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A new analytical approach for identifying market contagion

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Abstract

This study proposed a new analytical approach to identify the excessive comovement of two markets as contagion. This goal is achieved by linking latent-factor and single-equation error correction models and evaluating the breaks in the short- and long-term relationships and correlatedness in the linked model. The results demonstrated that a short-term relationship representing the market speed ratio between two markets plays a key role in contagion dynamics. When a long-term relationship or correlatedness is broken (comovement change) due to a break in the short-term relationship (market speed ratio), contagion is highly likely and should be formally declared. Bayesian posterior probabilities were calculated to determine the cause. Furthermore, this study applied this analytical Bayesian approach to empirically test the contagion effects of the U.S. stock market during the global financial crisis between 2007 and 2009 using 22 developed equity markets.

Keywords: Contagion test, Market integration, Bayesian approach, Comovement, Market speed ratio

Introduction

Over the past 40 years, financial markets have become increasingly integrated. Market integration began with open markets, in which asset prices were determined globally. The relevant literature emerged in the 1970s, with early theoretical developments in international asset-pricing models. Since then, a growing body of evidence supporting the argument that risk premia are determined globally has accumulated. These days, market integration is often tested using "equality" in risk pricing based on an asset-pricing benchmark or "the law of one price" to avoid arbitrage between variables, such as prices or interest rates. These definitions of financial market integration suggest that integration leads to the conversion of market-specific risk into common risk (Lehkonen 2015). It is strongly argued that an integrated market leads to enhanced economic growth and stability through direct financial channels, such as lower capital costs and increased investment opportunities (Carrieri et al. 2007). Global financial integration has some detractors. One major criticism is that it significantly raises global market volatility during a crisis and causes financial markets to be more vulnerable to global shocks. Since the onset of the global financial crisis (GFC) between 2007 and 2009, periodic local financial crises have occurred worldwide, and market contagion has become one of the most researched topics in the study of financial markets. Gagnon and Karolyi (2006) surveyed the history of financial market contagion and identified volatility spillover as the major driving force behind crisis contagion.

Extant literature has clearly demonstrated that disentangling contagion from market interdependence is crucial for singling out contagion in the event of a financial crisis. Market contagion is measured by a significant increase in the correlation between two markets. One major pitfall in correctly identifying contagion is that it is not simply shown by an increased correlation of performance indicators during a crisis period. The increasing correlation coefficient is somewhat flawed due to the interference of rising volatility, which is commonly associated with periods of financial stress (Forbes and Rigobon 2002). To overcome this, most contagion tests define contagion as "a correlation in excess of that expected via fundamentals" (Bekaert et al. 2005, p.40). However, Bekaert et al. (2005) acknowledged the difficulty in "identifying both the relevant fundamentals and how they are linked to correlation" (p.40).

Thus, contagion tests have been redefined continuously over the years, as the sources of correlation biases or correlation increases due to relevant fundamentals are identified by removing confounding effects.[1] This process started with Forbes and Rigobon (2002), who noted that cross-market correlation increases during crisis periods due to "common as well as market-specific factors" (p.2225). As the volatility of the target market may change due to common and market-specific factors, many studies use factor decomposition models to identify contagion (Bekaert et al. 2005; Corsetti et al. 2001, 2005; Dungey et al. 2005a, b; Dungey and Martin 2001; Forbes and Rigobon 2002).[2] Bekaert et al. (2014) defined contagion as "the comovement in excess of that implied by the factor model" (p.2598), which is an improvement on the old definition (Bekaert et al. 2005), as it is not limited to correlation increase as a contagion measurement.

To investigate the comovement defined by Bekaert et al. (2014) more precisely, this study linked the latent factor model (LFM) and the single equation error correction model (SEECM). The LFM assumes market-specific and common systematic volatilities to describe market returns under market integration. The SEECM describes the dynamic process of market returns to equilibrium from a state of disequilibrium due to a shock, assuming that two market return time series exhibit a linear equilibrium relationship that determines short- and long-term behavior (Banerjee et al. 1990, 1993; Davidson et al. 1978). By linking the two models via first-order autoregressive (AR[1]) errors, this study described stressful market situations more precisely. The major strength of linking the two models is the ability to establish the null market integration hypothesis (MIH) for tranquil periods and analyze the contagion structurally via a causal relation between two markets for crisis periods. This reveals that the short-term relationship, that is, the common-factor loading, is the main backbone of the linked model and is entirely embedded in the long-term relationship and correlatedness between two markets. These findings allow for an accurate and concise definition of contagion: *contagion is defined*

[1] A situation in which the effect between the target and source market is distorted by the presence of the relevant fundamentals. This situation is typically resolved by introducing a proper factor model.

[2] Gagnon and Karolyi (2006) surveyed the financial market contagion history.

as an occurrence of breaks in the long-term relationship or correlatedness between two markets mainly caused by breaks in the short-term relationship. This definition is reasonable, as contagion would be in effect when the short-term effect of a shock is not properly controlled at the beginning of the crisis period and continues to cause breaks in the long-term relationship or correlatedness between two markets. From the market behavior perspective, long-term relationships or correlatedness may be considered comovements, while short-term effects may be considered the market speed ratio (MSR) of two market movements under integration, which depends on liquidity and possibility of downside risk (Eq. 7).

This approach leads to the consideration of various situations as potential alternatives when a shock hits a source market (Table 1). The originality of this approach is the application of the Bayesian method to those situations in order to identify contagion. This study used the Bayesian method to analytically detect comovement in excess of that expected from fundamentals, identifying it as contagion. When comovement (long-term relationship or correlatedness) between the target and source markets is broken, the cause is determined by calculating posterior probabilities. If the cause is a short-term (MSR) break, the contagion is formally addressed.

Identifying contagion based on comovement involves two critical issues (Dungey and Renault 2018): (1) how correlation is related to common and market-specific factors during tranquil and crisis periods (Bekaert et al. 2005; Corsetti et al. 2005; Dungey and Martin 2007; Dungey et al. 2010, 2011; Forbes and Rigobon 2002) and (2) how all time-varying components of return volatilities are related to structural breaks during crisis times (Aït-Sahalia et al. 2015; Bae et al. 2003; Baur and Schulze 2005; Boyson et al. 2010; Bussetti and Harvey 2011; Favero and Giavazzi 2002; Rodriguez 2007). The present approach completely resolves these two critical issues, as demonstrated below.

The empirical part of this study involves applying the Bayesian approach to address the financial market contagion between the U.S. and other countries under market integration during the GFC between 2007 and 2009. This study evaluated the contagion effects of the U.S. stock market during the GFC using 22 developed equity markets. The results revealed that some countries suffered from financial contagion from the U.S. market crash, while others managed to escape contagion. Some key contagion parameters, such as short- and long-term effects, correlatedness between source and target markets, and contagion odds, must be monitored during a financial stress. An increase in short- and long-term effects or correlatedness suggests potential contagion. These contagion parameters could help policymakers and investors develop alternative and practical guidelines against contagion.

Identifying contagion is vital not only in financial markets but also in macroeconomic dynamics. Kalemli-Ozcan et al. (2013) and Beck (2021) demonstrated the necessity of identifying contagion to establish the link between financial integration and business cycle synchronization in European Union countries. The present approach to identify contagion is promising in its application to analyzing contagion phenomena in various macroeconomic and financial market variables across international markets.

The remainder of this paper is organized as follows. First, the market model and market integration hypothesis are discussed. Next, contagion dynamics and the Bayesian testing procedure are described. Following this, the major findings for the 2008 US market crisis are presented. Finally, concluding remarks are provided.

Methods

Market modeling under integration

Linking the latent factor and single equation error correction models

The latent factor model Eq. 10 (LFM, Appendix 1) was linked to the single equation error correction model Eq. 11 (SEECM, Appendix 2) to model the market dynamics between two markets under integration. This linking process yields a proper model for identifying contagion using a Bayesian approach. To link the two models, we introduced AR(1) errors $u_{x,t}$ and $u_{y,t}$ for the LFM, as in Eq. 1. AR(1) errors are useful for describing market behavior during periods of financial stress via the parameter η. For instance, providing more (or less) information to the market can make η close to zero (or close to 1), which indicates iid u. Phillips et al. (2011) investigated the near-explosive process with the AR(1) parameter η close to 1 during a crisis period.

Proposition 1 *Suppose that we employ an AR(1) model for the idiosyncratic factors $u_{x,t}$ and $u_{y,t}$ in Eq. 10:*

$$u_{x,t} = \eta_x u_{x,t-1} + a_{u,t} \quad u_{y,t} = \eta_y u_{y,t-1} + b_{u,t} \tag{1}$$

where $E\left(a_{u,t}W_t\right) = 0, E\left(a_{u,t}u_{x,t-1}\right) = 0, E\left(b_{u,t}W_t\right) = 0, E\left(b_{u,t}u_{y,t-1}\right) = 0,$ $E\left(a_{u,t}b_{u,t}\right) = 0$, $a_{u,t} \sim iid(0,1), and b_{u,t} \sim iid(0,1)$. *This links the LFM to the SEECM as follows:*

$$\Delta Y_t = \frac{\theta_y}{\theta_x}\Delta X_t - \left(1-\eta_y\right)\left(Y_{t-1} - \frac{\theta_y(1-\eta_x)}{\theta_x\left(1-\eta_y\right)}X_{t-1}\right) + \varepsilon_{Lt} \tag{2}$$

where $\varepsilon_{Lt} = \delta_y b_{u,t} - \frac{\theta_y}{\theta_x}\delta_x a_{u,t} + (\eta_x - \eta_y)\theta_y W_{t-1}$ *with iid over* t, $0 < \eta_y < 1$, *and* $Var(\varepsilon_{Lt}) = \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 + \left(\eta_x - \eta_y\right)^2\theta_y^2$.

Proof of Proposition 1 is provided in Appendix 3.

Remark 1 **Composition of the linked model.** Using Proposition 1, we obtain the following:

$$\begin{aligned}\Delta Y_t &= \frac{\theta_y}{\theta_x}\Delta X_t - \left(1-\eta_y\right)\left(Y_{t-1} - \frac{\theta_y(1-\eta_x)}{\theta_x\left(1-\eta_y\right)}X_{t-1}\right) + \varepsilon_{Lt} \\ &= \beta_{L0}\Delta X_t + \beta_{L1}(Y_{t-1} - \gamma_L X_{t-1}) + \varepsilon_{1t} = \beta_{L0}\Delta X_t + \beta_{L1}Y_{t-1} + \beta_{L2}X_{t-1} + \varepsilon_{Lt}.\end{aligned} \tag{3}$$

where $\beta_{L0} = \frac{\theta_y}{\theta_x}$, $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$, and $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ correspond to the short-term effect, long-term effect, and correlatedness of the linked model, respectively.[3] Thus, the factors that might cause the time-varying comovement (or long-term effect and correlatedness

[3] Note that β_{L1} represents the scaled inverse of market-specific volatility of market Y from $\beta_{L1} = -\left(1-\eta_y\right) = \frac{-(1-\eta_y)(1+\eta_y)}{(1+\eta_y)} = \frac{-1}{(1+\eta_y)}\left(1-\eta_y^2\right) = \frac{-1}{(1+\eta_y)}\frac{1}{Var\left(u_{y,t}\right)}$.

of Eq. 3) during a crisis include the common factors W_t via θ_x and θ_y and market-specific factors $u_{x,t}$ and $u_{y,t}$ via η_x and η_y. $\beta_{L0} = \frac{\theta_y}{\theta_x}$ is entirely embedded in γ_L, β_{L2} and ε_{Lt} in Eq. 2. *This feature not only plays a key role in formally defining contagion but also resolves various critical issues in identifying contagion, as described below.*

Remark 2 **Risk parameters**. The variances of the idiosyncratic factors of X and Y are $\mathrm{Var}(\delta_x u_{x,t}) = \delta_x^2/(1-\eta_x^2)$ and $\mathrm{Var}(\delta_y u_{y,t}) = \delta_y^2/\left(1-\eta_y^2\right)$ using Eq. 1, respectively. Thus, market-specific volatility becomes high as η_x and η_y are close to 1 or $|\delta_x|$ and $|\delta_y|$ are large. According to Proposition 1

$$Var(\varepsilon_{Lt}) = \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 + \left(\eta_x - \eta_y\right)^2\theta_y^2;$$

therefore, volatility of ΔY_t of the target market[4] depends on common and market-specific factors based on their parameters. Volatility increases not only when δ_y becomes large but also when θ_y,δ_x, and $\left(\eta_x - \eta_y\right)^2$ are large. Target market volatility may be decreased by reducing $\frac{\theta_y}{\theta_x}$ and $\left(\eta_x - \eta_y\right)^2\theta_y^2$ during the financial stress period.

Remark 3 **Causality issue.** During periods of financial stress with an increased possibility of downside risk, investors tend to exhibit more risk-adverse behavior due to imperfect information. Historically, fears have prompted investors to display herding behavior in the market, which increases the AR(1) parameters η_x *and* η_y to close to 1, making them different (Phillips et al. 2011). Different AR(1) parameters ($\eta_x \neq \eta_y$) make ε_{Lt} of Eq. 2 depend on W_{t-1}, suggesting *endogeneity, which usually leads to causality in both directions (from X to Y and from Y to X*; Dungey et al. 2005b). This could explain the various breakages in parameters during a crisis.

The market integration hypothesis

The market integration hypothesis (MIH) refers to a tranquil period situation in which market returns X and Y share a common factor with the respective level of risk, and hence, their normal comovement is expected. As Proposition 1 describes the general market dynamics between X and Y, the tranquil period may be based as a null hypothesis against the crisis period.

Proposition 2 *Assume the following null MIH:*

$$MIH: \eta_x = \eta_y = \eta_0 \text{ for some } 0 < \eta_0 < 1, \ \frac{\theta_y}{\theta_x} \neq 1, \text{ and } \theta_x\theta_y \neq 0 \tag{4}$$

Then, under the null MIH, Eq. 2 can be rewritten as follows:

$$\Delta Y_t = \frac{\theta_y}{\theta_x}\Delta X_t - (1-\eta_0)\left(Y_{t-1} - \frac{\theta_y}{\theta_x}X_{t-1}\right) + \varepsilon_{Mt} \tag{5}$$

[4] Volatility of ΔY_t is calculated as variance of $\Delta Y_t(\to 0)$ when $Y_{t-1} - \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}X_{t-1} \to 0$ and $\Delta X_t \to 0$. Refer to a simple example in Appendix 1.

where $\varepsilon_{Mt} = \delta_y b_{u,t} - \frac{\theta_y}{\theta_x}\delta_x a_{u,t}$ *is an iid error with* $E(\varepsilon_{Mt}) = 0$ *and finite variance of* $Var(\varepsilon_{Mt}) = \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2$.

Proof of Proposition 2, Eq. 5 easily follows by applying Eq. 4 to Eq. 2.

Remark 4 **Basic linear relationship under MIH.** Using Proposition 2, we obtain the following:

$$\Delta Y_t = \frac{\theta_y}{\theta_x}\Delta X_t - (1-\eta_0)\left(Y_{t-1} - \frac{\theta_y}{\theta_x}X_{t-1}\right) + \varepsilon_{Mt}$$
$$= \beta_{M0}\Delta X_t + \beta_{M1}(Y_{t-1} - \gamma_M X_{t-1}) + \varepsilon_{Mt} = \beta_{M0}\Delta X_t + \beta_{M1}Y_{t-1} + \beta_{M2}X_{t-1} + \varepsilon_{Mt} \quad (6)$$

where $\beta_{M0} = \frac{\theta_y}{\theta_x}$, $\gamma_M = \frac{\theta_y}{\theta_x}$, and $\beta_{M2} = \frac{\theta_y(1-\eta_0)}{\theta_x}$ correspond to the short-term effect, long-term effect, and correlatedness for the MIH, respectively, and $\left(Y_{t-1} - \frac{\theta_y}{\theta_x}X_{t-1}\right) = 0$ when X and Y are in equilibrium. Equation 6 subsequently implies a basic linear relationship[5] $Y_t = \frac{\theta_y}{\theta_x}X_t$ $Y_t = \frac{\theta_y}{\theta_x}X_t$ and the corresponding *market speed ratio (MSR)* between two markets during a tranquil period; that is,

$$\Delta Y_t / \Delta X_t = \frac{\theta_y}{\theta_x}. \quad (7)$$

In fact, the MSR equals the short-term effect and holds for general market situations (refer to footnote 5).

Remark 5 **Practical meaning of MIH.** The null MIH above implies a situation in which two markets are midway between the two extremes of perfect integration into one market and two completely independent markets (Appendix 4). This implies that market-specific volatility is the same in markets X and Y ($\eta_x = \eta_y = \eta_0$), and that they share a common factor $(\theta_x\theta_y \neq 0)$ with different levels of systematic risk $\left(\frac{\theta_y}{\theta_x} \neq 1\right)$. This notion is consistent with the financial market theory, which indicates that only global systematic risks are priced in two fully integrated markets and that integration leads to the conversion of a market-specific risk of each market into a common risk. This implies that local market-specific risks are fully diversified for integrated markets (Lehkonen 2015). In this sense, the MIH achieves two fully integrated markets without any endogeneity problem by allowing $\eta_x = \eta_y$ in Eq. 1. The parametric condition $0 < \eta_x = \eta_y = \eta_0 < 1$ is essential for the convergence to the equilibrium in Eq. 6 ($-1 < \beta_{M1} = -(1-\eta_0) < 0$).

Remark 6 **Contagion and the MSR.** It is reasonable to discuss market contagion by testing the MIH as a proper null hypothesis. Testing MIH against a crisis enables the detection of comovement change and breaks in long-term effect or correlatedness in excess of those implied by the MIH. As the MSR, being equal to the short-term effect, is actually the relative adjustment speed of market movements to new information from the source market, the MSR is a function of liquidity and possibility of downside risk.

[5] No endogeneity under the MIH does not mean that the contemporaneous causality direction runs only in one direction, from X to Y. It means that two markets are completely integrated or completely exchangeable via Eq. 5.

Therefore, if the MSR fails to decrease or slow down properly during a stress period, an increase in the comovement (long-term effect or correlatedness) would be unavoidable in excess of that implied by the MIH. *Thus, the MSR slowdown during crises is a critical issue in market contagion, as discussed below.*

Resolving the two critical issues

The present linked model resolved the two critical issues for identifying contagion based on comovements discussed in the Introduction: (1) how correlation is related to common and market-specific factors during tranquil and crisis periods and (2) how all time-varying components of return volatilities are related to structural breaks in crisis times. Regarding the first issue, Eq. 3 illustrates that correlatedness $\left(\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}\right)$ is related to a common factor via $\frac{\theta_y}{\theta_x}$ and to the market-specific factor of X through $(1-\eta_x)$ under general situations. Under MIH $\left(\eta_y = \eta_x = \eta_0\right)$, these factors are reduced to correlatedness $\beta_{M2} = \frac{\theta_y(1-\eta_0)}{\theta_x}$ in Eq. 6. During a financial crisis period, $\eta_y \neq \eta_x$ brings about endogeneity and changes in $\left(\frac{\theta_y}{\theta_x}, \eta_x, \eta_y\right)$, which, in turn, results in increases in correlatedness and long-term effects.

Regarding the second issue, the present linked model defines volatility spillover clearly and simply by considering systematic (common) and market-specific volatility. As $\frac{\theta_y}{\theta_x}$ is related to systematic volatility and η_x and η_y are related to market-specific volatility, volatility spillover occurs due to changes in $\left(\frac{\theta_y}{\theta_x}, \eta_x, \eta_y\right)$ caused by a shock. These volatility changes cause direct changes in the short-term effect $\left(\beta_{L0} = \frac{\theta_y}{\theta_x},\ \text{MSR}\right)$, long-term effect $\left(\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}\right)$, correlatedness $\left(\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}\right)$, or convergence speed to equilibrium $(\beta_{L1} = -\left(1-\eta_y\right))$. When the MIH is rejected or $\left(\frac{\theta_y}{\theta_x}, \eta_0, \eta_0\right)$ under the MIH changes to $\left(\frac{\theta_y'}{\theta_x'}, \eta_x', \eta_y'\right)$, the error volatility under the MIH changes from $\text{Var}(\varepsilon_{Mt}) = \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2$ to:

$$\text{Var}(\varepsilon_{Lt}) = \delta_y^2 + \left(\frac{\theta_y'}{\theta_x'}\delta_x\right)^2 + \left(\eta_x' - \eta_y'\right)^2 \theta_y'^2. \tag{8}$$

Thus, Eq. 8 defines the volatility spillover related to systematic $(\frac{\theta_y'}{\theta_x'})$ and market-specific element (η_x', η_y'). This explains how all time-varying components of return volatilities are related to structural breaks during crises.

Literature often treats volatility spillover as the tail dependence of market returns. Rodriguez (2007) noted that structural breaks in tail dependence are an actual dimension of contagion effects and tested them using copulas. Bussetti and Harvey (2011) used time-varying copulas to test financial contagion through tail events. Using coexceedance measures or quantiles, a number of studies have detected contagion using the tail (Bae et al. 2003; Baur and Schulze 2005; Boyson et al. 2010). Aït-Sahalia et al. (2015) measured contagion effects via the extreme tail events of mutually exciting jumps. Recently, Dungey and Renault (2018) applied a GARCH (Generalized Autoregressive Conditional Heteroskedasticity) common feature approach to identify contagion.

A simple illustrative market episode based on this discussion is provided in Appendix 5.

Defining and testing contagion

Defining contagion and testing breaks

A shock that occurs in source market X may cause a break in the established relationship with target market Y during tranquil periods, resulting in contagion. Three types of breaks in market relationships were considered: a break in a short-term relationship, or short-term break (SB), a break in a long-term relationship, or a long-term break (LB), and a break in correlatedness, or a correlatedness break (CRB). Contagion is expected to occur when a shock causes a break in the underlying relationship pertaining to β_{L0} (short-term), γ_{L} (long-term), and β_{L2} (correlatedness) in Eq. 3. As the long-term relationship $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ and correlatedness $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ contain the short-term effect $\beta_{L0} = \frac{\theta_y}{\theta_x}$, the short-term effect, which is the MSR, can be used to create the following formal definition of contagion.

Definition Contagion is declared if a long-term break (a break in $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$) or a correlatedness break (a break in $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$) is mainly caused by a short-term break (a break in $\beta_{L0} = \frac{\theta_y}{\theta_x}$).

This definition implies that contagion is declared when the short-term effect (the MSR) of a shock is not properly controlled in a target market at the beginning of the crisis period and continues to cause breaks in the long-term relationship or correlatedness between two markets.[6]

To derive the contagion test, first this study tested whether a shock causes a break in the established linear relationship under the MIH described in Remark 4. Therefore, whether a given period experiences breaks in the relationships pertaining to β_{M0} (short-term effect or MSR), γ_{M} (long-term effect), and β_{M2} (correlatedness) in Eq. 6 was tested. To test the null MIH for a given period, the following hypotheses were proposed:

$$H_0 : \beta_{\mathrm{L0}} = \beta_{\mathrm{M0}}, \beta_{\mathrm{L1}} = \beta_{\mathrm{M1}}, \beta_{\mathrm{L2}} = \beta_{\mathrm{M2}} \ \left(\beta_{\mathrm{L0}} = \beta_{\mathrm{M0}}, \gamma_L = \gamma_{\mathrm{M}}, \beta_{\mathrm{L2}} = \beta_{\mathrm{M2}}, \text{ equivalently}\right)$$

H_a: At least one of the following holds true:

$$\beta_{\mathrm{L0}} \neq \beta_{\mathrm{M0}}, \beta_{\mathrm{L1}} \neq \beta_{\mathrm{M1}}, \beta_{\mathrm{L2}} \neq \beta_{\mathrm{M2}} \left(\beta_{\mathrm{L0}} \neq \beta_{\mathrm{M0}}, \gamma_L \neq \gamma_{\mathrm{M}}, \beta_{\mathrm{L2}} \neq \beta_{\mathrm{M2}}, \text{ equivalently}\right)$$

The null hypothesis H_0 illustrates that β_{M0}, γ_M, and β_{M2} are fixed with iid error $\varepsilon_{\mathrm{Mt}}$, as noted in Eq. 6. In other words, for a given period, markets X and Y maintain $\beta_{L0} = \beta_{\mathrm{M0}}$, $\gamma_L = \gamma_{\mathrm{M}}$, and $\beta_{L2} = \beta_{\mathrm{M2}}$ under the MIH. If H_0 is rejected, then the MIH fails to hold. In this case, the markets fail to maintain their underlying relationship and experience breaks in the short- or long-term relationship or correlatedness.

[6] Note that a long-term break or a correlatedness break can be caused by changes in (η_x,η_y) rather than by changes in $\frac{\theta_y}{\theta_x}$. This case is not considered as contagion here because the break in the long-term relationship or correlatedness is caused by market specific factor.

We developed a test adopting quantile regression and a Z-test to evaluate such breaks (see Baur 2013 for a more detailed discussion on the advantages of quantile regression). This test is based on the idea that a random fluctuation of the slope estimates around a constant value (with only the intercept parameters systematically increasing as a function of quantile ϑ) provides evidence for the iid error of the classical linear regression under the null MIH specified by Eq. 6. If some of the slope coefficients change as a function of quantile $0 \leq \vartheta \leq 1$, it is detected via the Z-test, which is designed to assess the magnitude of the estimated changes in slope for a given quantile. To implement the test, quantile regression parameters were estimated across the entire range of conditional quantiles of ΔY_t given ΔX_t. $(\beta_{L0}, \gamma_L, \beta_{L2})$ was estimated using quantile regression across fixed N quantiles. Let $\left(\hat{\beta}_{L0j}, \hat{\gamma}_{Lj}, \hat{\beta}_{L2j}\right)$ be the slope estimates of $\left(\beta_{L0j}, \gamma_{Lj}, \beta_{L2j}\right)$ from the $\frac{j}{N+1}$th quantile regression for $j = 1, \ldots, N$. Using the Proposition 3 in the Appendix 6, the Z-test can be derived as follows. Let P_i be $\left(\hat{\varphi}_{i1}, \ldots, \hat{\varphi}_{iN}\right)$, where $\hat{\varphi}_{ij}\left(i = 1, 2, 3, j = 1, \ldots, N\right)$ is the ith slope estimate for one of the three slope estimates $(\hat{\beta}_{L0j}, \hat{\gamma}_{Lj}, \hat{\beta}_{L2j})$ from the $\frac{j}{N+1}$th quantile regression. Let $P_{i,-k}$ represent a vector constructed by excluding the kth element from $P_i = \left(\hat{\varphi}_{i1}, \ldots, \hat{\varphi}_{iN}\right)$. For instance, $P_{i,-1} = \left(\hat{\varphi}_{i2}, \ldots, \hat{\varphi}_{i,N}\right)$ and $P_{i,-N} = \left(\hat{\varphi}_{i1}, \ldots, \hat{\varphi}_{i,N-1}\right)$. Then,[7]

$$Z_{i,k} = \frac{\hat{\varphi}_{ik} - m_k}{s_k} \text{ is N(0, 1) asymptotically} \quad (9)$$

with mean m_k and standard deviation s_k from $P_{i,-k}$. This Z-test can be employed to test the following hypotheses.

H_0^{SB} There is no short-term (MSR) break between X and Y in the $\frac{k}{N+1}$th quantile

(or $\hat{\beta}_{L0k}$ originates from the same normal distribution as the others in P_1 under H_0).

H_1^{SB} There is a short-term (MSR) break between X and Y in the $\frac{k}{N+1}$th quantile

(or $\hat{\beta}_{L0k}$ does not originate from the same normal distribution as the others in P_1 under H_a).

H_0^{LB} There is no long-term break between X and Y in the $\frac{k}{N+1}$th quantile

(or $\hat{\gamma}_{Lk}$ originates from the same normal distribution as the others in P_2 under H_0).

H_1^{LB} There is a long-term break between X and Y in the $\frac{k}{N+1}$th quantile

(or $\hat{\gamma}_{Lk}$ does not originate from the same normal distribution as the others in P_2 under H_a).

H_0^{CRB} There is no correlatedness break between X and Y in the $\frac{k}{N+1}$th quantile

[7] This holds based on Proposition 3 in Appendix 6 and the weak law of large numbers under H_0.

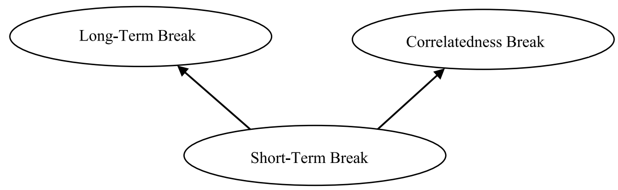

Fig. 1 Basic causal structure of testing contagion

(or $\hat{\beta}_{L2k}$ originates from the same normal distribution as the others in P_3 under H_0).

H_1^{CRB} There is a correlatedness break between X and Y in the $\frac{k}{N+1}$th quantile

(or $\hat{\beta}_{L2k}$ does not originate from the same normal distribution as the others in P_3 under H_a).

The basic structure of testing the hypotheses of our main concern in Fig. 1 shows that a long-term break (H_1^{LB}) or a correlatedness break (H_1^{CRB}) is caused by a short-term break (H_1^{SB}).

As a break in the established relationship is expected to occur at low quantiles corresponding to crisis periods, the contagion test can use the slope estimates at low quantiles, namely, $\hat{\beta}_{L01}$, $\hat{\gamma}_{L1}$, or $\hat{\beta}_{L21}$. The selection of the low quantile may be justified by the fact that during a crisis period, when ΔX_t decreases, ΔY_t tends to decrease more significantly than usual. Hence, a significant change is expected to occur in $\frac{\theta_y}{\theta_x}$ at low quantile (recall $\Delta Y_t/\Delta X_t = \frac{\theta_y}{\theta_x}$ under the MIH based on Eq. 7) or a structural break in (left) tail dependence, which is an actual dimension of the contagion effects (Bussetti and Harvey 2011; Rodriguez 2007). In summary, quantile regression is employed to handle contagion as a left-tail event. By rejecting H_0^{SB}, H_0^{LB}, and H_0^{CRB} using $\hat{\beta}_{L01}$, $\hat{\gamma}_{L1}$, and $\hat{\beta}_{L21}$, respectively, breaks can be identified in the short-term relationship, long-term relationship, and correlatedness during crisis periods, respectively.

Contagion dynamics against the MIH

Based on the null MIH and Eq. 6, various kinds of situations can occur when a shock hits market X. Changes in $\left(\frac{\theta_y}{\theta_x}, \eta_x, \eta_y\right)$ and volatility spillover based on Eq. 8 caused by a shock create structure breaks in short- or long-term effect or correlatedness (β_{L0},γ_L, β_{L2}). There are eight possible cases of breaks in short- or long-term effect or correlatedness (β_{L0}, γ_L, β_{L2}) resulting from the changes in $\left(\frac{\theta_y}{\theta_x}, \eta_x, \eta_y\right)$. Table 1 summarizes the eight possible situations (S1–S8) with causality relations between $\left(\frac{\theta_y}{\theta_x}, \eta_x, \eta_y\right)$ and (β_{L0}, γ_L, β_{L2}) when a shock hits market X. Appendix 7 verifies the causality relationships between the eight situations in Table 1. The last column of Table 1 addresses the contagion check. "No" and "Contained"[8] indicate avoidance of contagion. "Additional check required" indicates that a further check is needed to determine contagion.

[8] Difference between "NO" and "Contained" is whether volatility spillover given by Eq. 8 occurs. Refer to Verifications of Table 1 in Appendix 7.

Table 1 Contagion dynamics table

Classifi-cation	Causal volatility			Structure break			Contagion check
	$\frac{\theta_y}{\theta_x}$	η_x	η_y	Short-term effect (β_{L0})	Long-term effect (γ_L)	Correlated-ness (β_{L2})	
S-1	NC	NC	NC	NSB	NLB	NCRB	No
S-2	NC	C	C	NSB	NLB	CRB	No
S-3	NC	NC	C	NSB	LB	NCRB	Contained
S-4	C	C	NC	SB	NLB	NCRB	Contained
S-5	NC	C	UND	NSB	LB	CRB	Contained
S-6	C	UND	UND	SB	NLB	CRB	Additional check required
S-7	C	C	UND	SB	LB	NCRB	Additional check required
S-8	C	UND	UND	SB	LB	CRB	Additional check required

This table shows eight possible situations that can occur when a shock hits market X based on Eq. 6. In addition, it shows the causality relationship between volatility change and structural breaks due to a shock to market X. "C" and "NC" in the causal volatility columns denote "change" and "no change," respectively. "UND" denotes that "change" or "no change" are possible but undecided. "SB," "LB," "CRB," "NSB," "NLB," and "NCRB" in the structure break columns denote short-term break, long-term break, correlatedness break, no short-term break, no long-term break, and no correlatedness break, respectively. The last column shows a contagion check, where "No" and "Contained" indicate avoidance of contagion while "Additional check required" indicates that a further check is needed to determine contagion

Remark 7 **Handling the contagion test via classification.** As noted in Table 1, there is a one-to-one relationship between causal volatility and structural breaks for S-1 to S-4 (i.e., it contains no UND), while there is no such one-to-one relationship for S-5 to S-8 (i.e., it contains UNDs).[9] S-1 covers a normal MIH or a "fully integrated market." S-2 and S-3 handle the contagion bias from correlatedness break (CRB) or long-term break (LB), which were addressed by Forbes and Rigobon (2002). S-4 handles bias from a short-term break (SB), which is related to Favero and Giavazzi (2002), who employed a nonlinearity technique to detect a short-term effect in its opposite direction to observe "flight to quality" (p.241). S-5 contains contagion by exclusively keeping $\frac{\theta_y}{\theta_x}$ (*MSR*); hence, it might be considered to handle contagion bias from a long-term break (LB) and correlatedness break (CRB) simultaneously. S-5 is unlikely to occur in reality, as LB and CRB occur simultaneously with no short-term break (NSB). S-6, S-7, and S-8 are concerned with the possible bias from the SB, as noted in S-4. In these cases, further checks are necessary to identify contagion, as SB, CRB, SB, and LB occur simultaneously.

Remark 8 **S2–S5 in an economic or financial context.** According to Forbes and Rigobon (2002), CRB may be explained by aggregate or global shocks that simultaneously affect the fundamentals of several economies. For instance, a rise in the international interest rate, a contraction in the supply of international capital, or a decline in demand for international capital could simultaneously slow down economic growth in a number of countries. LB may be explained by how a shock to one country could affect the fundamentals in other countries. This case could work through a number of real linkages, such as trade or policy coordination. Trade could link economies, as a devaluation in one country would increase the competitiveness of that country's goods, potentially decreasing the competitiveness of other countries. This could have a direct effect on a

[9] Although causal volatility parts for S-6 and S-8 are identical, they are technically different in the sense that the UND for S-6 requires additional causal volatility restrictions for η_x and η_y. See verification of Table 1 in Appendix 7.

country's sales and output and, if the loss in competitiveness is severe enough, increase expectations of exchange rate devaluation and/or lead to an attack on the country's currency. In these cases, well-defined types of LB or CRB are handled, making it less difficult to handle contagion.

Remark 9 **S6–S8 in an economic or financial context.** These cases refer to any increased market comovement that cannot be explained by the S2–S5 cases. Masson (1998) presented a theory of multiple equilibria, which shows that a crisis in one country can be used as a sunspot for another. The shift from a good to a bad equilibrium is driven by a change in investor expectations rather than real linkages. Theories explaining contagion are based on multiple equilibria, capital market liquidity, investor psychology, and political economy. Valdés (2000) developed a model based on capital market liquidity and argued that a crisis in one country can cause a liquidity shock to market participants. This could force portfolio recomposition and drive a sell-off of certain asset classes, which would lower asset prices in countries not affected by the initial crisis. Mullainathan (2002) focused on investor psychology and argued that investors recall past events imperfectly. A crisis in one country could trigger memories of past crises, which would cause investors to recompute their priors regarding variables such as debt default and assign a higher probability to a bad state. The resulting downward comovement in prices would occur because memories rather than fundamentals are correlated. Drazen (2000) proposed that political economy can drive price comovements, such as during the European devaluations of 1993. For instance, if political pressure drives central bank presidents to maintain an existing exchange rate regime, when one country abandons its regime, the political costs of other countries would be reduced, changing their regime. This effect could generate bunching in the timing of the economic policy shifts. In these cases, relatively more complicated types of LB or CRB are handled; hence, it would be rather difficult to handle contagion.

Identifying contagion using a Bayesian approach

The change in the short-term effect $\beta_{L0} = \frac{\theta_y}{\theta_x}$ (or the MSR) is further checked to determine whether it is the main force behind a break in the long-term effect or correlatedness (LB or CRB). The *Bayesian approach* was used to calculate the related posterior probabilities. Following the definition of contagion, its occurrence can be investigated by calculating the posterior probabilities of $P\,(SB|LB) or P(SB|CRB)$. The posterior probability of $P\,(SB|LB)$ ($\mathrm{P}(SB|CRB)$) is interpreted as the probability that LB (CRB) is caused by SB when LB (CRB) is given or as the ratio of SB to LB (CRB). For S-1, S-2, S-3, and S-5, $P\,(SB|LB) = P(SB|CRB) = 0$, as they belong to NSB. For S-4, $P\,(SB|LB) = P(SB|CRB) = 0$, as it belongs to the NLB and NCRB simultaneously. Thus, either no or contained contagion was reported for S-1 to S-5.

For S-6, $\mathrm{P}\left(\mathrm{SB}|\mathrm{CRB}\right) > 0$ must be calculated, which belongs to SB and CRB. As the correlatedness $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ consists of independent multiplication components of $\frac{\theta_y}{\theta_x}$ and $(1 - \eta_x)$, an additional check is necessary to decide whether CRB is mainly due to the change in $\frac{\theta_y}{\theta_x}$ or in $(1 - \eta_x)$. The posterior probability CRB caused by changes in $\frac{\theta_y}{\theta_x}$ (or SB) is calculated as $\mathrm{P}(SB|CRB) = \frac{P(SB)P(CRB|SB)}{P(CRB)}$. For S-7, $\mathrm{P}\left(\mathrm{SB}|\mathrm{LB}\right) > 0$ must be calculated, which belongs to SB and LB. As the long-term effect $\gamma_{\mathrm{L}} = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ consists of independent multiplication components of $\frac{\theta_y}{\theta_x}$ and $\frac{(1-\eta_x)}{(1-\eta_y)}$, an additional check is required to determine whether LB is mainly due to the change in $\frac{\theta_y}{\theta_x}$ or in $\frac{(1-\eta_x)}{(1-\eta_y)}$. The posterior probability that LB is caused by changes in $\frac{\theta_y}{\theta_x}$ (or SB) is calculated as $\mathrm{P}(SB|LB) = \frac{P(SB)P(LB|SB)}{P(LB)}$. For S-8, it must be further verified that LB or CRB is due to the change in $\frac{\theta_y}{\theta_x}$ (or SB) by calculating the posterior probabilities of $P(SB|LB)\mathrm{and}P(SB|CRB)$.

The present contagion test procedure is summarized as follows:

(i) Assume Eq. 3 for target market Y and source market X,

$$\Delta Y_t = \frac{\theta_y}{\theta_x}\Delta X_t - \left(1-\eta_y\right)\left(Y_{t-1} - \frac{\theta_y(1-\eta_x)}{\theta_x\left(1-\eta_y\right)}X_{t-1}\right) + \varepsilon_{Lt}$$
$$= \beta_{L0}\Delta X_t + \beta_{L1}(Y_{t-1} - \gamma_L X_{t-1}) + \varepsilon_{1t} = \beta_{L0}\Delta X_t + \beta_{L1}Y_{t-1} + \beta_{L2}X_{t-1} + \varepsilon_{Lt}$$

(ii) Establish null hypotheses for coefficients in the $\frac{k}{N+1}$th quantile that correspond to the crisis period across fixed N quantiles: H_0^{SB} versus H_1^{SB}(short-term break), H_0^{LB} versus H_1^{LB}(long-term break), and H_0^{CRB} versus H_1^{CRB} (correlatedness break).

(iii) Apply the Z-test given by Eq. 9 with selected k for $(\beta_{\mathrm{L0}}, \gamma_{\mathrm{L}}, \beta_{\mathrm{L2}})$.

(iv) Using the test results from step (iii) and Table 1, complete the Bayesian test by calculating the posterior probabilities of $P(SB \mid LB)$ and $P(SB \mid CRB)$. If the posterior probability is greater than a specified value θ_0, contagion is declared.

To calculate the posterior probabilities of $P(SB|LB) and P(SB|CRB)$, the p values from step (iii) of the contagion test process were used to estimate the related probabilities. Recalling that the p value is the probability under H_0, to obtain a result equal to or greater than what was observed, p values from testing H_0^{SB}, H_0^{LB} and H_0^{CRB} were used as estimates of $P(SB), P(\mathrm{L}B), \mathrm{and} P(\mathrm{CR}B)$, respectively. In addition, $\hat{\mathrm{P}}(\mathrm{L}B|SB)$ and $\hat{\mathrm{P}}(\mathrm{CR}B|SB)$ could be estimated from the given testing results (Appendix 8, Eqs. 12 and 13).

Empirical results and discussion

Dynamic analysis of the GFC among developed markets

This study examined the dynamic transmission of the GFC to 22 developed equity markets between 2007 and 2009 using the analytical Bayesian approach developed above.

Data

Data from the Thompson Reuters DataStream were used, which provided equity market indices for most countries. Using the U.S. financial market as the origin of the GFC, 22 developed markets were examined (see Appendix 9 for equity market indices by country and DataStream code). These countries were selected based on the developed market lists provided by Dow Jones, the FTSE Group, MSCI, Russell, and S&P. These data providers classify the market status of countries as "developed" or "emerging" based on their economic size, wealth, and the quality, depth, and breadth of their markets. This study selected countries included in all the developed market lists provided by these data providers.[10] The data consisted of the daily prices of the aggregate stock indices. This study used a 2-day moving average return to overcome the issue posed by geographical time differences in the analysis of the U.S. financial contagion to other countries (Forbes and Rigobon 2002). The sample period, spanning approximately 6 years, from January 2004 to October 2009, focused on the GFC (2007–2009). The crisis began in the U.S. subprime mortgage market in 2007 and led to sharp declines in equity markets worldwide, affecting both developed and emerging markets (Aloui et al. 2011; Dungey and Gajurel 2014; Frank and Hesse 2009; Horta et al. 2010; Hwang et al. 2013; Kenourgios et al. 2011; Samarakoon 2011). As the purpose of this study was to examine the spread of the U.S. financial crisis to developed countries during the GFC, this study did not consider data from November 2009 onwards, when Greece revealed that its budget deficit was more than twice the value it previously disclosed, which signaled the beginning of the European sovereign debt crisis. Including data from November 2009 onward may lead to unexpected results owing to the effect of the European sovereign debt crisis. Thus, the sample period was ended in October 2009 to alleviate the possible noise from other financial market crises. Following the crisis related to the bankruptcy of WorldCom in June 2002, no significant financial crisis was reported until the beginning of the subprime crisis in the U.S. Therefore, this sample period, which began in January 2004 and continued through October 2009, covered an extended period of tranquility followed by the GFC. The sample included 1522 daily observations. According to Brière et al. (2012), it is difficult to identify precise dates that correspond to a crisis, and previous studies on financial contagion during the GFC have used slightly different crisis periods. To identify the crisis period, this study selected observation dates included within most crisis periods considered in previous studies on the GFC. This study assumed that the crisis period began on August 1, 2007 and continued until March 31, 2009. Table 2 reports the summary statistics of the aggregate stock market indices' daily returns for developed markets during the crisis and tranquil periods. Compared with the tranquil period, the crisis period had smaller mean returns (− 0.0015 vs. 0.0010) and greater variation (0.0225 vs. 0.0099).

Country-wise contagion analysis of the GFC spread

To explore the dynamic spread of the GFC from U.S. aggregate equity market returns to developed equity market returns, this study utilized the daily returns reported by the DataStream equity index for each country. Each country's equity market index returns were treated as the outcome variable and U.S. equity market returns as a predictor in the SEECM in Eq. 4.[11] To implement the test procedure, it was observed that the crisis period belonged to the 5th quantile (Appendix 10); hence, $k = 1, N = 19$ in the test procedure.

[10] The emerging markets will be considered in a separate work because they seem to have been less affected by the GFC and hence it requires quantile regression on different quantile for each emerging market.

[11] We test for unit roots in each return series using the augmented Dickey–Fuller test (Dickey and Fuller 1979) and identify their stationarity. A series without unit root problems is regarded as stationary. Our result shows that no return series has a unit root at the 1% significance level, thereby satisfying the stationarity assumption. Thus, we apply the SEECM to the stationary case.

Table 2 Descriptive statistics of aggregate stock market indices

	Crisis period (August 1, 2007 to March 31, 2009, $n = 435$)				Tranquil period (January 1, 2004 to October 30, 2009, excluding crisis period, $n = 1087$)			
	Mean	Std. Dev	Maximum	Minimum	Mean	Std. Dev	Maximum	Minimum
Australia	−0.0013	0.0287	0.0874	−0.1477	0.0012	0.0116	0.0603	−0.0514
Austria	−0.0021	0.0218	0.1015	−0.0779	0.0013	0.0101	0.0404	−0.0594
Belgium	−0.0020	0.0198	0.0829	−0.0783	0.0010	0.0078	0.0355	−0.0375
Canada	−0.0008	0.0211	0.0982	−0.0932	0.0007	0.0088	0.0393	−0.0441
Denmark	−0.0014	0.0213	0.1024	−0.1065	0.0011	0.0093	0.0576	−0.0424
Finland	−0.0018	0.0215	0.0925	−0.0762	0.0009	0.0116	0.0645	−0.0882
France	−0.0016	0.0220	0.0993	−0.0988	0.0010	0.0096	0.0623	−0.0430
Germany	−0.0012	0.0209	0.1140	−0.0716	0.0009	0.0104	0.0607	−0.0407
Hong Kong	−0.0008	0.0283	0.1435	−0.1270	0.0011	0.0111	0.0741	−0.0400
Ireland	−0.0028	0.0272	0.0977	−0.1354	0.0010	0.0121	0.0551	−0.0633
Israel	−0.0010	0.0188	0.1028	−0.0708	0.0010	0.0102	0.0506	−0.0454
Italy	−0.0018	0.0239	0.1191	−0.1033	0.0009	0.0103	0.0591	−0.0473
Japan	−0.0016	0.0227	0.1373	−0.0952	0.0006	0.0104	0.0461	−0.0568
Netherlands	−0.0020	0.0212	0.0975	−0.0881	0.0009	0.0088	0.0520	−0.0402
New Zealand	−0.0017	0.0198	0.0970	−0.0833	0.0009	0.0099	0.0382	−0.0453
Norway	−0.0015	0.0339	0.1489	−0.1270	0.0016	0.0162	0.0816	−0.0813
Portugal	−0.0016	0.0174	0.1023	−0.1010	0.0009	0.0064	0.0336	−0.0276
Singapore	−0.0016	0.0187	0.0928	−0.0763	0.0012	0.0091	0.0611	−0.0391
Spain	−0.0014	0.0209	0.1037	−0.0923	0.0010	0.0088	0.0474	−0.0404
Sweden	−0.0014	0.0215	0.0901	−0.0712	0.0011	0.0104	0.0551	−0.0534
Switzerland	−0.0012	0.0173	0.1031	−0.0700	0.0007	0.0072	0.0314	−0.0345
UK	−0.0017	0.0251	0.1254	−0.0987	0.0009	0.0100	0.0697	−0.0447
US	−0.0011	0.0229	0.1158	−0.0903	0.0005	0.0081	0.0381	−0.0428
Average	−0.0015	0.0225	0.1068	−0.0948	0.0010	0.0099	0.0528	−0.0482

This table shows the summary statistics of the aggregate stock market indices' daily returns for the developed markets during the crisis and tranquil periods. The crisis period spans August 1, 2007 to March 31, 2009, and the tranquil period spans January 1, 2004 to October 31, 2009, excluding the crisis period

Next, the SEECM was estimated at 5% increments, from the 5th to 95th quantiles, via quantile regression for 22 developed markets. Given the estimates of the quantile regression parameters across the entire range of quantiles of each country's equity market returns, this study tested whether the crisis period at the 5th quantile experiences short-term (MSR, β_{L0}), long-term (γ_L), or correlatedness (β_{L2}) breaks from the aggregate U.S. equity market owing to excess and significant shocks. Table 3 reports the Z-test statistics with the corresponding p values in parentheses for $\hat{\beta}_{L0}$, $\hat{\gamma}_L$, and $\hat{\beta}_{L2}$ for the 22 developed countries. Their standard errors were computed using the Markov chain marginal bootstrap resampling method, which is robust to data that are not iid (He and Hu 2002). This study posited hypotheses H_0^{SB} versus H_1^{SB} to test β_{L0}, H_0^{LB} versus H_1^{LB} to test γ_L, and H_0^{CRB} versus H_1^{CRB} to test β_{L2} with $k = 1$ and $N = 19$.

According to the S-classifications in Table 1, Figs. 2, 3, 4, 5, 6, 7 and 8 present plots of $\hat{\beta}_{L0}$, $\hat{\gamma}_L$, and $\hat{\beta}_{L2}$ across the entire range of quantiles for the markets. The U.S. equity market returns had positive short- and long-term effects on the selected developed market returns ($\hat{\beta}_{L0} > 0, \hat{\gamma}_L > 0$) and converge to equilibrium ($-1 < \hat{\beta}_{L1} < 0$). Blue, red, and green lines indicate $\hat{\beta}_{L0}$ (short-term, MSR), $\hat{\gamma}_L$(long term), and $\hat{\beta}_{L2}$(correlatedness), respectively. Before discussing the test results by country, it should be mentioned that $\eta_x = \eta_y = \eta_0$ under MIH holds for a specific value of η_0 (say, η_{j0}) for the jth pair of a

Table 3 Results of testing breaks in market structure and aggregate equity contagion

Country/Region	Short-term slope $\left(\hat{\beta}_{L0}\right)$		Long-term slope ($\hat{\gamma}_L$)		Correlatedness slope $\left(\hat{\beta}_{L2}\right)$		S-classification
	Z-statistics (*p* value)	Test result	*Z*-statistics (*p* value)	Test result	*Z*-statistics (*p* value)	Test result	
Austria	0.5696 (0.5689)	NSB	0.9804 (0.3269)	NLB	1.8002 (0.0718)	NCRB	S-1
Finland	−0.3412 (0.7329)	NSB	0.9995 (0.3175)	NLB	−1.7255 (0.0844)	NCRB	S-1
Singapore	−0.3309 (0.7407)	NSB	0.3752 (0.7075)	NLB	−0.8731 (0.3826)	NCRB	S-1
Switzerland	0.9030 (0.3665)	NSB	−0.0959 (0.9236)	NLB	0.5191 (0.6037)	NCRB	S-1
Germany	0.3992 (0.6897)	NSB	0.2844 (0.7761)	NLB	−2.5688 (0.0102)	CRB	S-2
Sweden	1.1001 (0.2713)	NSB	1.1967 (0.2314)	NLB	3.2396 (0.0012)	CRB	S-2
Canada	0.7982 (0.4247)	NSB	1.9949 (0.0461)	LB	1.9332 (0.0532)	NCRB	S-3
Israel	1.0349 (0.3007)	NSB	5.5676 (0.0001)	LB	1.8911 (0.0586)	NCRB	S-3
Norway	1.4517 (0.1466)	NSB	2.7502 (0.0060)	LB	0.2120 (0.8321)	NCRB	S-3
Japan	3.2586 (0.0011)	SB	1.2034 (0.2288)	NLB	−0.1057 (0.9158)	NCRB	S-4
Hong Kong	2.1743 (0.0297)	SB	1.7097 (0.0873)	NLB	2.1155 (0.0344)	CRB	S-6
Ireland	3.2761 (0.0011)	SB	1.2718 (0.2034)	NLB	5.7732 (0.0001)	CRB	S-6*C*
Australia	2.7523 (0.0059)	SB	2.4111 (0.0159)	LB	1.0472 (0.2950)	NCRB	S-7
Belgium	2.5232 (0.0116)	SB	2.6750 (0.0075)	LB	0.6477 (0.5172)	NCRB	S-7*C*
Italy	2.3155 (0.0206)	SB	2.1128 (0.0346)	LB	1.4285 (0.1532)	NCRB	S-7
Netherlands	3.2921 (0.0010)	SB	3.1973 (0.0014)	LB	−1.4982 (0.1341)	NCRB	S-7*C*
UK	4.6282 (0.0001)	SB	2.0424 (0.0411)	LB	1.1758 (0.2397)	NCRB	S-7
Denmark	7.9276 (0.0001)	SB	4.8248 (0.0001)	LB	2.1290 (0.0333)	CRB	S-8*C*
France	3.7543 (0.0002)	SB	2.5161 (0.0119)	LB	1.9690 (0.0489)	CRB	S-8
New Zealand	2.7614 (0.0058)	SB	2.5294 (0.0114)	LB	3.8960 (0.0001)	CRB	S-8*C*
Portugal	4.0425 (0.0001)	SB	3.2373 (0.0012)	LB	3.9412 (0.0001)	CRB	S-8*C*
Spain	3.2326 (0.0012)	SB	5.9617 (0.0001)	LB	3.3640 (0.0008)	CRB	S-8*C*

This table reports the *Z*-test statistics with the corresponding *p* values in parentheses for the short-term (MSR), long-term, and correlatedness parameters at the 5th quantile by equity market in developed countries. "SB," "LB," "CRB," "NSB," "NLB," and "NCRB" denote short-term break, long-term break, correlatedness break, no short-term break, no long-term break, and no correlatedness break, respectively. The test results are based on the 5% significance level. In the last column, additional italicized capital letter *C* is attached to S-classification to denote that the country turns out to have suffered from contagion by Bayesian test

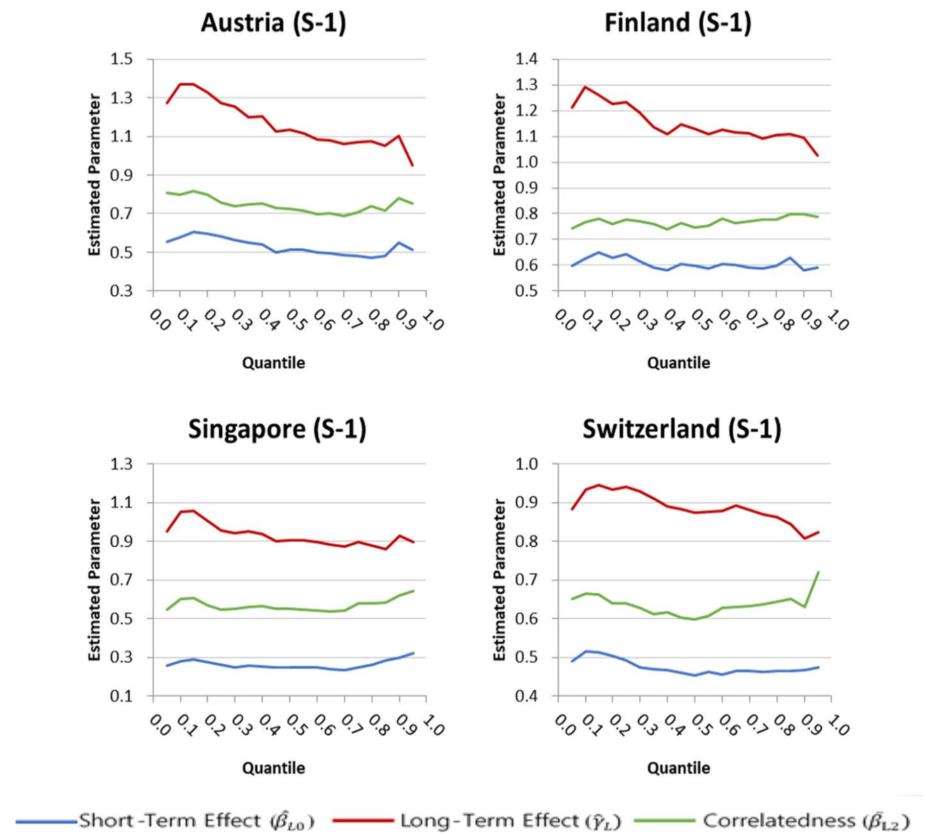

Fig. 2 Estimated parameters by quantile for S-1 markets (no break)

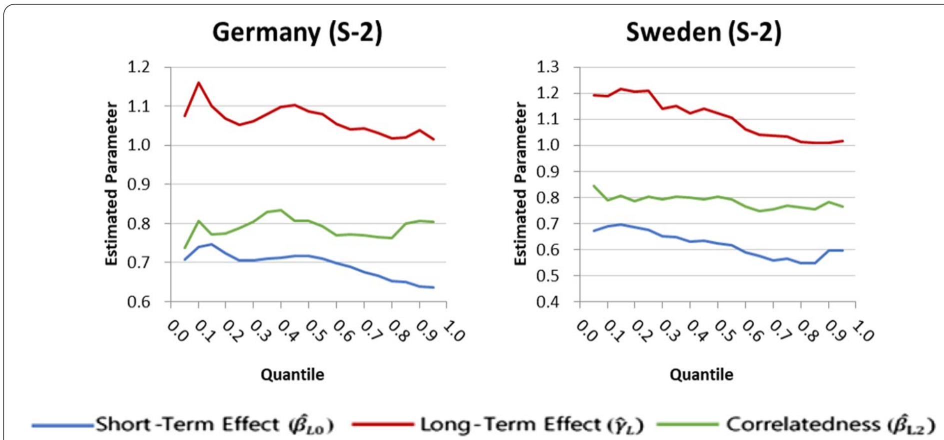

Fig. 3 Estimated parameters by quantile for S-2 markets (CRB)

selected country and the U.S. market. Each market had a unique factor common with the U.S. market; hence, the market-specific risk of the U.S. market (η_x) showed a different value depending on the selected market, namely, $\eta_x = \eta_y = \eta_{j0}$ under the MIH for the *j*th pair of a selected country and the U.S. market.

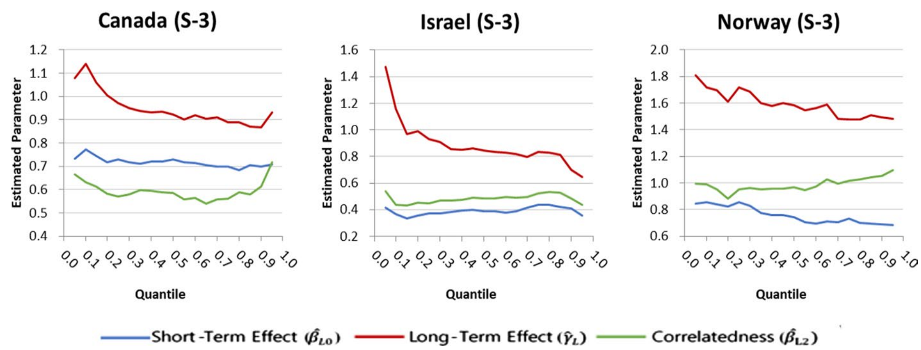

Fig. 4 Estimated parameters by quantile for S-3 markets (LB)

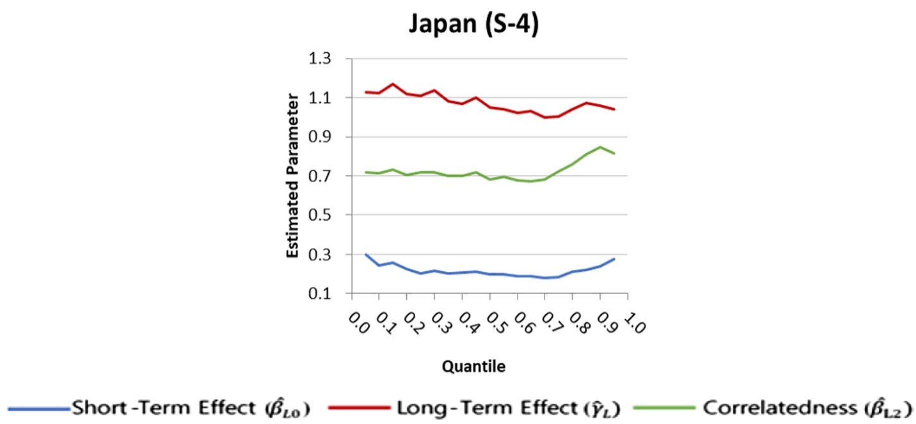

Fig. 5 Estimated parameters by quantile for S-4 markets (SB)

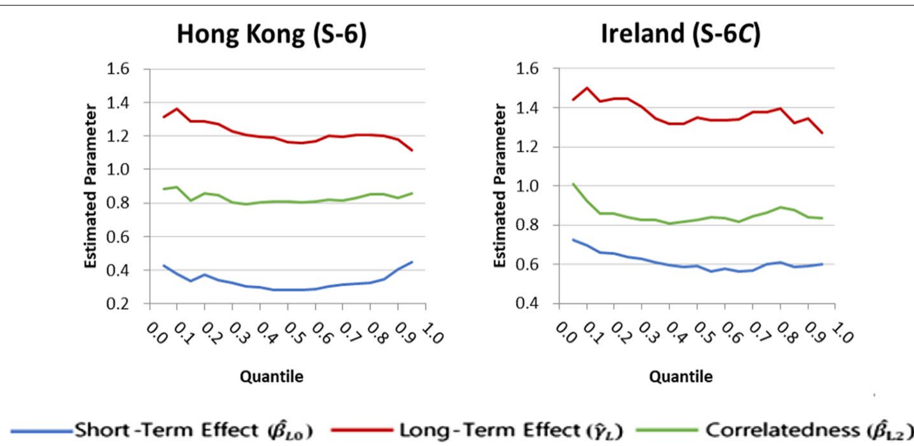

Fig. 6 Estimated parameters by quantile for S-6 markets (SB and CRB)

S-1 markets Austria, Finland, Singapore, and Switzerland maintained established short-and long-term relationships and correlatedness with the U.S. market. According to the S-1 classification rule in Table 1, no contagion was reported. Figure 2 might serve as an example of markets escaping contagion. *Overall, it shows that at the 5th quantile, the*

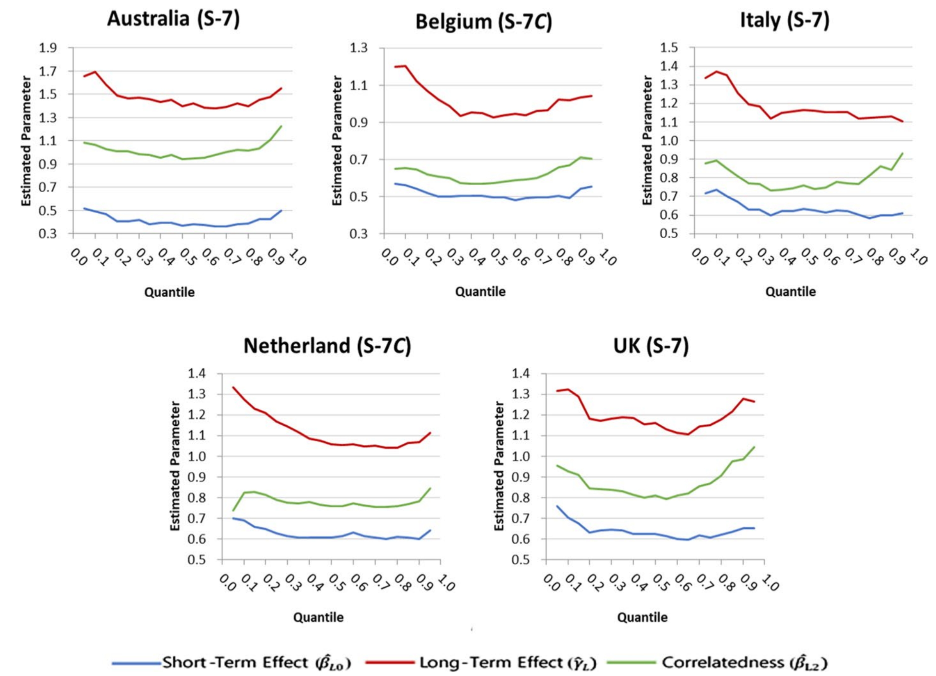

Fig. 7 Estimated parameters by quantile for S-7 markets (SB and LB)

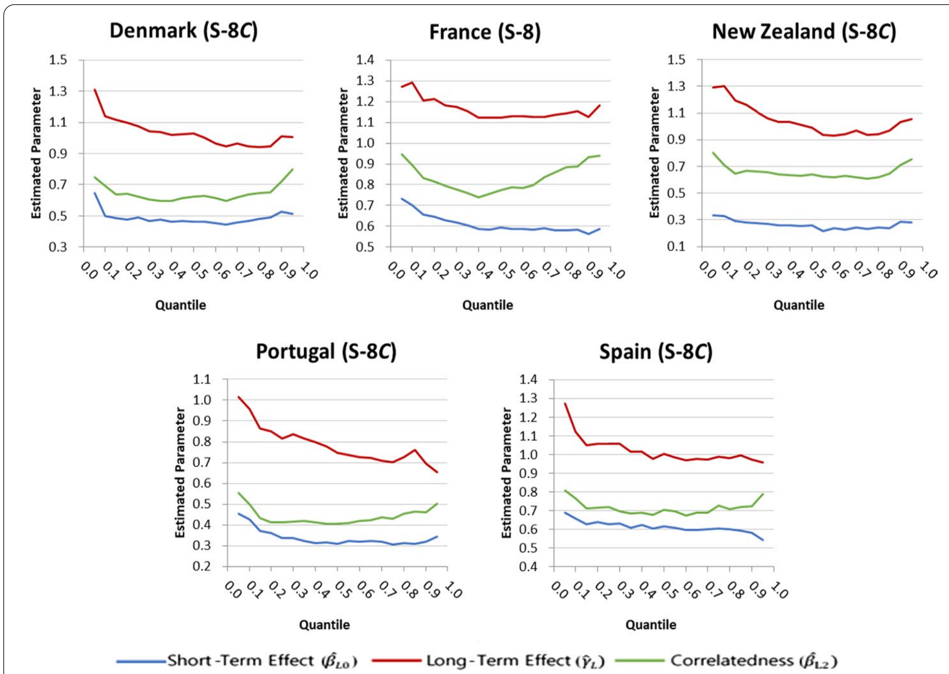

Fig. 8 Estimated parameters by quantile for S-8 countries (all breaks)

long-term effect (red lines), correlatedness (green lines), and short-term effect (blue lines) tended to show a decrease, that is, slowdown in comovement and MSR at the 5th quantile.

S-2 markets The equity market returns for Germany and Sweden can be categorized as S-2. These markets suffered from a correlatedness break without short- or long-term breaks during crisis periods. According to the S-2 classification rule in Table 1, firms escape contagion by effectively containing volatility spillovers. Figure 3 *shows that at the 5th quantile, the long-term (red lines) and short-term effects (blue lines) tended to decrease (i.e., slowdown in comovement and MSR).*

S-3 markets The test results for Canada, Israel, and Norway showed no breaks in the short-term and correlatedness and break in the long-term. According to the S-3 classification rule in Table 1, they escaped contagion by keeping the short-term effect unchanged. As shown in Fig. 4, the long-term effect for these markets at the 5th quantile increases compared with the other periods. *Other than Israel, Fig. 4 shows that at the 5th quantile correlatedness (green lines) and short-term effect (blue lines) tended to show decreases (slowdown in comovement and MSR). For Israel, the short-term effects and correlatedness were as low as 0.4–0.5, although they suffer from a small increase.*

S-4 markets The equity market in Japan was classified as S-4, as it suffered from a short-term break without long-term and correlatedness breaks. As shown in Fig. 5, the shock appeared to increase the short-term effect. According to the S-4 classification rule in Table 1, Japan escapes contagion by maintaining a longer-term equilibrating mechanism. *In Japan, the short-term effect (MSR) was as low as 0.3, although it suffers from a small increase.*

For the S-6, S-7, and S-8 cases below, the posterior probabilities were estimated to determine contagion, as described in Appendix 8. To obtain the posterior probability, this study estimated the related probabilities using the p values given in Table 3. For instance, Hong Kong had a posterior probability of

$$\hat{\mathrm{P}}_0(SB|CRB) = \frac{\hat{\mathrm{P}}(SB)\hat{\mathrm{P}}(CRB|SB)}{\hat{\mathrm{P}}(CRB)} = \frac{0.0297 \times \frac{7}{13}}{0.0344} = 0.4649,$$

where $\hat{\mathrm{P}}(SB)$ and $\hat{\mathrm{P}}(\mathrm{CR}B)$ were p values for testing short-term and correlatedness break, respectively, in Table 3, and $\hat{\mathrm{P}}(CRB|SB) = 7/13$ was estimated from Table 3 (i.e., given the 13 SB markets, seven countries show CRB). Thus, the posterior probability that CRB was caused by SB in Hong Kong was estimated as $\hat{\mathrm{P}}(SB|CRB) = \min\left(\frac{0.0297 \times \frac{7}{13}}{0.0344}, 1\right) = 0.4649$ using Eq. 13 of Appendix 8.

S-6 markets The Hong Kong and Irish markets were classified as S-6. The shock affected the short-term effect and correlatedness, while the long-term effect during the crisis period was maintained. According to the classification rule of S-6 and Eq. 13, the posterior probabilities that CRB was caused by SB for Hong Kong and Ireland were estimated as follows:

$$\text{Hong Kong: } \hat{\mathrm{P}}(SB|CRB) = \min\left(\frac{0.0297 \times \frac{7}{13}}{0.0344}, 1\right) = 0.4649$$

$$\text{Ireland: } \hat{\mathrm{P}}(SB|CRB) = \min\left(\frac{0.0011 \times \frac{7}{13}}{0.0001}, 1\right) = 1$$

For the Hong Kong market, the posterior probability that CRB was caused by SB was relatively low (less than 0.5), indicating that CRB was mainly caused by a change in the market-specific volatility of X (η_x). On the other hand, the Irish market revealed

$\hat{\mathrm{P}}(SB|IB) = 1$; in other words, the CRB was caused by SB. Therefore, according to the S-6 classification rule in Table 1, the Hong Kong market appears to have barely escaped contagion by properly containing the break in the short-term effect, while the Irish market appears to have failed to escape contagion. *Notice from* Fig. 6 *that the short-term effect (MSR) and correlatedness (comovement) increased at the 5th quantile; however, Hong Kong had less correlatedness increase than Ireland.* Clarke and Hardiman (2012) reported that a number of Irish financial institutions faced imminent collapse during the GFC, and the Irish government instigated a €63 billion bank bailout.

S-7 markets The markets in Australia, Belgium, Italy, the Netherlands, and the UK were categorized as S-7. They suffered from short- and long-term breaks but maintained correlatedness with the U.S. market during the crisis period. Using the S-7 classification rule of Table 1 and Eq. 12 of Appendix 8, the posterior probabilities that LB was caused by SB for these countries were calculated as follows:

Australia: $\hat{\mathrm{P}}(SB|LB) = \min\left(\frac{0.0059 \times \frac{10}{13}}{0.0159}, 1\right) = 0.2854.$

Belgium: $\hat{\mathrm{P}}(SB|LB) = \min\left(\frac{0.0116 \times \frac{10}{13}}{0.0075}, 1\right) = 1.$

Italy: $\hat{\mathrm{P}}(SB|LB) = \min\left(\frac{0.0206 \times \frac{10}{13}}{0.0346}, 1\right) = 0.4580.$

Netherlands: $\hat{\mathrm{P}}(SB|LB) = \min\left(\frac{0.0010 \times \frac{10}{13}}{0.0014}, 1\right) = 0.5495.$

UK: $\hat{\mathrm{P}}(SB|LB) = \min\left(\frac{0.0001 \times \frac{10}{13}}{0.0411}, 1\right) = 0.0019.$

$\hat{\mathrm{P}}(LB|SB) = 10/13$ was calculated using Table 3. Indeed, ten countries showed LB, given the 13 SB markets. From the posterior probabilities above, the LB of markets in Belgium and the Netherlands were mainly caused by SB; hence, they appear to have suffered from contagion ($\hat{\mathrm{P}}(SB|LB) > 0.5$). According to the S-7 classification rule in Table 1, Australia, Italy, and the UK escaped contagion by containing a break in the short-term effect. *Notice from* Fig. 7 *that both the long-term (comovement) and short-term effects (MSR) increased at the 5th quantile for Belgium and the Netherlands.*

The National Bank of Belgium (2017) reported that the 2008–2009 Belgian financial crisis was caused by two of Belgian's largest banks, Fortis and Dexia, and was exacerbated by the GFC. Masselink and van den Noord (2009) also stated that the negative effects of the financial crisis became more apparent and economic growth came to a grinding halt in the Netherlands in the second quarter of 2008. Such a large contraction was driven not only by the strong fall in world trade but also by negative developments in domestic demand associated with an adverse wealth shock. Specifically, typical Dutch strengths, such as the country's funded pension system and strong position in world trade, turned out to be vulnerabilities in the wake of the crisis and negatively impacted consumption and investment. This appears to have noticeably increased the long-term effect at 5th quantile for the Netherlands.

S-8 markets Denmark, France, New Zealand, Portugal, and Spain appear to have suffered from all types of breaks (SB, LB, and CRB) during the crisis period. These results correspond to those for S-8. As seen in Fig. 8, the shock directly increased the short- and long-term effects and correlatedness between markets during the crisis period; hence, the posterior probabilities were calculated for these markets using the classification rule of S-8. $\hat{\mathrm{P}}(SB|LB)$ and $\hat{\mathrm{P}}(SB|CRB)$ for these countries were calculated using Eqs. 12 and 13 presented in Appendix 8, as follows:

Denmark:

$$\hat{\mathrm{P}}(SB|LB) = \min\left(\frac{0.0001 \times \frac{10}{13}}{0.0001}, 1\right) = 0.7692,\ \hat{\mathrm{P}}(SB|CRB) = \min\left(\frac{0.0001 \times \frac{7}{13}}{0.0333}, 1\right) = 0.0016$$

France:

$$\hat{\mathrm{P}}(SB|LB) = \min\left(\frac{0.0002 \times \frac{10}{13}}{0.0119}, 1\right) = 0.0129,\ \hat{\mathrm{P}}(SB|CRB) = \min\left(\frac{0.0002 \times \frac{7}{13}}{0.0489}, 1\right) = 0.0022$$

New Zealand:

$$\hat{\mathrm{P}}(SB|LB) = \min\left(\frac{0.0058 \times \frac{10}{13}}{0.0114}, 1\right) = 0.3914,\ \hat{\mathrm{P}}(SB|CRB) = \min\left(\frac{0.0058 \times \frac{7}{13}}{0.0001}, 1\right) = 1$$

Portugal:

$$\hat{\mathrm{P}}(SB|LB) = \min\left(\frac{0.0001 \times \frac{10}{13}}{0.0012}, 1\right) = 0.0641,\ \hat{\mathrm{P}}(SB|CRB) = \min\left(\frac{0.0001 \times \frac{7}{13}}{0.0001}, 1\right) = 0.5385$$

Spain:

$$\hat{\mathrm{P}}(SB|LB) = \min\left(\frac{0.0012 \times \frac{10}{13}}{0.0001}, 1\right) = 1,\ \hat{\mathrm{P}}(SB|CRB) = \min\left(\frac{0.0012 \times \frac{7}{13}}{0.0008}, 1\right) = 0.8077$$

Based on the posterior probabilities above, the LB or CRB of markets in Denmark, New Zealand, Portugal, and Spain were mainly caused by SB; hence, they appear to have suffer from contagion ($\hat{\mathrm{P}}(SB|LB) > 0.5$ or $\hat{\mathrm{P}}(SB|CRB) > 0.5$). By contrast, France was found to have escaped contagion by properly containing a break in the long-term effect. *Notice a sudden long-term effect decrease for France and increases in all three parameters for the other countries at the 5th quantile* (see Fig. 8)*. Note that the short-term effect was MSR and the long-term effect or correlatedness was comovement.*

GFC contagion was reported in Denmark, New Zealand, Portugal, and Spain. The financial crisis in Denmark started in the summer of 2008 with the collapse of the Roskilde Bank, and Denmark experienced a systemic financial crisis in the banking sector until 2010 (Kickert 2013). Several studies have reported significant effects of the GFC on the New Zealand economy (Murphy 2011; Spencer 2012). It has also been reported that the international recession hit Portugal in 2008, and eventually, the country was unable to repay or refine its government debt without the assistance of third parties (Pereira and Wemans 2012). The Great Spanish Depression began in 2008, and Spain was unable to bail out its financial sector; therefore, it had to apply for a €100 billion rescue package under the European Stability Mechanism. Of all European countries, Spain was affected the worst by the GFC (Worldview Report 2009). Although the French financial system also suffered contagion effects from the GFC, it recovered resiliently owing to regulations limiting banks' debt exposure (Xiao 2009).

These empirical results can be compared with those of Bekeart et al. (2014), in which GFC transmission to 415 country-industry equity portfolios was examined using a factor model. The empirical results of equity market contagion for some countries in Bekeart et al. (2014) reported similar findings as the present results. In sum, the empirical contagion studies of developed markets during crisis presented here indicated that *long-term effect, correlatedness, and short-term effect should be decreased to properly contain contagion. During a financial crisis, comovement and MSR determine the behavior of stock returns and residual series. MSR is closely related to liquidity and the possibility of downside risks.*

Other contagion cases in the literatures

The literature contains contagion case studies outside those in developed countries. By employing the volatility impulse response function (VIRF) approach, Jin and An (2016) addressed the extent to which the effects of contagion occurred between the BRIC (Brazil, Russia, India, China) and the U.S. stock markets and demonstrated how the BRIC stock markets were influenced in the context of the 2007–2009 GFC. Their discussion on the extent and influence of contagion could be made more precise by finding the corresponding S-classification category, as presented in Table 3. Dimitriou et al. (2013) examined the U.S. and BRIC markets and found that decoupling was indicated in the early stages of the crisis; however, linkages reemerged after the collapse of Lehman Brothers. Their findings suggested that BRIC might belong to S-4 of Table 1 (SB, NLB, NCRB) and escape contagion. Luchtenberg and Vu (2015) demonstrated that economic fundamentals, such as trade structure, interest rates, inflation rates, industrial production, regional effects, and investors' risk aversion, contributed to international contagion, including emerging markets. Such economic fundamentals were included in the common factor W_t or the market-specific factor $u_{x,t}$ in the present LFM.

Chiang et al. (2007) used the dynamic conditional correlation (DCC) model developed by Engle (2002) to examine the Asian crisis and found that in the early stage of contagion, there was a negative correlation, which was followed by herding behavior that dominated the latter stages of the crisis. A negative correlation at the early stage of contagion indicated NSB, and herding behavior in the later stages of the crisis indicated possible LB or CRB.

Investigating the entire group of emerging markets via the present approach is beyond the scope of this study, as it requires a technically more cautious approach than the developed markets of this study, necessitating regressions at different quantiles from the 5th quantile for each country.

Summary and policy implications

Figure 9 summarizes the test results presented in Table 3. It shows four S-1, five S-7, and five S-8 markets but no S-5 market. The low frequency of S-5 is consistent with Remark 7: long-term and correlatedness breaks hardly occur without a short-term break.

More detailed probabilistic contagion assessments are possible by counting the relative frequencies of contagion in each scenario presented in Fig. 9. The contagion odds

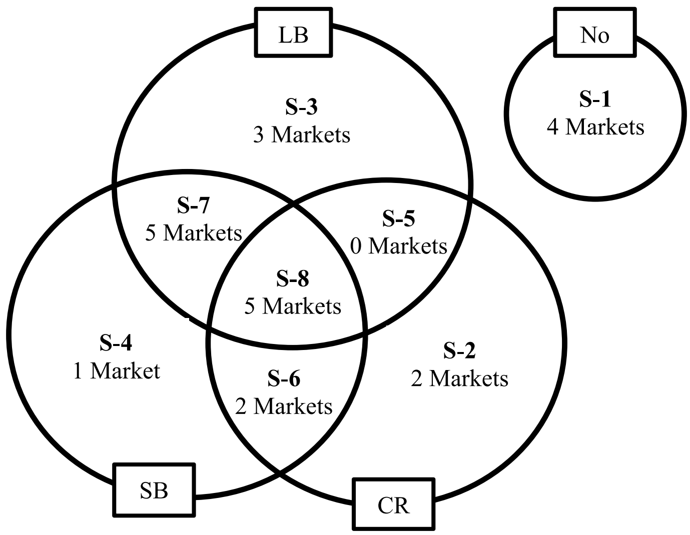

Fig. 9 Classifications of the test results from Table 3. This figure shows the number of developed markets in the S class. The figures in parenthesis denote the number of markets that have suffered contagion

Table 4 The contagion odds

Market experience	Contagion odds
SB (exclusively)	0.54 (0)
LB (exclusively)	0.46 (0)
CRB (exclusively)	0.56 (0)
SB and LB (exclusively)	0.60 (0.4)
SB and CRB (exclusively)	0.70 (0.5)
LB and CRB (exclusively)	0.80 (0)
SB, LB, and CRB (exclusively)	0.80 (0.80)

This table shows the contagion odds for markets that experience SB, LB, CRB, and their combinations. The numbers in parentheses indicate the contagion odds for markets that experience SB, LB, or CRB, and their combinations, exclusively

for markets that experienced SB, LB, or CRB were calculated by estimating the cardinality in relation to each case. For instance, the contagion odds for markets that experienced SB (or SB exclusively) equaled the number of markets with contagion in SB (or SB exclusively) divided by the total number of markets in SB (or SB exclusively). Table 4 reports the contagion odds for the developed markets in various scenarios.

The contagion odds in each cell served as the conditional probability of contagion given market experience. Examining the odds for markets that experience SB, LB, CRB, and their combinations exclusively, indicated within parentheses, revealed that (1) a developed market escaped contagion unless it suffered from SB and LB, SB and CRB, and SB, LB, and CRB exclusively, and (2) a single exclusive break did not cause any contagion regardless of SB, LB, or CRB. These empirical results, presented in Table 4, provided useful implications for policies against market contagion.

When a source market is hit by a significant shock, policymakers of a target market should carefully monitor the short-term effect ($\hat{\beta}_{L0}$, MSR) between their market and source market. Monitoring could be done by checking changes in the target market's relative speed to the source market in the model. When the short-term effect appears to break (probably increase), it must be recognized as a warning sign that contagion is highly possible. Decision-makers should observe the long-term effect ($\hat{\gamma}_L$) and correlatedness ($\hat{\beta}_{L2}$), classify them into one of the S-types, and establish an economic policy that checks the short-term effect (MSR). As a short-term effect $\left(\frac{\theta_y}{\theta_x}, MSR\right)$ is determined by the common factor W_t in Eq. 11, the target market can escape contagion by implementing a proper economic plan that influences MSR directly via θ_y, such as adjusting liquidity or providing more information. Target market Y in S-4, S-6, and S-7 appears to contain breaks in short-term effect properly via θ_y when it escapes contagion with no break in the long-term effect or correlatedness. Finally, S-8 for some target market Y may occasionally be interpreted as a successful recovery from contagion, in which market Y successfully establishes a new sound relationship with market X by restructuring all ties to market X (or via SB, LB, and CRB; see the test results for France S-8).

Conclusions

The literature indicates that contagion tests suffer from various biases, such as the "interdependence, not contagion issue" noted by Forbes and Rigobon (2002, p.2251). Numerous studies have been performed to remove confounding effects or contagion biases from comovement in excess. Solutions have been developed using various models. This study structurally resolved these difficulties by linking LFM and SEECM and applying the Bayesian approach, which determines the identification of the breaks in long-term relationship or correlatedness (or comovement). In addition, the present contagion test defined contagion as a long-term or correlatedness break (or comovement break) mainly caused by a short-term break (or MSR change). Furthermore, this test is easy to implement.

Employing the Bayesian approach, this study successfully addressed various episodes of market contagion. The results revealed the key contagion parameters to be monitored in time-varying situations: contagion odds, short-term effects, long-term effects, and correlatedness. Figures 2, 3, 4, 5, 6, 7 and 8 strongly suggest that the significant increases in these parameters should be handled with caution. The results presented in Fig. 9 and Table 4 help predict contagion odds in time varying situations. In addition, these results should allow policymakers to develop alternative economic safeguards against contagion via MSR, which depends on liquidity or the possibility of downside risk. Future studies should apply this approach to emerging markets, including the BRIC, during the GFC.

Appendix 1. Latent factor model (LFM)

Assume that there are two market stationary returns modeled as follows:

$$X_t = \theta_x W_t + \delta_x u_{x,t},\ Y_t = \theta_y W_t + \delta_y u_{y,t} \quad (10)$$

where W_t represents a common factor with loadings θ_x and θ_y. The common factors W_t is assumed to follow a stochastic process with zero mean and unit variance, i.e.,$W_t \sim (0,1)$. It represents fundamental market volatilities, and its loadings θ_x and θ_y indicate the systematic risk levels of markets X and Y, respectively. Systematic risk refers to the risk due to integrated market factors and affects the both markets. The terms $u_{x,t}$ and $u_{y,t}$ in Eq. 10 are idiosyncratic factors unique to markets X and Y with the loadings δ_x and δ_y, respectively, and are assumed to follow stochastic processes with zero mean and unit variance, i.e.,$u_{x,t} \sim (0,1), u_{y,t} \sim (0,1)$. To complete the specification of the model, all factors are assumed to be independent and as a consequence:

$$\mathrm{E}\left(u_{x,t}u_{y,t}\right) = 0, \mathrm{E}\left(u_{x,t}W_t\right) = 0, \mathrm{E}\left(u_{y,t}W_t\right) = 0.$$

To highlight the interrelationships between the two market returns in Eq. 10, the variances and covariance are represented as follows:

$$\mathrm{Cov}(X_t, Y_t) = \theta_x\theta_y, \quad \mathrm{Var}(X_t) = \theta_x^2 + \delta_x^2, \quad \mathrm{Var}(Y_t) = \theta_y^2 + \delta_y^2.$$

Note that the following equilibrium relationship between X_t and Y_t exists by assuming LFM Eq. 10:

$$Y_t = \frac{\theta_y}{\theta_x}X_t - \frac{\theta_y}{\theta_x}\delta_x u_{x,t} + \delta_y u_{y,t}.$$

Appendix 2. Single equation error correction model (SEECM)

Assuming that X_t and Y_t are two stationary market returns, the SEECM is specified as

$$\Delta Y_t = \alpha + \beta_0\Delta X_t + \beta_1 Y_{t-1} + \beta_2 X_{t-1} + \varepsilon_t = \alpha + \beta_0\Delta X_t + \beta_1(Y_{t-1} - \gamma X_{t-1}) + \varepsilon_t \tag{11}$$

where $\gamma = -\frac{\beta_2}{\beta_1}, \Delta Y_t \equiv Y_t - Y_{t-1}, \Delta X_t \equiv X_t - X_{t-1}$, and ε_t is the independent and identically distributed (iid) error. The part of the equation within the parentheses in Eq. 11 is the error correction mechanism, where $(Y_{t-1} - \gamma X_{t-1}) = 0$ when X and Y are in equilibrium. The coefficient β_0 *specifies the short-term effect* of an increase in X on an increase in Y, while β_1 describes the speed at which X and Y return to equilibrium from a state of disequilibrium. The coefficient γ *specifies the long-term effect* of X on Y. Note that when $-1 < \beta_1 < 0$ ($\beta_1 > 0$), the system converges to equilibrium (diverges from equilibrium). Since β_1 represents the speed of return to equilibrium (and is therefore the scaled inverse of market-specific volatility of market Y, refer to footnote 2 for its details) and $\beta_2 = -\gamma\beta_1$ is the long-term relationship adjusted by the corresponding volatility, β_2 measures the *correlatedness* between markets X and Y. Recall that the correlatedness together with long-term effect is considered as comovement and the correlation analysis alone suffers from bias in testing contagion (Forbes and Rigobon 2002). Forbes and Rigobon (2002) note that cross-market correlations increase during a crisis period without breaks in the long-term relationship between markets. Long-term effect is related to causation, not correlation.

Dynamic specifications by SEECM allow us to estimate and test for both short- and long-term effects that help us better understand contagion dynamics between two markets. When a shock hits a market, we expect an immediate short-term effect of the shock on the other market. If the shock effect continues, it tends to have a long-term effect and may cause deviation from the equilibrium (or breaks in long-term relationship or correlatedness, i.e., breaks in comovement) between two markets. Thus, SEECM Eq. 11 is suited to dynamically model how a shock occurring in one market influences the other market during crisis. Engle and Granger (1987) propose Eq. 11 as a two-step error correction model for two or more cointegrated time series, whereas the SEECM employed herein applies to two stationary time series (see De Boef and Keele (2008) for details). The concepts of error correction, short-term effect, and equilibrium are not unique to cointegrated data. We assume that a long run equilibrium or relationship exists between stationary Y_t and stationary X_t during a tranquil period. Assuming stationary (X_t, Y_t) is technically sound in the sense that their relationship is invariant over time during a tranquil period. Note that when deviation from the long run equilibrium occurs, its recovery process, which involves dynamism between X and Y, is modeled by SEECM.

For a simple illustration of stationary SEECM, let's say we regress the first difference of given one market returns (ΔY_t) on one lag of the market returns (Y_{t-1}), one lag of the other market returns (X_{t-1}), and the first difference of the other market returns (ΔX_t) as noted in Eq. 11. The coefficients are $\beta_0 = 0.3$, $\beta_1 = -0.5$, and $\beta_2 = 1.0$, which implies the long-term effect of X on Y, $\gamma = 2$. If X market return were to increase five points ($\Delta X_t = 5$), market Y return will first increase 1.5 points immediately (5×0.3, the coefficient of β_0). Although this increase in X market return might disturb the equilibrium, the SEECM implies that market Y return and market X return have an equilibrium relationship through the error correction process, that is, a 10-point increase in Y (5×2, the coefficient of γ). However, the increase in market Y return (or re-equilibration) is not immediate, occurring over future time periods at a rate dictated by β_1. The largest portion of the movement in market Y return will occur in the next time period when a 5-point increase in Y (10×0.5, the coefficient of β_1) will occur. In the following time period, $(t+1)$, market Y return will increase 2.5 points, increasing 1.25 points at $t+2$ and 0.63 points in $t+3$ and so on until market Y return has increased 10 points. Thus, market X return has two effects on market Y return: one effect that occurs immediately and another effect that is dispersed across future time periods. This error correction process might be in trouble during crisis periods. For instance, an occurrence of $\beta_1 > 0$ or $\beta_1 < -1$ during crisis periods might lead to extreme divergence of market returns.

Appendix 3. Proof of Proposition 1

The following may easily be derived from Eq. 10.

$$\Delta Y_t = \frac{\theta_y}{\theta_x}\Delta X_t - \left(Y_{t-1} - \frac{\theta_y}{\theta_x}X_{t-1}\right) - \frac{\theta_y}{\theta_x}\delta_x u_{x,t} + \delta_y u_{y,t}$$

Then, it is easy to verify that

$$\begin{aligned}\Delta Y_t &= \frac{\theta_y}{\theta_x}\Delta X_t - \left(1-\eta_y\right)\left(Y_{t-1} - \frac{\theta_y(1-\eta_x)}{\theta_x\left(1-\eta_y\right)}X_{t-1}\right)\\ &\quad - \frac{\theta_y}{\theta_x}\delta_x u_{x,t} + \delta_y u_{y,t} - \eta_y Y_{t-1} + \eta_x\frac{\theta_y}{\theta_x}X_{t-1}\\ &= \frac{\theta_y}{\theta_x}\Delta X_t - \left(1-\eta_y\right)\left(Y_{t-1} - \frac{\theta_y(1-\eta_x)}{\theta_x\left(1-\eta_y\right)}X_{t-1}\right)\\ &\quad - \frac{\theta_y}{\theta_x}\delta_x u_{x,t} + \delta_y u_{y,t} - \eta_y(\theta_y W_{t-1} + \delta_y u_{y,t-1}) + \eta_x\frac{\theta_y}{\theta_x}(\theta_x W_{t-1} + \delta_x u_{x,t-1})\\ &= \frac{\theta_y}{\theta_x}\Delta X_t - \left(1-\eta_y\right)\left(Y_{t-1} - \frac{\theta_y(1-\eta_x)}{\theta_x\left(1-\eta_y\right)}X_{t-1}\right)\\ &\quad - \frac{\theta_y}{\theta_x}\delta_x a_{u,t} + \delta_y b_{u,t} - \eta_y\theta_y W_{t-1} + \eta_x\theta_y W_{t-1}\\ &= \frac{\theta_y}{\theta_x}\Delta X_t - \left(1-\eta_y\right)\left(Y_{t-1} - \frac{\theta_y(1-\eta_x)}{\theta_x\left(1-\eta_y\right)}X_{t-1}\right)\\ &\quad + \delta_y b_{u,t} - \frac{\theta_y}{\theta_x}\delta_x a_{u,t} + (\eta_x - \eta_y)\theta_y W_{t-1}.\end{aligned}$$

Thus $\epsilon_{Lt} = \delta_y b_{u,t} - \frac{\theta_y}{\theta_x}\delta_x a_{u,t} + (\eta_x - \eta_y)\theta_y W_{t-1}$. Now it is easy to see that $Var(\varepsilon_{Lt}) = \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 + \left(\eta_x - \eta_y\right)^2\theta_y^2$ and $Cov(\varepsilon_{Lt}, \varepsilon_{L(t+1)}) = 0$. The proof is complete. Q.E.D

Appendix 4. Two extreme cases of the linked model

If two markets are perfectly integrated into one, then $\delta_x = \delta_y, a_{u,t} = b_{u,t}$ and parameters $\theta_x = \theta_y,$ and $\eta_x = \eta_y = \eta_0$ for any $0 \le \eta_0 < 1$. Then, $\Delta Y_t = \Delta X_t$. In contrast, if two markets are completely independent, then $\theta_y = 0$ or $\theta_x = 0$. When $\theta_y = 0$ ($\theta_x = 0$), we have $\Delta Y_t = -\left(1-\eta_y\right)Y_{t-1} + \delta_y b_{u,t}$ ($\Delta \mathrm{X_t} = -(1-\eta_x)X_{t-1} + \delta_x a_{u,t}$), where ΔY_t ($\Delta \mathrm{X_t}$) is not affected by any factor related to $X_t(Y_t)$. In other words, the markets do not have a common factor. Of course, $X_t = \delta_x u_{x,t}$ and $Y_t = \delta_y u_{y,t}$ when $\theta_x = \theta_y = 0$.

Appendix 5. Simple illustration of model break scenario from MIH during crisis

In Remark 1, we have the following composition of our linked model

$$\begin{aligned}\Delta Y_t &= \frac{\theta_y}{\theta_x}\Delta X_t - \left(1-\eta_y\right)\left(Y_{t-1} - \frac{\theta_y(1-\eta_x)}{\theta_x\left(1-\eta_y\right)}X_{t-1}\right) + \varepsilon_{Lt}\\ &= \beta_{L0}\Delta X_t + \beta_{L1}(Y_{t-1} - \gamma_L X_{t-1}) + \varepsilon_{1t} = \beta_{L0}\Delta X_t + \beta_{L1}Y_{t-1} + \beta_{L2}X_{t-1} + \varepsilon_{Lt}.\end{aligned}$$

where $\beta_{L0} = \frac{\theta_y}{\theta_x}$ (short-term effect), $\beta_{L1} = \eta_y - 1, \gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ (long-term effect), and $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ (correlatedness). Errors, $\varepsilon_{Lt} = \delta_y b_{u,t} - \frac{\theta_y}{\theta_x}\delta_x a_{u,t} + (\eta_x - \eta_y)\theta_y W_{t-1}$, are iid over t, $0 < \eta_y < 1$, and $Var(\varepsilon_{Lt}) = \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 + \left(\eta_x - \eta_y\right)^2\theta_y^2$.

For a simple illustration of model break scenario from MIH during crisis, we assume MIH with $\mathrm{\eta_y} = \eta_x = 0.5$, $\theta_x = 1$, $\theta_y = 0.3$, $\delta_y = 2, \delta_x = 1$. Then $\mathrm{\beta_{M0}} = 0.3$, $\mathrm{\beta_{M1}} = -0.5$, and $\beta_{M2} = 0.15$, which implies the long-term effect of X on Y, $\mathrm{\gamma_M} = 0.3$. Note that $\varepsilon_{Mt} = 2b_{u,t} - 0.3a_{u,t}$ and $Var(\varepsilon_{Mt}) = 4 + (0.3)^2 = 4.09$.

During a stress period, *herding behavior* in both markets might happen which increases η_x, η_y, δ_x and δ_y. We assume that $\eta_x = 0.6, \eta_y = 0.7 (\eta_x \neq \eta_y)$, $\delta_x = 1.1$ and $\delta_y = 2.2$. This immediately brings up volatility increase of market Y via

$$\varepsilon_{Lt} = 2.2 b_{u,t} - 0.3 \times 1.1 a_{u,t} + (0.6 - 0.7) \times 0.3 \times W_{t-1}$$

and

$$Var(\varepsilon_{Lt}) = 4.84 + (0.3 \times 1.1)^2 + (0.1)^2 (0.3)^2 = 4.9498.$$

Recall that the increase of $\eta_y = 0.7$ from 0.5 might influence the error correction process*.* Then the above ε_{Lt} brings up *endogeneity* issue (see Remark 3) which leads to *causality between two financial markets* (from X to Y and from Y to X). These together in turn increase MSR $\left(e.g., \theta_y/\theta_x = 1,\right)$ and hence short-term effect ($\beta_{L0} = 1$). In other words, MSR immediately responds to *herding behavior* during a crisis period. Then the change in short-term effect β_{L0} might bring about break(s) of long term effect $\left(\gamma_L = 4/3 = 1 \times 4/3 = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}\right)$ or correlatedness $\left(\beta_{L2} = 0.4 = 1 \times 0.4 = \frac{\theta_y(1-\eta_x)}{\theta_x}\right)$. This illustrates how an increase of short-term effect or MSR due to herding behavior, the historically well-known event during a stress period in financial or economic context, brings increases of long- term effect as well as correlatedness. In this episode, by controlling η_y close to 0.6 via economic policies, one may contain the break in MSR.

Appendix 6. Proposition 3

Under H_0, with some regular conditions, the quantile slope estimates $\hat{\beta}_{MN} = (\hat{\beta}_{Mj})_{j=1}^{N}$ have an asymptotically multivariate normal distribution $N(\boldsymbol{\beta}_{MN}, n\mathbf{V})$ for $\boldsymbol{\beta}_{MN} = (\beta_j)_{j=1}^{N}$ and a given variance covariance matrix $\mathbf{V}_{3N \times 3N}$ as the number of observations n increases (Refer to (3.7) of Koenker 2005, p.73)*.*

Appendix 7. Verification of Table 1

Below, we assume that Eq. 3 always holds as general case (i.e., Eq. 6 from the null MIH is a special case of Eq. 3, with $\eta_x = \eta_y = \eta_0$). Based on Eq. 3, it is straightforward to observe that $\beta_{L0} = \frac{\theta_y}{\theta_x}, \gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$, and $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$. Acceptance of β_{L0}, γ_L, or β_{L2} in a selected quantile (or acceptance of H_0^{SB}, H_0^{LB}, or H_0^{CRB} equivalently) indicates no change in β_{L0}, γ_L, or β_{L2} (or NSB, NLB, or NCRB) in the quantile, respectively, when a shock hits market X. For each situation, we discuss possible volatility spillover.

(S-1) Acceptance of $\beta_{L0} = \frac{\theta_y}{\theta_x}$ (short-term), $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ (long-term) and $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ (correlatedness) together implies no change in $\frac{\theta_y}{\theta_x}$ and $\eta_x = \eta_y = \eta_0$. In this case, there is no volatility spillover at all by Eq. 8. Thus, S-1 follows. Clearly, S-1 escapes the contagion, and the contagion check is "NO".

(S-2) Acceptance of $\beta_{L0} = \frac{\theta_y}{\theta_x}$ (short-term) and rejection of $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ (correlatedness) implies no change in $\frac{\theta_y}{\theta_x}$ and change of η_x(or η_x'). Acceptance of $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ (long-term) necessarily implies a change of η_y(or η_y') such that $\eta_x' = \eta_y'$. Thus, S-2 follows. In this case, there is no volatility spillover at all by Eq. 8, that is,

$$Var(\varepsilon_{\mathrm{Lt}}) = \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 + \left(\eta_x' - \eta_y'\right)^2\theta_y^2 = \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 = \mathrm{Var}(\varepsilon_{Mt}).$$

Clearly, S-2 escapes contagion, and the contagion check is "NO".

(S-3) Acceptance of $\beta_{L0} = \frac{\theta_y}{\theta_x}$ (short-term) and $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ (correlatedness) implies no change in $\frac{\theta_y}{\theta_x}$ and η_x. The rejection of $\gamma_{\mathrm{L}} = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ (long-term) necessarily implies a change of η_y(say, η_y'). Thus, S-3 follows. In this case, there is volatility spillover

$$Var(\varepsilon_{\mathrm{Lt}}) = \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 + \left(\eta_x - \eta_y'\right)^2\theta_y^2 > \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 = \mathrm{Var}(\varepsilon_{Mt})$$

and

$$\Delta Y_t = \frac{\theta_y}{\theta_x}\Delta X_t - \left(1 - \eta_y'\right)\left(Y_{t-1} - \frac{\theta_y\left(1-\eta_x'\right)}{\theta_x\left(1-\eta_y'\right)}X_{t-1}\right) + \varepsilon_{Lt}.$$

Since a break in short-term effect (MSR) does not occur (or NSB), S-3 escapes the contagion even though there is volatility spillover and the contagion check is "Contained". In this situation, market Y maintains market stability via controlling η_y' (or its own market-specific volatility) properly.

(S-4) Rejection of $\beta_{L0} = \frac{\theta_y}{\theta_x}$ (short-term) and acceptance of $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ (correlatedness) necessarily implies changes in $\frac{\theta_y}{\theta_x}$ and η_x (or $\frac{\theta_y'}{\theta_x'}$ and η_x'). Acceptance of $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ (long-term) and $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ (correlatedness) implies no change in η_y. Thus, S-4 follows. In this case, there might be volatility change by Eq. 8, that is,

$$\mathrm{Var}(\varepsilon_{\mathrm{Lt}}) = \delta_y^2 + \left(\frac{\theta_y'}{\theta_x'}\delta_x\right)^2 + (\eta_x' - \eta_y)^2\theta_y'^2 \neq \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 = \mathrm{Var}(\varepsilon_{Mt})$$

and

$$\Delta Y_t = \frac{\theta_y'}{\theta_x'}\Delta X_t - \left(1 - \eta_y\right)\left(Y_{t-1} - \frac{\theta_y'\left(1-\eta_x'\right)}{\theta_x'\left(1-\eta_y\right)}X_{t-1}\right) + \varepsilon_{Lt}.$$

Since a break in the long-term effect and correlatedness (comovement) does not occur (or NLB and NCRB), S-4 escapes the contagion even though there could be volatility spillover and the contagion check is "Contained". In this situation, market Y maintains market stability via keeping η_y unchanged.

(S-5) Acceptance of $\beta_{L0} = \frac{\theta_y}{\theta_x}$ (short-term) and rejection of $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ (correlatedness) necessarily implies no change in $\frac{\theta_y}{\theta_x}$ and a change in η_x (or η_x'). In this case, rejection of $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ (long-term) do not produce any required situation for η_y (i.e., UND) but $\eta_y' \neq \eta_x'$. Thus, S-5 follows. In this case, volatility spillover clearly occurs by Eq. 8 as follows:

$$\mathrm{Var}(\varepsilon_{\mathrm{Lt}}) = \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 + \left(\eta_x' - \eta_y'\right)^2\theta_y^2 > \delta_y^2 + \left(\frac{\theta_y}{\theta_x}\delta_x\right)^2 = \mathrm{Var}(\varepsilon_{Mt})$$

and

$$\Delta Y_t = \frac{\theta_y}{\theta_x}\Delta X_t - \left(1 - \eta_y'\right)\left(Y_{t-1} - \frac{\theta_y\left(1 - \eta_x'\right)}{\theta_x\left(1 - \eta_y'\right)}X_{t-1}\right) + \varepsilon_{Lt}.$$

Since a break in short-term effect (MSR) does not occur (or NSB), S-5 escapes the contagion even though there is volatility spillover and the contagion check is "Contained". In this situation, market Y maintains market stability by keeping $\frac{\theta_y}{\theta_x}$ (MSR) unchanged.

(S-6) Rejection of $\beta_{L0} = \frac{\theta_y}{\theta_x}$ (short-term) implies a change in $\frac{\theta_y}{\theta_x}$. In this case, acceptance of $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ (long-term) and rejection of $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ (correlatedness) produce some required condition for η_y and η_x (e.g., $\frac{\theta_y'(1-\eta_x')}{\theta_x'\left(1-\eta_y'\right)} = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$) although they are allowed to vary. Thus, (S-6) follows. Since the statuses of η_x and η_y are unknown (i.e., UND) with breaks in short-term effect and correlatedness (or SB and CRB), this case needs a further check to conclude contagion, and the contagion check is "Additional check required". Following the definition of the contagion, contagion occurs if the correlatedness break (or break in $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$) is mainly caused by a change in $\frac{\theta_y}{\theta_x}$(MSR) not by the change in $(1 - \eta_x)$.

(S-7) Rejection of $\beta_{L0} = \frac{\theta_y}{\theta_x}$ (short-term) and acceptance of $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ (correlatedness) necessarily implies a change in $\frac{\theta_y}{\theta_x}$ and η_x (or $\frac{\theta_y'}{\theta_x'}$ and η_x'). In this case, rejection of $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ (long-term) does not produce any required situation for η_y. Thus, S-7 follows. Since the status of market-specific volatility of Y (η_y) is unknown (i.e., UND) with breaks in short- and long-term effects (or SB and LB), this case needs a further check to conclude contagion, and the contagion check is "Additional check required". Following the definition of the contagion, contagion occurs if the long-term break (or break in $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$) is mainly caused by the change in $\frac{\theta_y}{\theta_x}$(MSR) not by the change either in $(1 - \eta_x)$ or $\left(1 - \eta_y\right)$.

(S-8) Rejection of $\beta_{L0} = \frac{\theta_y}{\theta_x}$ (short-term) implies a change in $\frac{\theta_y}{\theta_x}$ (or $\frac{\theta_y'}{\theta_x'}$). It is trivial to see that rejection of $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$ (correlatedness) and rejection of $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ (long-term) do not produce any required situation for η_y and η_x. Thus, S-8 follows. Since the statuses of η_x and η_y are unknown (i.e., UND) with breaks in short- and long-term effects and correlatedness (or SB, LB, and CRB), this case needs a further check to conclude contagion, and the contagion check is "Additional check required". Following the definition of the contagion, contagion occurs if the long-term break or correlatedness break (or break in $\gamma_L = \frac{\theta_y(1-\eta_x)}{\theta_x(1-\eta_y)}$ or break in $\beta_{L2} = \frac{\theta_y(1-\eta_x)}{\theta_x}$) is mainly caused by the change in $\frac{\theta_y}{\theta_x}$ (MSR) not by the change either in $(1 - \eta_x)$ or $\left(1 - \eta_y\right)$.

Appendix 8. Estimating posterior probabilities $\hat{P}(SB|LB)$ and $\hat{P}(SB|CRB)$

For $\hat{P}(LB|SB)$ and $\hat{P}(CRB|SB)$, one may estimate them from given testing results, which test H_0^{SB}, H_0^{LB} and H_0^{CRB} across m different target market Ys given the same source market X. Suppose that tests yield the m_1Y s having experienced SB, m_2 Ys having experienced LB, m_3 Ys having experienced CRB, m_{12}Y s having experienced SB and LB, and m_{13} Ys having experienced SB and CRB. Then, $\hat{P}(LB|SB) = \frac{m_{12}}{m_1}$ and $\hat{P}(CRB|SB) = \frac{m_{13}}{m_1}$. Using those estimates of conditional probabilities we obtain $\hat{P}_0(SB|LB) = \frac{\hat{P}(SB)\hat{P}(LB|SB)}{\hat{P}(LB)}$ and $\hat{P}_0(SB|CRB) = \frac{\hat{P}(SB)\hat{P}(CRB|SB)}{\hat{P}(CRB)}$. If $\hat{P}_0(SB|LB)$ or $\hat{P}_0(SB|CRB)$ exceeds one, the posterior probability is set equal to one. Thus, we have posterior probability estimators as follows:

$$\hat{P}(SB|LB) = min\left(\hat{P}_0(SB|LB), 1\right) \tag{12}$$

$$\hat{P}(SB|CRB) = min\left(\hat{P}_0(SB|CRB), 1\right) \tag{13}$$

Appendix 9. Names of equity market indices by country and Datastream code

Country/Region	Index name	Code
Australia	AUSTRALIA-DS MARKET $—PRICE INDEX	TOTMAU$
Austria	AUSTRIA-DS Market—PRICE INDEX	TOTMKOE
Belgium	BELGIUM-DS Market—PRICE INDEX	TOTMKBG
Canada	S&P/TSX COMPOSITE INDEX—PRICE INDEX	TTOCOMP
Denmark	MSCI DENMARK—PRICE INDEX	MSDNMKL
Finland	OMX HELSINKI (OMXH)—PRICE INDEX	HEXINDX
France	EUROPE-DS Market—PRICE INDEX	TOTMKER
Germany	DAX 30 PERFORMANCE—PRICE INDEX	DAXINDX
Hong Kong	HANG SENG—PRICE INDEX	HNGKNGI
Ireland	IRELAND-DS MARKET $—PRICE INDEX	TOTMIR$
Israel	ISRAEL TA 100—PRICE INDEX	ISTA100
Italy	ITALY-DS MARKET $—PRICE INDEX	TOTMIT$
Japan	TOPIX—PRICE INDEX	TOKYOSE
Netherland	NETHERLAND-DS Market—PRICE INDEX	TOTMKNL
New Zealand	NEW ZEALAND-DS MARKET $—PRICE INDEX	TOTMNZ$
Norway	NORWAY-DS MARKET $—PRICE INDEX	TOTMNW$
Portugal	PORTUGAL PSI ALL-SHARE—PRICE INDEX	POPSIGN
Singapore	SINGAPORE-DS DS-MARKET EX TMT—PRICE INDEX	TOTXTSG
Spain	MADRID SE GENERAL (IGBM)—PRICE INDEX	MADRIDI
Sweden	OMX STOCKHOLM (OMXS)—PRICE INDEX	SWSEALI
Switzerland	SWITZ-DS Market—PRICE INDEX	TOTMKSW
UK	UK-DS MARKET $—PRICE INDEX	TOTMUK$
US	S&P 500 COMPOSITE—PRICE INDEX	S&PCOMP

Appendix 10. 5th or lower percentile daily returns against the sample periods

See Fig. 10.

Fig. 10 Distribution of 5th percentile daily returns by country. This figure shows 5th or lower percentile daily returns by country against the sample period from January 2004 to October 2009. The crisis period from August 2007 to March 2009 is contained within the dotted line box

Abbreviations
MSR: Market speed ratio; GFC: Global financial crisis; LFM: Latent factor model; SEECM: Single equation error correction model; MIH: Market integration hypothesis; SB: Short-term break; LB: Long-term break; CRB: Correlatedness break.

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HSL conducted the formal analysis, validation and wrote original draft. TYK proposed methodology and revised the paper. Both authors read and approved the final manuscript.

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Availability of data and materials
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Declarations

Competing interests
The authors declare that they have no competing interests.

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