drivers identified in this study and all the techniques adopted being "fooled" by some "invisible hand." Although this is possible in principle, this seems a rather remote possibility. The techniques used enjoy a good scientific reputation and, unless grossly misused, are expected to yield sound results. Indeed, all the results converge to the same conclusion. Moreover, although usually not considered a good argument at the academic level, common sense seems to align itself with the outcome of the scientific results. Informal behavioral analysis would confirm that, at least in principle, previous volatility has the potential to provoke nervous reactions among traders, to generate fear and anguish and sometimes even panic, exacerbating the volatility already present in the market. Scarce liquidity is a mechanical factor leading to an easily understandable consequence on price volatility. Practitioners fear low liquidity as do exchange managers and regulatory authorities. If liquidity is low, minor trading activity has the potential to move prices abruptly in the same way that a small movement may cause dramatic vibrations in a glass of water while a small disturbance will not display the same effect in the open sea. Finally, SL orders could trigger large price movements if enough of them, ready to be fired, unchain a sequence of events. This explanation does not aim to replace contrary scientific evidence, but the underlying point is that in this case, scientific evidence—as demonstrated by Granger testing, audit-trail data analysis, and simulation—confirms and corroborates common sense.

Conclusion

The goal of this paper is to identify the main driver of price changes at VST. Although finance research has long attempted to spot unambiguous market drivers, research on drivers at VST seems to be missing in the existing literature. Therefore, we assumed that establishing a theory dealing with such a topic in a market in which speed is more and more important would fill this gap. Along with well-known financial crises, a new kind of market disruption has emerged: flash crashes. Academic research needs to carefully investigate the causes of such phenomena if future crises are to be prevented. Indeed, the relatively recent entrance of HFT into the marketplace has further decoupled finance from the economy, a relationship assumed to hold until then, although with exceptions. Ultrafast financial trading denies all relations to economic variables, and when the three main causes investigated earlier are nonlinearly combined, it further reduces any residual hope of market equilibrium and stability. Obviously, not all three factors carry the same weight and play the same role in an excess volatility crisis. Out of the three simultaneous conditions identified as the main contributors to VSTPC, SL orders are the most common in day-to-day operations; they are normal practice, and no regulatory authority is concerned about them. Nevertheless, they can be important contributors, especially when many of them accumulate, ready to be triggered as price movements take a definitive direction. It is also intuitive that in a nervous market, most investors would be cautious enough to protect their trading with SL orders, even against small swings. Volatility is also a not an uncommon occurrence. Sharply falling as well as rising prices are occasional but not infrequent events; they are intrinsic to market practice and fortunately so—frozen markets are not desirable from any participant's perspective. Investors look for price dynamics and a lack of them would make financial activities unappealing. Scarce liquidity is a different kind of beast; it is widely considered a major threat in itself. Regulators and exchanges are engaged in a full-time struggle to ensure more and more abundant liquidity and fight illiquidity.

HFT has found several supporters, even within the academic community, on the basis that this practice tends to increase market liquidity. However, even scarce liquidity in itself is not the automatic cause of a major crisis: if prices are stable, the macro effect would scarcely be noticeable, for example, when prices move up one tick and then down one tick and then up one again, and so forth. There are securities and even entire markets that are frequently or permanently affected by scarce liquidity, but they do not necessarily experience daily crises. From all previous considerations, it sounds sensible to state that the nonlinear input–output transformation effect is the real cause of excess volatility at VST. Had markets been capable of preventing apparently innocuous causes to become violent outcomes, volatility would not have become a critical event causing, according to some authors, daily mini flash crashes. The butterfly effect seems to be at the root of most unsolved problems that the markets are currently facing. Systems have apparently grown too complex and too rapidly for systems theory to cope with them. The VSTPC theory is an attempt to better understand their behavior.

Appendix: Description of the simulation

The simulation can either run under the Base Case or the Trend Case. In the former, all simulation parameters follow a completely random path whereas in the Trend Case certain values of the parameters occur with more or less probabilities and so cause prices to follow an either upward or downward trend.

The simulation runs for a certain number of cycles in order to produce sufficient data for statistical purposes (100 in the case of this simulation). At each time unit the algorithm executes an order.

At each cycle the simulation produces 2500 orders to be executed in sequence. For each order the algorithm randomly selects a BookType (either Bid or Ask), an OrderType (either Limit or Market), and a TraderType (either Slow or HFT). There are 15 fast traders and 11,859 slow traders (source: CFTC-SEC 2010a). Moreover, if OrderType is Market, the algorithms also select the trade Volume, which can either be equal to 1 or 10 securities (with 250-to-1 probabilities). If Volume is equal to 10, then execution of the Market order repeats 10 times. Moreover, in the following trade Volume will be equal to 5 and the one following this, it will be equal to 3. This simulates a wave of high, yet decreasing, volume of trades.

Yet, two consecutive Market orders are never allowed.

If the routine runs under the Trend condition, it generates a trade direction (either buy or sell) and in case of Market orders the trade direction is selected with higher probability (60% in this simulation).

If the TraderType is HFT then the order is being executed immediately, otherwise it is queued for a predetermined number of time units (650 in this simulation) in order to simulate latency due to slowness of the human trader, hardware, software and networking. Then, at each repetition, the queue advances one step and when the queueing time expires the slow order is being executed as well.

If OrderType is Limit and the book depth is less than maximum (in this simulation the maximum depth was set to 5), then the routine adds one order to the book at the highest Bid or lowest Ask (according to the value of BookType). If the depth is already at the maximum, this condition is being interpreted as sufficient consensus by the investor community of the soundness of the price such that a jump to the next price level is acceptable. This mechanism allows to move the limit order book to the next available price but only if the bid-ask spread is greater than one tick. Otherwise, the new order is ignored. Under the modality SL, a Limit order also insert a StopLoss order in the appropriate StandBy book.

If OrderType is Market, then the order is executed against the best price on the opposite book (a Bid Market order is executed against the lowest price on the Ask book and viceversa). If the modality SL is active, execution of a Market order triggers activation of StopLoss orders linked to that order. A StopLoss order gets activated by moving it from the StandBy book to the Executable book. StopLoss orders in the Executable book are associated with a stop-loss execution price, in this case 2 ticks below or above trade execution (according to whether the corresponding Limit order is either a Bid or Ask order). When a new execution price is reached, all outstanding StopLoss orders activated at that price get executed at the best available price as Market orders. Market orders consume liquidity and potentially decrease the best Bid or increase the best Ask price. This means that, if more than 5 active StopLoss orders exist at a certain price level, as soon as that price level is reached, the StopLoss orders get immediately executed as Market orders. Therefore, the first StopLoss orders will be executed at the expected price whereas the remaining ones might executed at the same or at a worse price, Let us suppose that 7 outstanding StopLoss orders are active at price 100. If trading price drops to 100, they all become executable. The first 4 will execute at 100 but the remaining 3 will no longer find a Limit Bid order at 100. At that point the best Bid will be 99.75. However, since price has dropped to 99.75, all active StopLoss orders at that price will also turn to executable, potentially reducing the price even further and executing more StopLoss orders at 99.50, and so on. All this is being executed automatically by the rules of the exchange, with no external intervention, either human or silicon. This explains the impact of StopLoss orders as a factor of VSTPC.

Every operation is being logged onto the database so that at the end of each cycle it is possible to collect information about volatility and later on comparing the measurement between runs of the simulation at different levels of HFT activity and for different combination of the parameters (Random Walk vs. Trend; Volume bursts, Stop Loss orders).

Abbreviations

ABM	Agent-based model
QTY	Quantity
SL	Stop-loss
HFT	High-frequency trading
RW	Random walk

V	Volatility (in formulae)
L	Liquidity (in formulae)
RWM	Random walk model
VST	Very-short time
Q	Quantity (in formulae)
S	Stop-loss (in formulae)
VSTPC	Very short-time price change

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Declarations

Competing interests
The authors declare that they have no competing interests.

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