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The credit card-augmented Divisia monetary aggregates: an analysis based on recurrence plots and visual boundary recurrence plots

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Abstract

In this paper, we compare the dynamics of the growth rates of the original Divisia monetary aggregates, the credit card-augmented Divisia monetary aggregates, and the credit card-augmented Divisia inside monetary aggregates. This analysis is based on the methods of recurrence plots, recurrence quantification analysis, and visual boundary recurrence plots which are phase space methods designed to depict the underlying dynamics of the system under study. We identify the events that affected Divisia money growth and point out the differences among the different Divisia monetary aggregates based on the recurrence and visual boundary recurrence plots. We argue that the broad Divisia monetary aggregates could be used for monetary policy and business cycle analysis as they are exhibiting less fluctuation compared to the narrow Divisia monetary aggregates. They could positively affect policy decisions regarding environmental choices and sustainability. We also point out the changes in the monetary dynamics locating the 2008 global financial crisis and the Covid-19 pandemic.

Introduction

In a recent paper, Andreadis et al. (2023) investigate the dynamics properties of the Divisia monetary aggregates for the United States using recurrence plots and recurrence quantification analysis. They use monthly data, from January 1967 to December 2020, from the Center for Financial Stability (CFS) in New York City and make comparisons between the narrow and broad Divisia monetary aggregates. They find evidence of nonlinearity but a reservation of a possible chaotic explanation of their origin.

The Divisia monetary aggregates are superior to the simple-sum aggregates constructed by the Federal Reserve in the United States by many other central banks around the world. See, for example, Barnett (1978, 1980), Belongia (1996), Hendrickson (2014), Serletis and Gogas (2014), Belongia and Ireland (2014, 2015, 2016), Ellington (2018), Dai and Serletis (2019), Serletis and Xu (2020, 2021), and Xu and Serletis (2022), among others. Moreover, recently Barnett (2016), Jadidzadeh and Serletis (2019), and Dery and Serletis (2021) argue that we should be using the broad Divisia monetary aggregates, as opposed to the narrow Divisia aggregates.

The Divisia monetary aggregates used by Andreadis et al. (2023) exclude the transaction services provided by credit cards. However, as noted by Liu et al. (2020, pp. 3), "the volume of credit card transaction services has more than doubled in the past decade. Over 80% of American households with credit cards are currently borrowing and paying interest on credit cards." Motivated by these developments in the financial services industry, recently Barnett and Su (2016, 2018, 2019) and Barnett et al. (2023) introduced Divisia monetary aggregates that jointly aggregate the services provided by credit cards and the services provided by monetary assets. The new aggregates are known as the credit card-augmented Divisia and credit card-augmented Divisia inside monetary aggregates. Data on these aggregates are also available from the Center for Financial Stability, but these series start in July 2006.

In a recent paper, Liu et al. (2020) compare the inference ability of the credit card-augmented Divisia monetary aggregates and credit card-augmented Divisia inside monetary aggregates to the conventional Divisia monetary aggregates, at eight levels of monetary aggregation, M1, M2, M2M, MZM, ALL, M3, M4-, and M4. They use cyclical correlation analysis and Granger causality tests and find that both the conventional Divisia monetary aggregates and the credit card-augmented Divisia monetary aggregates are informative in predicting output. They find that in the aftermath of the 2007–2009 financial crisis, the credit card-augmented Divisia measures of money are more informative when predicting real economic activity than the conventional Divisia monetary aggregates. They also find that broad the Divisia monetary aggregates (at the M3, M4-, and M4 levels of aggregation) provide better measures of the flow of monetary services generated in the economy relative to the narrow Divisia monetary aggregates (at the M1, M2, M2M, MZM, and ALL levels of aggregation).

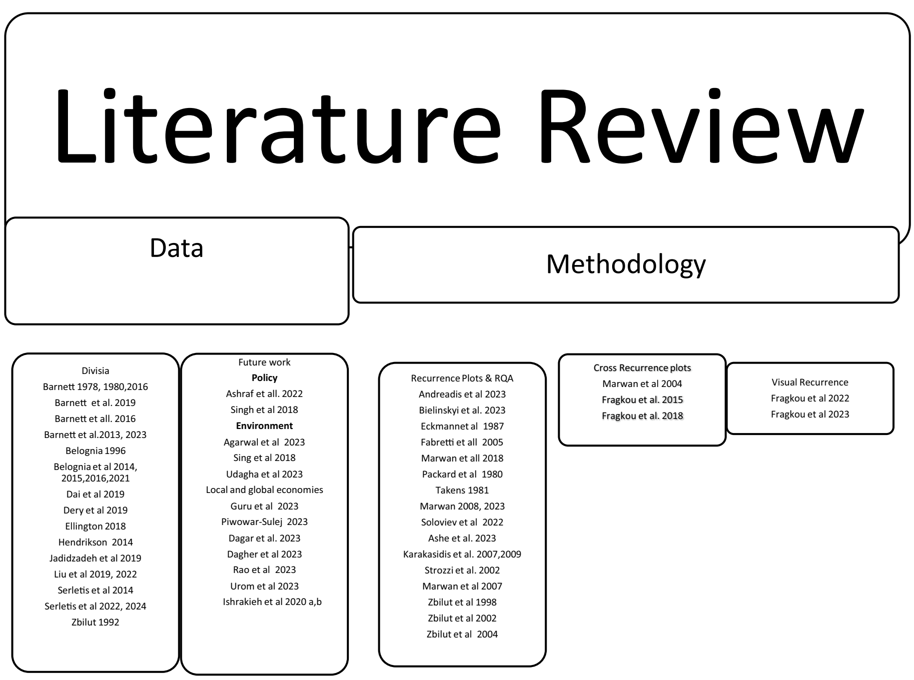
More recently, Serletis and Xu (2024), building on the work by Belongia and Ireland (2021), investigate the role that the credit card-augmented Divisia monetary aggregates could play in monetary policy and business cycle analysis. They use Bayesian methods to estimate a structural VAR under priors that reflect Keynesian channels of monetary policy transmission, but produce posterior distributions for the structural parameters consistent with classical channels. They also find that valuable information is contained in the credit-augmented Divisia monetary aggregates and that they perform even better than the conventional Divisia aggregates.

The research objective is to explain the different behavior across the Divisia monetary aggregates and to point out the years when changes in the monetary dynamics took place. The novelty of our approach is to use tools from dynamical systems theory to provide insights in characterizing the underlying system and its dynamics. Times series analysis includes methods such as autocorrelation, average mutual information, Fourier analysis, and rescaled range analysis, among others. An alternative class of methods based on the system's phase space reconstruction provides the possibility to extract information about system dynamics. Recurrence plots and recurrence quantification analysis constitute such methods (see Eckmann et al. (1987), Zbilut and Webber (1992), Zbilut et al. (2002), Marwan (2008, 2023), Marwan et al. (2007), Bielinskyi et al. (2023), and Andreadis et al. (2023), among others). Using recurrence plots and recurrence quantification analysis, significant qualitative information can be extracted about a dynamical system. Recurrence quantification analysis has been applied to dynamical systems from a wide range of scientific fields such as biology (Zbilut et al. (2004) and Giuliani et al. (2002)), economics and finance (Strozzi et al. (2002), Fabretti et al. (2005), and Soloviev et al. (2022)), transportation systems (Fragkou et al. (2018)), and molecular systems (Karakasidis et al. (2007) and Karakasidis et al. (2009)). A recent extension of recurrence plots, the visual boundary recurrence plots (see Fragkou et al. (2022)), has been applied in the detailed differentiation of the results of recurrence plots applied to magnetohydrodynamic turbulent channel flows (see Fragkou et al. (2023)). In this paper the main objective is to provide support for the use of the credit card-augmented Divisia monetary aggregates by the monetary authorities for developing monetary policies, as they could potential better relate to macroeconomic variations. Moreover, the use of these aggregates could better address environmental, climate change, and sustainability concerns.

In this paper, our methodology is to extend the application of the recurrence plots analysis in Andreadis et al. (2023) to the new credit-card augmented Divisia monetary aggregate and the credit-card augmented Divisia inside monetary aggregates. We also employ the methods of visual boundary recurrence plots, recently introduced by Fragkou et al. (2022), to get a better understanding of the dynamics of the Divisia monetary aggregates. This method is an extension of the method of the recurrence plots and can be used to identify detailed differences in the dynamics as represented in the recurrence plots.

We use monthly data for the United States, from 2006:7 to 2022:12, from the Center for Financial Stability. The sample period is dictated by the availability of the credit-card augmented Divisia monetary aggregate and the credit-card augmented Divisia inside monetary aggregates (which start in July 2006). For a detailed discussion of the data and the methodology regarding the calculation of the Divisia monetary aggregates, see

Table 1 Literature review



Barnett et al. (2013), Barnett et al. (2023), Barnett and Su (2019), and http://www.centerforfinancialstability.org.

In Table 1, we provide a flow chart with the literature review.

We organize the paper as follows. In Sect. "The Different Kinds of Divisia Monetary Aggregates", we briefly provide the theoretical foundations of the original Divisia monetary aggregates, the credit-card augmented Divisia monetary aggregates, and the credit-card augmented inside money Divisia monetary aggregates. In Sect. "The Data", we discuss the data and provide a graphical representation of the growth rates of the narrow and broad Divisia aggregates. In Sect. "Recurrence Plots of the Divisia Monetary Aggregates", we present the recurrence plots of the Divisia money growth rates, and in Sects. "Recurrence Quantification Analysis" and "Visual Boundary Recurrence Plots of the Divisia Monetary Aggregates" we present the visual boundary recurrence plots and the visual boundary recurrence plot rates. The final section concludes.

The different kinds of Divisia monetary aggregates

As noted by Andreadis et al. (2023, pp. 3), "unlike the simple sum monetary aggregates currently in use by most central banks around the world, the Divisia monetary aggregates, invented by Barnett (1978, 1980), do not assume perfect substitutability among their component assets and allow for different user costs of their components." There are three kinds of the Divisia monetary aggregates, currently available by the Center for Financial Stability in New York City, the conventional Divisia monetary aggregates constructed by Barnett (1978, 1980) and the credit card-augmented Divisia monetary aggregates and credit card-augmented Divisia inside monetary aggregates, the latter two constructed more recently by Barnett and Su (2016, 2018, 2019) and Barnett et al. (2023). A further extension to the case of risk with intertemporal nonseparability can be found in Barnett and Liu (2019). As already noted, Liu et al. (2020) and Liu and Serletis (2020) are the first papers to empirically examine the inference ability of the credit card-augmented Divisia monetary aggregates. More recent investigations of the merits of the credit-augmented Divisia monetary aggregates include Liu and Serletis (2022), Serletis and Xu (2024), and Barnett et al. (2023).

They examine the relative information content of the credit card-augmented Divisia monetary aggregates and credit card-augmented Divisia inside monetary aggregates in comparison to conventional Divisia monetary aggregates. They find that both the conventional Divisia monetary aggregates and the credit card-augmented Divisia monetary aggregates have predictive power on output, and that the predictive power of the credit card-augmented Divisia monetary aggregates is stronger.

Also, Liu and Serletis (2020) examine the effects of the variability of money growth on output in the context of a bivariate VARMA, GARCH-in-Mean, asymmetric BEKK model, and find that the volatility of the credit card-augmented Divisia M4 monetary aggregate has a statistically significant negative impact on output from 2006:7 to 2019:3.

However, there is no effect of the traditional Divisia M4 growth volatility on real economic activity. They conclude that "the balance sheet targeting monetary policies after the financial crisis in 2007–2009 should pay more attention on the broad credit card-augmented Divisia M4 aggregate to address economic and financial stability."

In what follows, we briefly review each of these Divisia aggregates.

The conventional Divisia monetary aggregates

The original Divisia monetary aggregates do not include the services provided by credit cards. They aggregate over the different monetary assets treating them as durable goods. In this regard, Barnett (1978) uses the following formula for the user cost (in real terms) of an asset *i*, denoted by π_{it}^{a},

$$\pi_{it}^{a} = \frac{R_t - r_{it}^{a}}{1 + R_t} \tag{1}$$

where R_t is the rate of return on the benchmark asset and r_{it} is the rate of return on asset *i*, where 'a' is used as a superscript to indicate assets.

With data on quantities and user costs of the component monetary assets, the expenditure share of asset *I* can be calculated as follows

$$s_{it} = \frac{\pi_{it}^{a} m_{it}^{a}}{\sum_{i=1}^{I} \pi_{it}^{a} m_{it}^{a}} \tag{2}$$

where m_{it}^{a} is real balances of asset *I* in period *t*. Then the (discrete time) growth rate of a Divisia aggregate is given by

$$dlogM_t = \sum_{i=1}^{I} s_{it} dlog\, m_{it}^{a} \tag{3}$$

The credit card-augmented Divisia monetary aggregates

The conventional Divisia monetary aggregates discussed so far do not include the transaction services of credit cards. It was only recently that Barnett et al. (2023) used economic aggregation and index number theory to jointly aggregate monetary assets and the transaction services of credit cards and constructed what are now known in the literature as the credit card-augmented Divisia monetary aggregates. In doing so, under the assumption of risk neutrality, Barnett et al. (2023) first derives the user cost of credit card transaction services, π_{lt}^{c}, as

$$\pi_{lt}^{c} = \frac{e_{lt} - R_t}{1 + R_t} \tag{4}$$

where R_t is as before (the benchmark interest rate) and e_{lt} is the expected interest on the credit card transaction *l*, where 'c' is used as a superscript to denote credit card transaction services. Then the credit card-augmented Divisia monetary aggregate is

$$dlogM_t = \sum_{i=1}^{I} s_{it} dlog\, m_{it}^{a} + \sum_{l=1}^{L} s_{lt} dlog\, m_{it}^{c} \tag{5}$$

where s_{it} is the user-cost-evaluated expenditure share of monetary asset i and s_{lt} is the user-cost-evaluated expenditure share of credit card transaction *l.*

The credit card-augmented Divisia inside monetary aggregates

More recently, Barnett and Su (2019) introduced the credit card-augmented Divisia inside monetary aggregates, based on the concept of inside money from the supply side of liquidity services, defined as the monetary services produced by financial firms. As Liu et al. (2020, pp. 4) put it, "conditions from the supply side of the liquidity services are at least equally important. In fact, inside money is highly relevant to the transmission mechanism of monetary policy and to the indicator value of the resulting service flows. The Federal Reserve's policy of quantitative easing during the financial crisis, with its goal of affecting the supply of liquid assets, appears to impact the inside money directly. Therefore, it is important to account for the monetary services produced by deposit-based financial firms."

The credit card-augmented Divisia inside monetary aggregates are calculated using the same formula as for the credit card-augmented Divisia monetary aggregates but the following formula for the user cost of monetary assets

$$\pi_{it}^{a} = \frac{(1 - ki)R_t - r_{it}^{a}}{1 + R_t} \tag{6}$$

where k_i is the required reserve ratio on monetary asset i. See Barnett and Su (2019) and Liu et al. (2020) for more details.

The data

We use monthly data for the United States, from 2006:7 to 2022:8 (a total of 195 monthly observations). Although the original Divisia monetary aggregates are available since 1967, the credit card-augmented Divisia monetary aggregates and the credit card-augmented Divisia inside monetary aggregates are only available since July of 2006. For this reason, and for purposes of providing a comparison among the three different kinds of Divisia aggregates, we use monthly data for the United States, from 2006:7 to 2022:8 (a total of 195 observations). The sample period includes the extreme economic events of the 2007–2009 financial crisis and the Covid-19 recession. The data are from the AMFM program of the Center for Financial Stability (CFS).

We focus on the growth rates of the Divisia monetary aggregates and show the growth rates in Fig. 1, in panel (a) for the conventional Divisia aggregates, in panel (b) for the credit card-augmented Divisia monetary aggregates, and in panel (c) for the credit card-augmented Divisia inside monetary aggregates.

As can be seen in Fig. 1, there are significant fluctuations in the Divisia aggregates from the middle of 2008 until the beginning of 2009, during 2011, and from 2014 to 2020. During the Covid-19 period from mid-2020 to mid-2021, the fluctuations are stronger.

We also observe that there are differences in the fluctuations of the conventional Divisia monetary aggregates relative to the credit card-augmented Divisia monetary aggregates and the credit card-augmented Divisia inside monetary aggregates. In particular, the fluctuations of the conventional Divisia aggregates are stronger than those of the credit card-augmented Divisia aggregates and the credit card-augmented Divisia inside

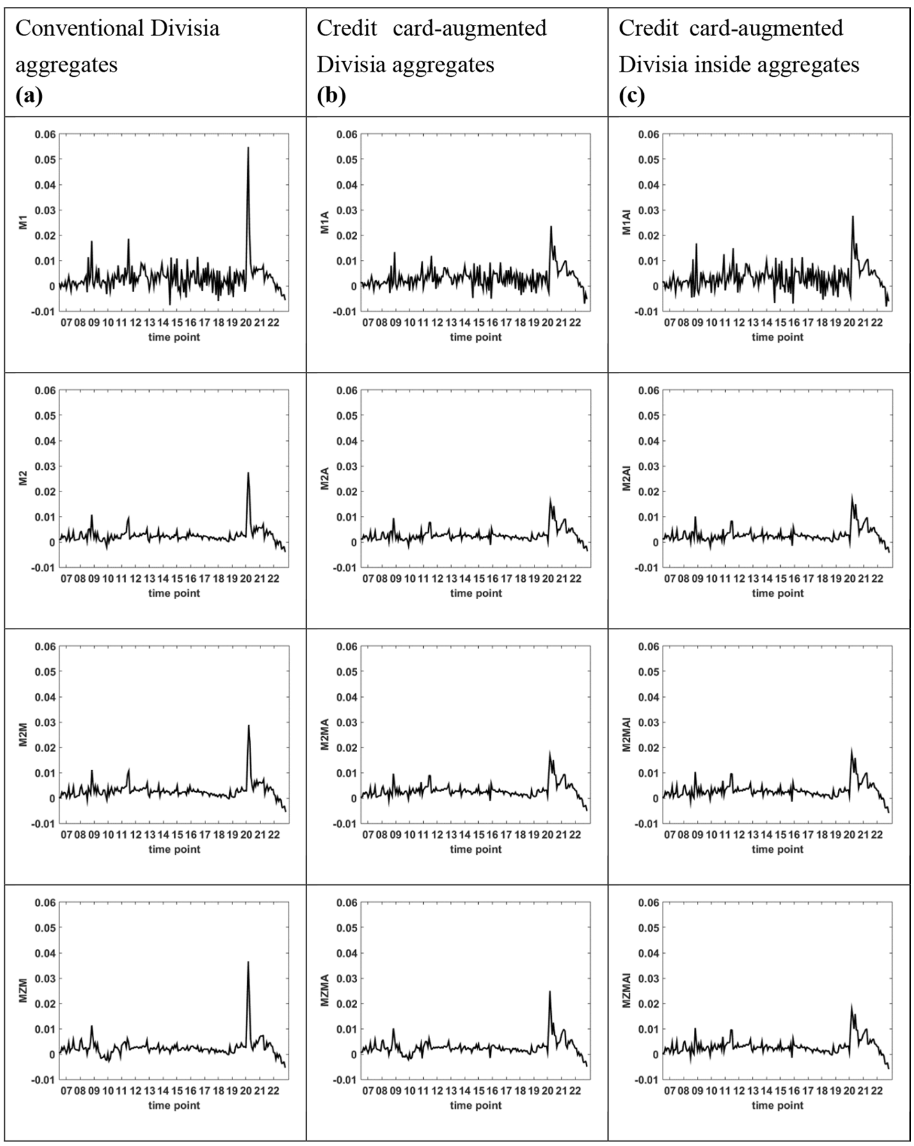


Fig. 1 Growth rates of the **a** original Divisia monetary aggregates, **b** the credit card-augmented Divisia monetary aggregates, and **c** the credit card-augmented Divisia inside monetary aggregates

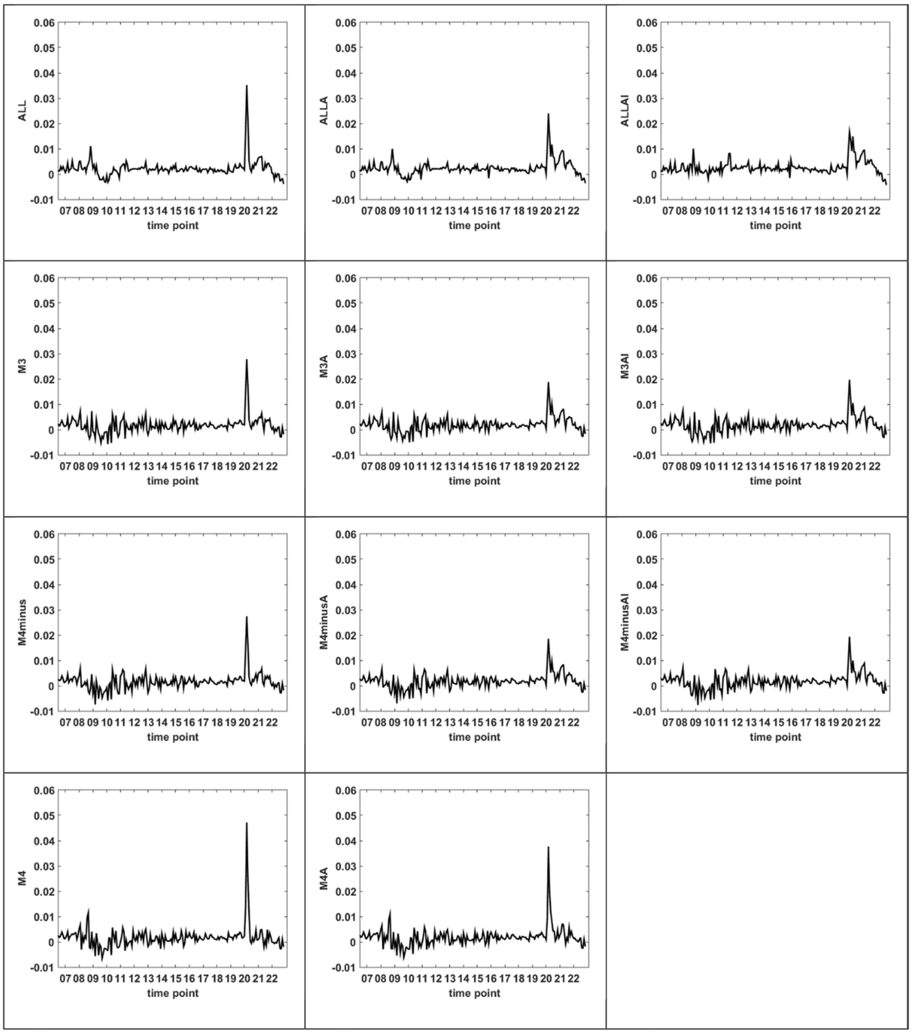


Fig. 1 continued

monetary aggregates at almost all levels of aggregation, M1, M2, M2M, MZM, ALL, M3, M4-, and M4.

Recurrence plots of the Divisia monetary aggregates

Let us briefly review the method of constructing the recurrence plots used by Andreadis et al. (2023) in their study of the conventional Divisia monetary aggregates. Recurrence plots were first introduced by Eckmann et al. (1987) to extract the qualitative characteristics of a dynamical system. It is a graphical tool associated with the trajectory of the underlying system's phase space. Since their introduction, recurrence plots have been applied in various areas. See Marwan et al. (2008) for a review.

Initially, we consider a time series T of K points. It is denoted as $T = \{T_1, T_2, \ldots, T_K\}$, with $T_i \in \mathbb{R}$, and $1 \leq i \leq K$. Firstly, we define an embedding of the time series T, see Packard et al. (1980) and Takens (1981), by fixing an embedding dimension $d_{E,T}$ and a delay time τ_T. Thereafter, we define the reconstructed time series S that possesses L_T points, $L_T = K - \left(d_{E,T} - 1\right)\tau_T$. It is defined as $S = \left\{S_1, S_2, \ldots, S_i, \ldots, S_{L_T}\right\}$, with

$S_i = \left(T_i, T_{i+\tau_T}, \ldots, T_{i+(d_{E,T}-1)\tau_T}\right) \in R^{d_{E,T}}, 1 \leq i \leq L_T$. Then, we choose a value of a threshold, r_T, that is also well known as a cut-off distance, see Zbilut and Webber (1992). After, we consider the recurrence plot Lattice $L_T \times L_T$ with points $(i,j), 1 \leq i,j \leq L_T$. Finally, we define for each value of (i,j) of the recurrence plot Lattice and a positive value of a threshold r_T, the function $f : L \times L \to \{1, 0\}$:

$$f(i,j) = \begin{cases} 1, S_j \in B(S_i, r) & i, j recurrent \\ 0, S_j \notin B(S_i, r) & otherwise \end{cases} \tag{7}$$

where $B(S_i, r)$ denotes the neighbourhood with centre S_i and threshold r_T.

In the recurrence plot Lattice, we plot with black the recurrent points and with white the non-recurrent points. The distribution of those black and white points in the recurrence plots creates various patterns called textures, see Eckmann et al. (1987), that reflect the various dynamical properties of the system. In this work, we are using the Matlab (MATLAB, 2008) software for the plotting of the recurrence plots.

In Table 2 we provide the values of the parameters chosen so that the recurrence matrix could be sparse enough in order to construct the recurrence plots.

In Fig. 2 we present the resulting recurrence plots of the growth rates of the narrow Divisia monetary aggregates. Similar plots for the broad Divisia monetary aggregates are presented in Fig. 3.

The texture of the recurrence plots presents similarities and differences across the three different kinds of the Divisia monetary aggregates and across the different levels of monetary aggregation. We will discuss below for a selection of time intervals the

Table 2 The parameter values for the construction of the recurrence plots of the growth rates of the Divisia aggregates: time delay, τ_T, embedding dimension, $d_{E,T}$, and threshold, r_T

Parameter values for the construction of the Recurrence plots									
	Original Divisia aggregates			Credit card-augmented Divisia aggregates			Credit card-augmented Divisia inside aggregates		
	τ_T	$d_{E,T}$	r_T	τ_T	$d_{E,T}$	r_T	τ_T	$d_{E,T}$	r_T
A. Narrow monetary aggregates									
M1	2	5	0.0061	1	3	0.003	1	4	0.005
M2	4	3	0.0014	3	4	0.0018	3	4	0.0022
M2M	5	5	0.0025	5	6	0.0025	5	4	0.0021
MZM	4	4	0.0019	5	3	0.0014	5	4	0.0022
ALL	4	3	0.0016	4	4	0.0016	3	4	0.0019
B. Broad monetary aggregates									
M3	4	3	0.002	3	3	0.0019	5	4	0.0027
M4-	4	3	0.002	5	3	0.0018	5	4	0.0027
M4	4	3	0,02	5	3	0.0018	NA	NA	NA

significant changes in the following three groups of the Divisia monetary aggregates, group A = M1, group B = (M2, M2M, MZM and ALL), and group C = Broad aggregates (M3, M4-, and M4).

2006:7 to 2008:6

For the Divisia monetary aggregates M1, M1A, and M1AI, we observe dense dark areas, indicating deterministic regions with high concentration of recurrent points. For the Divisia aggregates (M2, M2M, MZM and ALL), (M2A, M2MA, MZMA and ALLA), and (M2AI, M2MAI, MZMAI, and ALLAI) we observe fewer dark areas, indicating deterministic regions with less concentration of recurrent points. For the broad monetary aggregates (M3, M4-, and M4), (M3A, M4A- and M4A), and (M3AI and M4AI-) we observe fewer dark areas; that indicates deterministic regions with less concentration of recurrent points.

2008:8 to 2009:6

For the Divisia aggregates M1, M1A, and M1AI, there are regions with small and large white areas corresponding to the global financial crisis period. For the Divisia aggregates (M2, M2M, MZM, and ALL), (M2A, M2MA, MZMA, and ALLA) and (M2AI, M2MAI, MZMAI, and ALLAI), we observe large white areas corresponding to the global financial crisis. For the broad Divisia aggregates ((M3, M4-, and M4), (M3A, M4A-, and M4A), and (M3AI, M4AI-) we observe large white areas corresponding to the financial crisis.

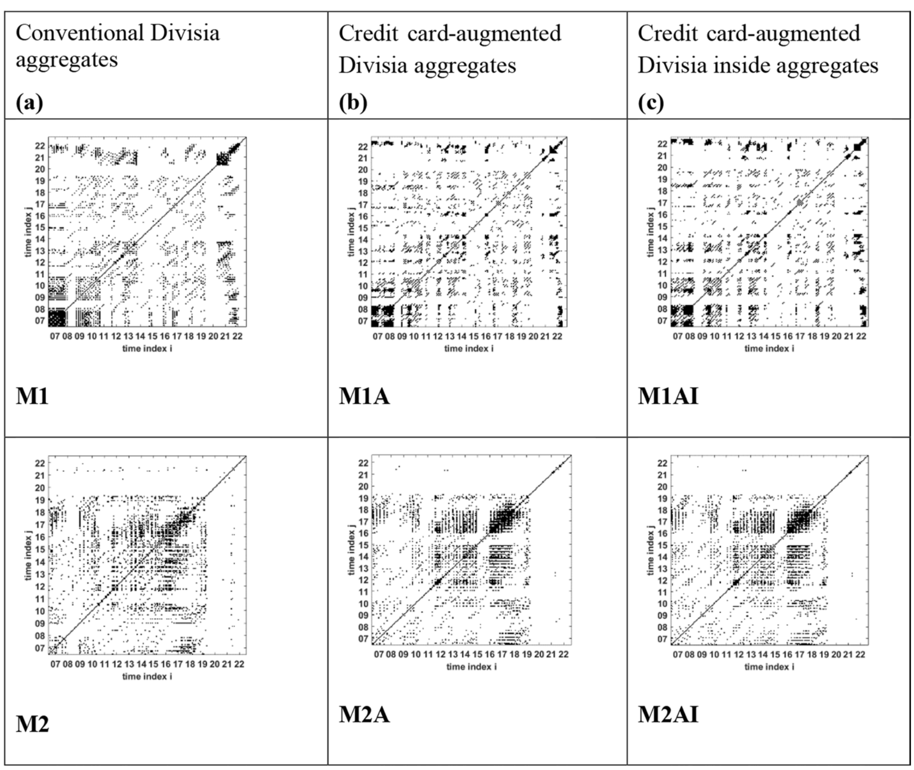


Fig. 2 The recurrence plots of the Narrow monetary aggregates of the **a** original Divisia monetary aggregates, **b** the credit card-augmented Divisia monetary aggregates, and **c** the credit card-augmented Divisia inside monetary aggregates

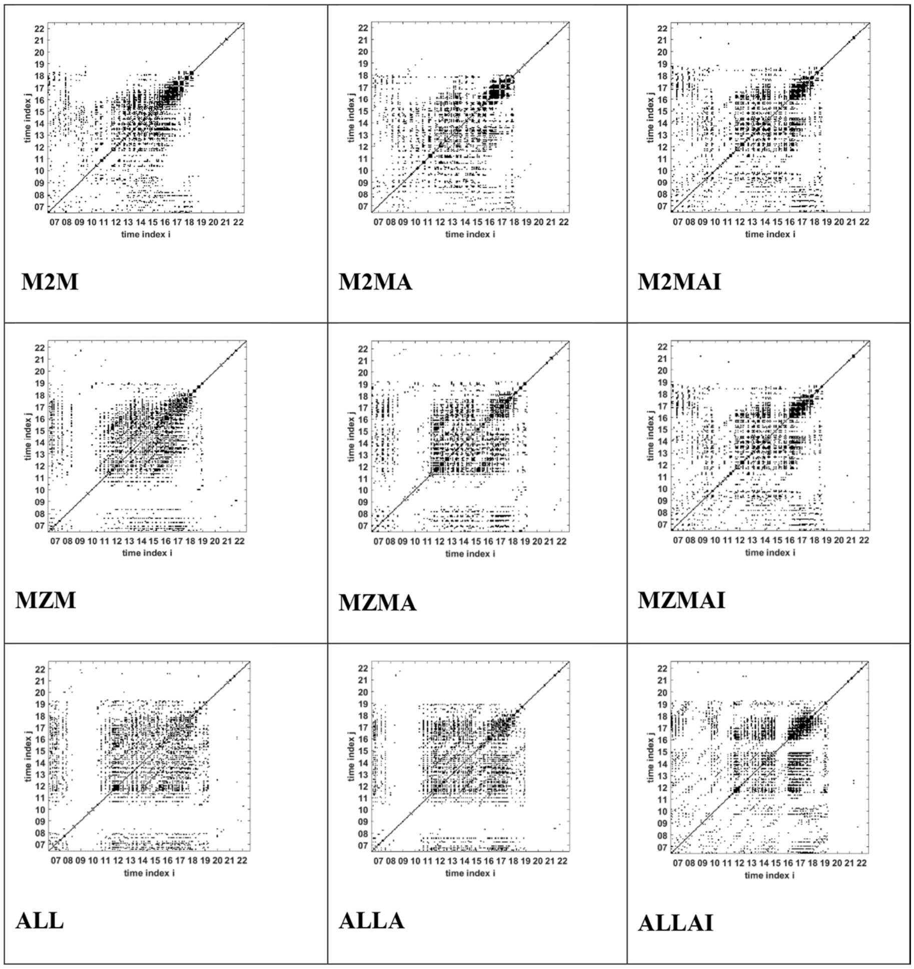


Fig. 2 continued

2009:7 to 2011:6,

In this region the Divisia aggregates M1, M1A, and M1AI present a different behavior. In the recurrence plot of M1 we can see in general small lines parallel to the diagonal with the majority of isolated-like points indicating loss of correlation with the past and large fluctuations while in the recurrence plot of M1A and M1AI we have longer lines that form parallel to the diagonal indicating less fluctuation and more correlation with the past.

For the Divisia aggregates (M2, M2M, MZM, and ALL), (M2A, M2MA, MZMA, and ALLA) and (M2AI, M2MAI, MZMAI, and ALLAI) we observe different behavior. In the recurrence plots of (M2, M2M, MZM, and ALL) we can see in general small lines parallel to the diagonal showing deterministic behavior with some fluctuations. Moreover, a considerable number of isolated points are observed indicating loss of correlation with the past and large fluctuations while in the recurrence plot of (M2A, M2MA, MZMA, and ALLA) and (M2AI, M2MAI, MZMAI, and ALLAI) there are longer lines that form parallel to the diagonal, indicating less fluctuation

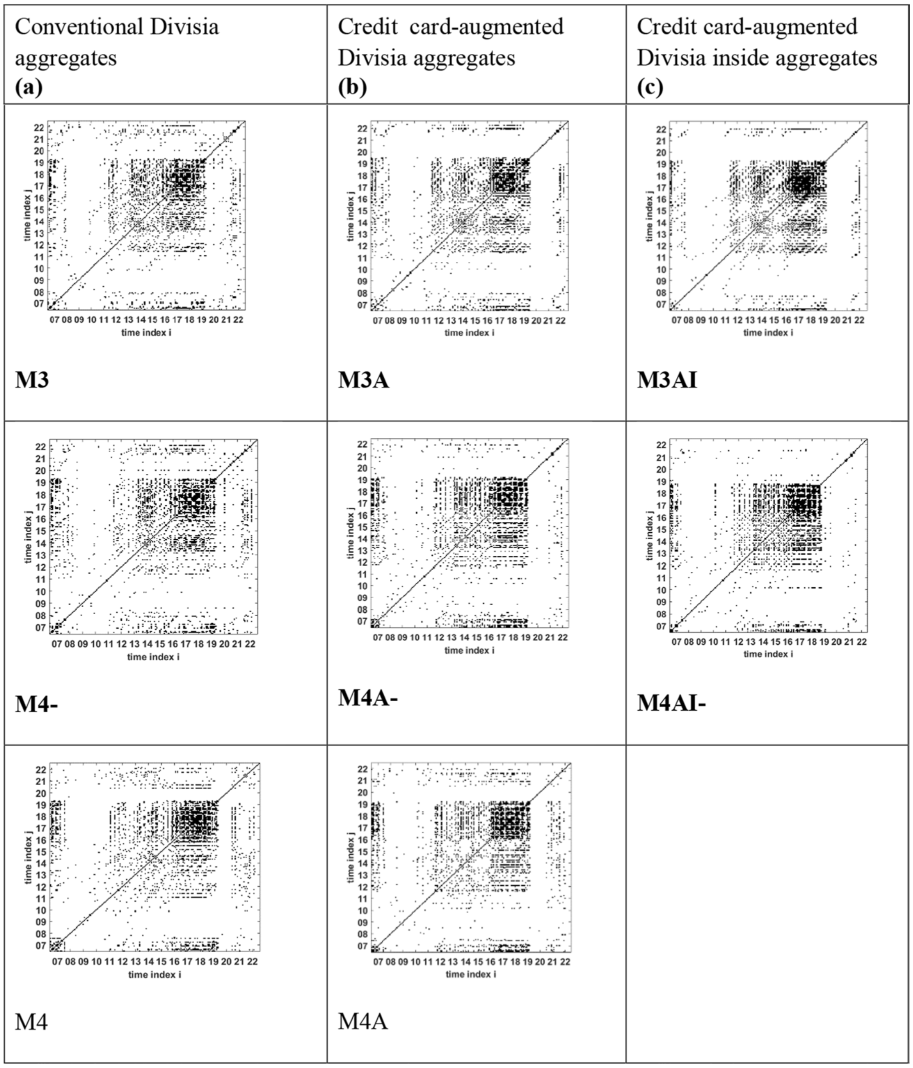


Fig. 3 The recurrence plots of the Broad monetary aggregates of the **a** original Divisia monetary aggregates, **b** the credit card-augmented Divisia monetary aggregates, and **c** the credit card-augmented Divisia inside monetary aggregates

and more correlation with the past. For the broad Divisia aggregates (M3, M4-, and M4), (M3A, M4A-, and M4A) and (M3AI, M4AI-) we observe large white areas which correspond to strong fluctuations that we believe correspond to the financial crisis.

2012:6 to 2014:6

In this region we observe again more isolated lines in the recurrence plot of M1 compared to the recurrence plots of M1A and M1AI, indicating similar effect as in the time region of 2009:m7 to 2011:m6. We also observe more isolated lines in the recurrence plots of the Divisia (M2, M2M, MZM, and ALL) aggregates compared to the recurrence plots of the (M2A, M2MA, MZMA, and ALLA) and (M2AI, M2MAI, MZMAI, and ALLAI) Divisia monetary aggregates, indicating similar effects as in the time region of 2009:m7 to 2011:m6.

For the broad Divisia aggregates (M3, M4-, and M4), (M3A, M4A-, and M4A), and (M3AI and M4AI-), we observe isolated points, depicting loss of correlation. The same isolated points seem to appear during other time periods, indicating uncorrelated states and sudden changes in the dynamics (not necessary abruptly).

2014:7 to 2016:6

During those years, for the Divisia M1, M1A, and M1AI aggregates, we observe isolated points, depicting loss of correlation. The same isolated points seem to appear during other time periods denoting uncorrelated states as a change in dynamics takes place (not necessary abruptly). For the Divisia aggregates (M2, M2M, MZM, and ALL), (M2A, M2MA, MZMA, and ALLA), and (M2AI, M2MAI, MZMAI, and ALLAI) we observe dark areas during this time interval with deterministic regions with high concentration of points. For the broad Divisia aggregates (M3, M4-, and M4), (M3A, M4A-, and M4A), and (M3AI and M4AI-) we observe isolated points, depicting loss of correlation. The same isolated points seem to appear during other time periods denoting uncorrelated states as a change in dynamics takes place (not necessary abruptly).

2019:6 to 2021:6

During these years, we observe large fluctuations in the M1, M1A, and M1AI Divisia aggregates, the (M2, M2M, MZM, and ALL), (M2A, M2MA, MZMA, and ALLA), and (M2AI, M2MAI, MZMAI, and ALLAI) Divisia aggregates, as well as in the (M3, M4-, and M4), (M3A, M4A-, and M4A), and (M3AI and M4AI-) Divisia aggregates. These fluctuations are represented by large white areas in the plots and correspond to the Covid-19 recession.

2021:7 to 2022:12

Over this period, we observe more continuous lines in the recurrence plot of Divisia M1 and more white areas in the recurrence plots of Divisia M1A and Divisia M1AI, indicating larger fluctuations. For the Divisia aggregates (M2, M2M, MZM, and ALL), (M2A, M2MA, MZMA, and ALLA), and (M2AI, M2MAI, MZMAI, and ALLAI), we also observe white areas indicating larger fluctuations. Finally, for the broad Divisia aggregates (M3, M4-, and M4), (M3A, M4A-, and M4A), and (M3AI and M4AI-) we observe large fluctuations which correspond to the post Covid-19 period. Thus, the application of the recurrence plots provides an additional support to the economics of the post Covid 19 period, as it was indicated by Liu and Serletis (2022, pp. 2194) that “the change in payment methods and demand for monetary services during the pandemic could be driven by the Covid-19 circumstances along with the changes in consumption patterns.”

Recurrence quantification analysis

Let us briefly review the method of Recurrence Quantification Analysis, introduced by Zbilut and Webber (1992), as a tool for detecting deterministic or chaotic behavior based on quantifying the presence of patterns in recurrence plots. It was used by Andreadis et al. (2023) in their study of the original Divisia monetary aggregates.

The **Recurrence Rate** (**RR**) or **%recurrence** is the ratio of the number of recurrent points to the total number of points of the plot.

$$RR = \frac{1}{N^2}\sum_{i,j=1}^{N} R_{i,j}R_{i,j} = \begin{cases} 1, & (i,j)\,recurrent \\ 0, & otherwise \end{cases} \tag{8}$$

High Recurrence Rate values indicate recurrent states that could be due to a deterministic behavior; chaotic states are associated with low values of the Recurrence Rate.

The **Determinism** (**DET**) is the ratio of the number of recurrent points forming line segments parallel to the main diagonal to the total number of the points of the plot:

$$DET = \frac{\sum_{l=l_{min}}^{N} l \cdot Pd(l)}{\sum_{i,j}^{N} R_{i,j}} \tag{9}$$

where *Pd(l)* is the histogram of the lengths *l* of the diagonal lines. The presence of diagonal lines indicates the existence of a deterministic structure. DET reveals also information relevant to the duration of a stable interaction. In financial data when time series values have periodicities, the DET parameter has high values otherwise when data fluctuates the DET parameter tends to take very small values. The DET value in combination with the RR value are very sensitive parameters reflecting abrupt changes of the dynamics of the system. A big drop in the RR and DET values is a reliable indicator of an abrupt change in the dynamics of the series.

The **Average Length** (**L**) of the diagonal line segments in the plot without including the main diagonal is defined as

$$L = \frac{\sum_{l=l_{min}}^{N} l \cdot Pd(l)}{\sum_{l=l_{min}}^{N} Pd(l)} \tag{10}$$

This measure refers to the diagonal line length. Small L values reveal processes with stochastic or chaotic behavior while big values indicate a deterministic process. The L value is related to the DET value, as both count the number of recurrent points on the diagonal structures.

The **Laminarity** (**LAM**) is defined as the percentage of recurrence points that are part of vertical lines:

$$LAM = \frac{\sum_{l=l_{min}}^{N} l \cdot Pv(l)}{\sum_{l=1}^{N} lPv(l)} \tag{11}$$

where *Pv(l)* is the histogram of the lengths of the vertical lines included in the recurrence plot. Laminarity is a measure of the appearance of laminar states indicative of intermittency. The lower the LAM value, the more stable the system is.

The **Trapping Time** (**TT**) is defined as the average time that the system has been trapped in the same state. It is defined as the average length of the vertical lines.

$$TT = \frac{\sum_{l=l_{min}}^{N} lPv(l)}{\sum_{l=l_{min}}^{N} Pv(l)} \quad (12)$$

where *Pv(l)* is the histogram of the lengths of the vertical lines. The Trapping Time measure indicates the slowing variation of the values of the time series through time. High TT values indicate a non-fluctuating, slow changing series.

In Fig. 4 we represent the recurrence quantification measures of the (a) original Divisia monetary aggregates, (b) the credit card-augmented Divisia monetary aggregates, and (c) the credit card-augmented Divisia inside monetary aggregates.

The results in Fig. 4 indicate that in the case of the broad credit card-augmented Divisia monetary aggregates the values of all the recurrence quantification measures are higher than in the original Divisia ones. That could be explained due to their constructions to consider credit card transactions. The largest values of the DET for the credit card aggregates MA and MAI further support a possible explanation of a deterministic structure. The largest value of DET appears in the case of M1AI, 0.7842, which combined with the highest value of L and Trapping time shows correlated states. That supports the thesis of Liu et al. (2020) that the credit card Divisia aggregated could be used for forecasting the financial process. In addition, the application of the recurrence quantification analysis, via the largest TT values for the M4A aggregate, supports the work by Liu and Serletis (2020) that the Divisia M4 monetary aggregate has stronger inference ability on economic activity in the United States compared to the traditional Divisia M4 aggregate.

Visual boundary recurrence plots of the Divisia monetary aggregates

In this section we apply the visual boundary recurrence plot (VBRP), introduced in Fragkou et al. (2022), to provide a closer look at the dynamics of the Divisia money growth rates whose recurrence plots were presented in Figs. 3 & 4. In this approach, each point of the recurrence plot is colored based on whether its first neighbors in the vertical and horizontal directions are recurrent or not. This plot is used to study the stability of the textures of the recurrence plots viewing them as geometrical shapes under horizontal and vertical translations. For each point **p** of the recurrence plot we consider the four points in the vertical and horizontal directions around it and we denote this set as **V(p).**

Firstly, we define a partition of the recurrent points **R(T)** as follows:

a) The 'recurrent strong points,' denoted with **RS(T),** are the recurrent points **p** such that the four points of **V(p)** are also recurrent points. The system is staying always trapped around the recurrent strong points in all the vertical and horizontal directions around them.
b) The 'recurrent weak points,' denoted with **RW(T)**, are the recurrent points **p** such that all the four points of **V(p)** are non-recurrent points. The system is not staying trapped around the recurrent weak points in all the vertical and horizontal directions around them.

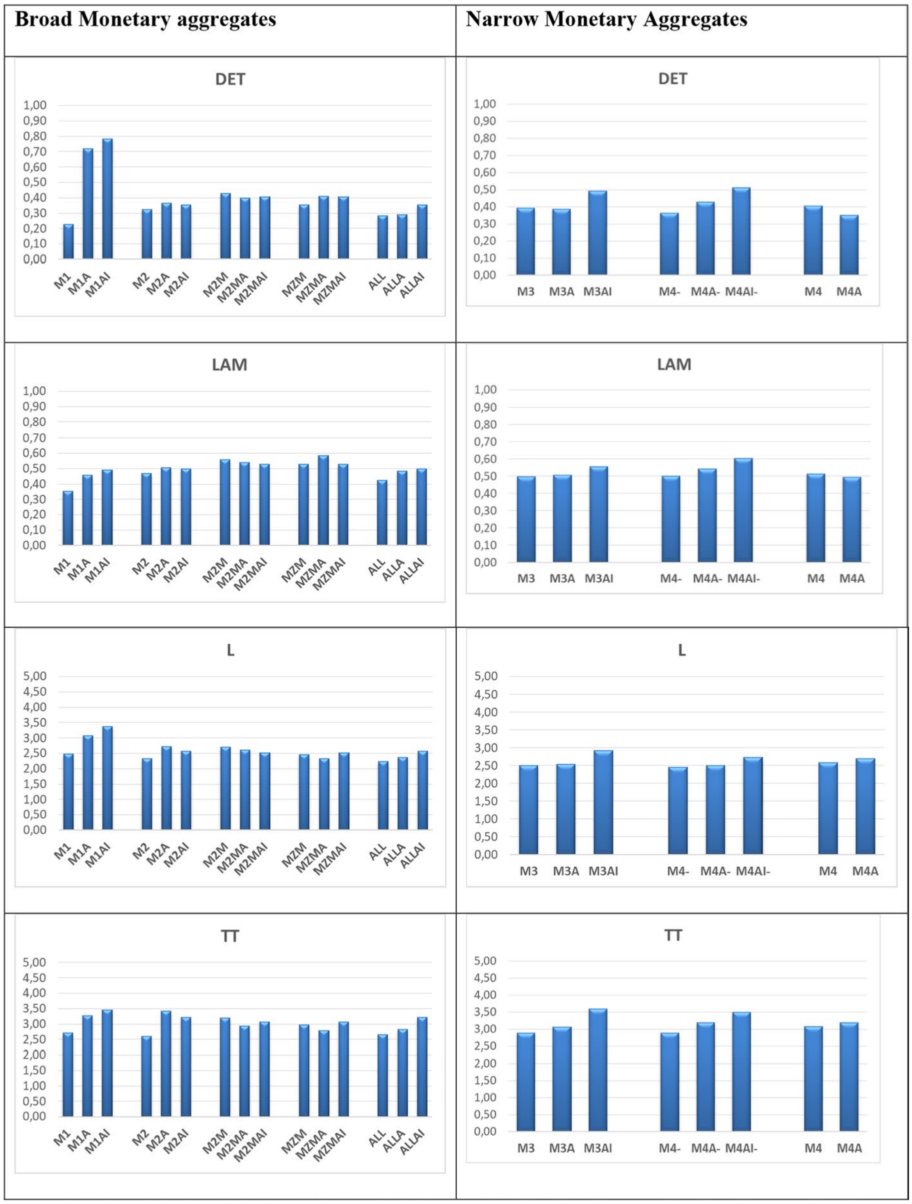


Fig. 4 The recurrence quantification measures of **a** the original Divisia monetary aggregates, **b** the credit card-augmented Divisia monetary aggregates, and **c** the credit card-augmented Divisia inside monetary aggregates

c) The 'recurrent middle points,' denoted with **RM(T)**, is the complement of the recurrent weak points or recurrent strong points in the set of all recurrent points. The system is staying trapped around those points in some of the vertical and horizontal directions around them.

Secondly, we apply the same approach as above to define a partition of the non-recurrent points **NR(T)** as follows:

d) The 'non-recurrent strong points,' denoted with **NRS(T)**, are the non-recurrent points **p** such that all the four points of **V(p)** are also non-recurrent points. The system is staying trapped around the non-recurrent weak points in all of the vertical and horizontal directions around them.
e) The 'non-recurrent weak points,' denoted with **NRW(T)**, are the non-recurrent points **p** such that all the four points of **V(p)** are recurrent points. The system is staying trapped around the non-recurrent weak points in all of the vertical and horizontal directions around them.
f) The 'non-recurrent middle points,' denoted with **NRM(T)**, is the complement of the non-recurrent weak points or non-recurrent strong points in the set of all recurrent points. The system is staying trapped around the non-recurrent middle points in some of the vertical and horizontal directions around them.

This internal partition of the recurrent points of a recurrence plot allows us to create a 'visual boundary recurrence plot,' denoted as VBRP(T), defined as follows:

$$\boldsymbol{VBRP(T)} = \begin{cases} black\left(\mathbf{recurrentstrong}\right), & P \in RS(T) \\ green\left(\mathbf{recurrentweak}\right), & P \in RW(T) \\ \boldsymbol{orange}\left(\mathbf{recurrentmiddle}\right), & P \in \mathbf{RM(T)} \\ \boldsymbol{white}\left(\mathbf{non-recurrentstrong}\right), & P \in \boldsymbol{N}\mathbf{RS(T)} \\ \boldsymbol{brown}\left(\mathbf{non-recurrentweak}\right), & P \in \boldsymbol{N}\mathbf{RW(T)} \\ \boldsymbol{magneta}\left(\mathbf{non-recurrentmiddle}\right), & P \in \boldsymbol{N}\mathbf{RM(T)} \end{cases} \tag{13}$$

In Figs. 5 and 6 we present the visual boundary recurrence plots of the growth rates of the narrow and broad (respectively) Divisia monetary aggregates, in a similar fashion as we did in Figs. 2 and 3.

The visual boundary recurrence plots provide a more detailed analysis and classification of recurrent and non-recurrent points and allow us to better visualize the changes in the dynamics of the system under study. We observe that there are recurrent strong points during the years 2006, 2007, and 2022 which correspond to 'time locked situations,' when the system stays trapped around a given state. During the years 2009–2011 and 2013, we observe the existence of a number of recurrent middle points and non-recurrent middle points which are surrounded by recurrent weak points. In addition, in the 2015–2019 period we observe a significant number of recurrent weak points which indicates less deterministic situations.

In Table 3, we provide a summary of whether in every year there are recurrent strong points, indicated with y, or not, indicated with n. We observe that all the broad monetary aggregates present similar behavior as the Divisia M2, M2A, M2AI, and M2MA aggregates. Other series present also close similarities as it is indicated by the same colors in the corresponding columns of Table 3.

Visual boundary recurrence rates of the Divisia monetary aggregates

Finally, we calculate the 'visual boundary recurrence rates,' introduced in Fragkou et al. (2022), to quantify the various colored areas that are defined in the visual boundary recurrence plots.

r^1: recurrent strong points (RS) rate

r^2: recurrent weak points (RW) rate

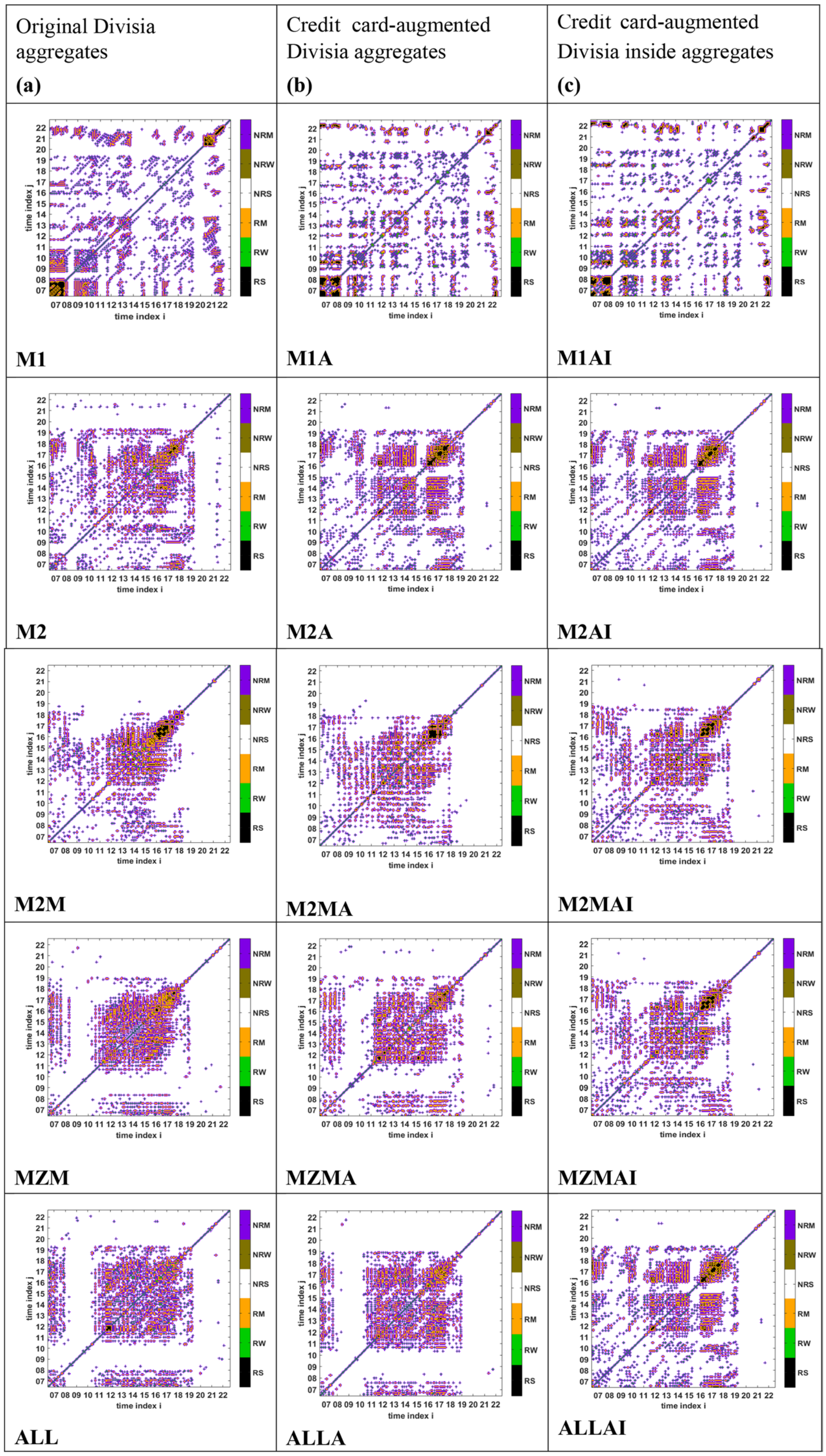


Fig. 5 The visual boundary recurrence plots of the of the Narrow monetary aggregates of the **a** original Divisia monetary aggregates, **b** the credit card-augmented Divisia monetary aggregates, and **c** the credit card-augmented Divisia inside monetary aggregates

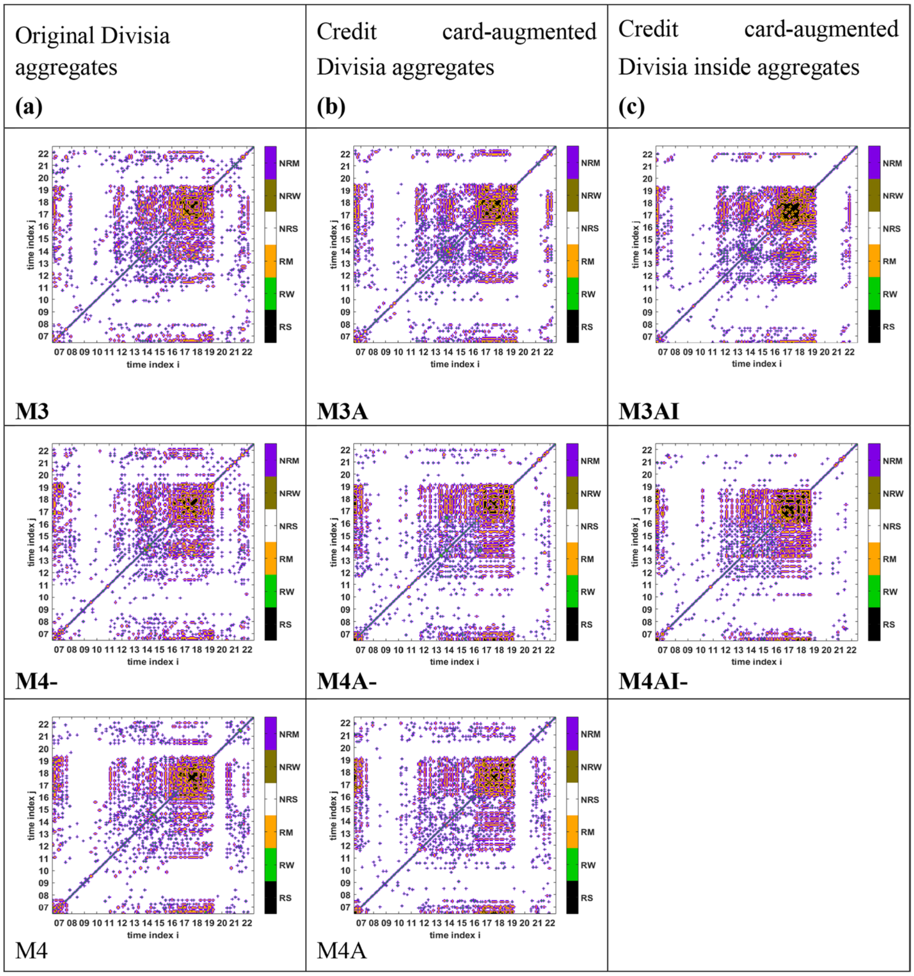


Fig. 6 The visual boundary recurrence plots of the broad monetary aggregates of the **a** original Divisia monetary aggregates, **b** the credit card-augmented Divisia monetary aggregates, and **c** the credit card-augmented Divisia inside monetary aggregates

r^3: recurrent middle points (RM) rate

r^4: non recurrent strong points (NRS) rate

r^5: non recurrent weak points (NRW) rate

r^6: non recurrent middle points (NRM) rate

These rates are defined as the ratio of the corresponding points over the total number of points of the recurrence plot.

As can be seen in Table 4, for all the Divisia monetary aggregates, the boundary recurrence rate r^4 of the non-recurrent strong points is larger than the other visual boundary recurrence rates, suggesting that most of the points are non-recurrent points. In addition, the boundary recurrence rate r^5 of the non-recurrent weak points is the lowest one. The values of the visual boundary recurrence rate r^2 of the recurrent weak points, although small, show the tendency to have isolated recurrent points indicative strong fluctuations. The value of r^3 and r^6 may be indicative of the transitions of the dynamics of the system (see Table 4). For example, the small values of r^1, r^3, and r^6 could be indicating more abrupt transitions.

The following inequalities apply from the value of the visual boundary recurrence rates r^1 of the recurrent strong points:

$$r^1(\mathrm{M1}) < r^1(\mathrm{M1A}) < r^1(\mathrm{M1AI})$$

$$r^1(\mathrm{All}) < r^1(\mathrm{AllA}) < r^1(\mathrm{AllAI})$$

$$r^1(\mathrm{M3}) < r^1(\mathrm{M3A}) < r^1(\mathrm{M3AI})$$

$$r^1(\mathrm{M4-}) < r^1(\mathrm{M4A-}) < r^1(\mathrm{M4AI-})$$

Table 3 Years of the existence of recurrence strong points

A. Narrow monetary aggregates															
Years	M1	M1A	M1AI	M2	M2A	M2AI	M2M	M2MA	M2MAI	MZM	MZMA	MZMAI	ALL	ALLA	ALLAI
2007	y	y	y	n	n	n	n	n	n	n	n	n	n	n	n
2008	y	y	y	n	n	n	n	n	n	n	n	n	n	n	n
2012	n	n	n	n	n	n	n	n	y	y	y	y	y	n	y
2016	n	n	n	n	n	n	y	n	n	y	y	y	n	n	y
2017	n	n	n	y	y	y	y	y	y	n	y	y	n	n	y
2018	n	n	n	y	y	y	y	y	y	y	y	y	n	n	y
2019	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
2021	y	y	y	n	n	n	n	n	n	n	n	n	n	n	n
2022	y	y	y	n	n	n	n	n	n	n	n	n	n	n	n

Table 3 (continued)

B. Broad monetary aggregates								
Years	M3	M3A	M3AI	M4-	M4A-	M4AI-	M4	M4A
2007	n	n	n	n	n	n	n	n
2008	n	n	n	n	n	n	n	n
2012	n	n	n	n	n	n	n	n
2016	n	n	n	n	n	n	n	n
2017	y	y	y	y	y	y	y	y
2018	y	y	y	y	y	y	y	y
2019	y	y	y	y	y	y	y	y
2021	n	n	n	n	n	n	n	n
2022	n	n	n	n	n	n	n	n

$$r^1(M4) < r^1(M4A)$$

It is also worth mentioning that the value of r^5(M1) is greater than the values of r^5(M1A) and r^5(M1AI). That might be because of changes in the fluctuations of these monetary aggregates around the year 2020. Hence, the application of the visual boundary recurrence plots provides an additional support to the arguments of Barnett and Liu (2019), Liu et al. (2020), and Barnett et al. (2023) that much of the policy relevance of the Divisia monetary aggregates could be strengthened by the use of the credit-augmented Divisia aggregates.

Extensions and future work

In future work, it will be of interest to apply the method of Cross Recurrence Plots introduced by (Zbilut et al. (1998), Marwan et al. (2007), Marwan et al. (2008)) to analyze the dependencies between two different system states. This method is used in the analysis of systems of turbulent flows (see Fragkou et al. (2015)), energy markets (see Bielenskyi et al. (2023)), financial and business cycles (see Ashe et al. 2023)), and climate change and geological data (see Marwan et al. (2004)).

Let us briefly recall the analysis. We use the same method to embed the two systems that are represented by their trajectories $\vec{x_i}$, and $\vec{y_j}$ in a d-dimensional phase space, as it is used in the construction of the recurrence plot, see paragraph 4 of this work. If the embedding parameters are not equal then, we then we have to choose the set of parameters with the higher embedding dimension (Marwan et al. 2007). Then, the distances d_{ij} between the reconstructed $\vec{x_i}$, $\vec{y_j}$ vectors are calculated using the Euclidean norm,

Table 4 Visual boundary recurrence rates, $r^{(i)}, i = 1, 2, 3, 4, 5, 6$, for the growth rates of the Divisia monetary aggregates

Visual Boundary Recurrence Rates						
	r^1	r^2	r^3	r^4	r^5	r^6
A. Narrow Divisia monetary aggregates						
M1	0,005496	0,033925	0,041026	0,727258	0,000947	0,191348
M1A	0,009085	0,039124	0,036808	0,740056	0,002082	0,172845
M1AI	0,011356	0,037531	0,037907	0,75057	0,003544	0,159092
M2	0,001664	0,027356	0,053746	0,719912	0,001235	0,196086
M2A	0,005473	0,024764	0,050909	0,740514	0,00085	0,17749
M2AI	0,004358	0,024923	0,051334	0,739877	0,000903	0,178606
M2M	0,006212	0,017822	0,049208	0,775119	0,001736	0,149902
M2MA	0,004964	0,019016	0,046495	0,77436	0,000597	0,154568
M2MAI	0,005154	0,020399	0,051812	0,75179	0,001031	0,169813
MZM	0,002228	0,019115	0,057398	0,757497	0,00102	0,162743
MZMA	0,002816	0,016633	0,060846	0,753428	0,000797	0,16548
MZMAI	0,005154	0,020399	0,051812	0,75179	0,001031	0,169813
ALL	0,000903	0,027793	0,0533	0,723084	0,000903	0,194,016
ALLA	0,001074	0,021343	0,057129	0,746275	0,000752	0,173,427
ALLAI	0,004403	0,025155	0,051867	0,737362	0,000913	0,1803
B. Broad Divisia monetary aggregates						
M3	0,003785	0,027007	0,051813	0,727858	0,000537	0,188998
M3A	0,004437	0,024843	0,053885	0,731321	0,000797	0,184717
M3AI	0,008609	0,023276	0,048889	0,759539	0,001169	0,158518
M4-	0,004161	0,025558	0,05235	0,727268	0,000537	0,190126
M4A-	0,005371	0,022624	0,053277	0,743707	0,00038	0,174642
M4AI-	0,008464	0,018609	0,050456	0,773166	0,000868	0,148438
M4	0,004886	0,026766	0,050579	0,732208	0,000913	0,184649
M4A	0,005181	0,027317	0,049262	0,734484	0,000488	0,183268

Euclidean ($\| \|_2$). Then, by setting off a cut off distance, (threshold ε) a cross recurrence matrix $\mathbf{CR}_{i,j}$ is constructed, as follows

$$CR_{i,j}(\varepsilon) = \Theta(\varepsilon - \| \overrightarrow{x_i} - \overrightarrow{y_j} \|) \tag{14}$$

where $1 \leq i \leq M, 1 \leq j \leq N$, and Θ is the Heaviside function. The elements of the cross recurrence matrix CR$i, j$ are either 0's or 1's. The white areas of the cross-recurrence plot correspond to the 0's and the black areas of the plots correspond to the 1's. As such, the cross-recurrence matrix is very similar to the recurrence matrix and both systems are represented in the same phase space. The cross-recurrence plot searches for those periods of time when a state of the first system recurs to one of the other systems.

It will be also interesting in future works to apply these tools in time series of other countries, such as for example India (see Guru et al. (2023), Rao et al. (2023), Agarwal et al. (2023), Singh et al. (2018)), Poland (see Piwowar-Sulej et al. (2023)), Pakistan (see Dagar et al. (2023)), Lebanon (see Ishrakieh et al. (2020a, b), South Africa (see Udeagha et al. (2023)), and global economies (see Dagher and Hasanov (2023) and Urom et al. (2023)).

Policy suggestions

The Divisia monetary aggregates that we used are provided by the Center for Financial Stability in New York City and through Bloomberg terminals. They can be used to address monetary policy and business cycle issues.

Many central banks around the world also produce Divisia monetary data. They are the European Central Bank, the Bank of Israel, the Bank of Japan, the National Bank of Poland, the Bank of England, the International Monetary Fund, and the Federal Reserve Bank of St. Louis. In fact, as an indication of the level of international acceptance of the Divisia approach to monetary aggregation, see the International Monetary Fund's official data document, *Monetary and Financial Statistics: Compilation Guide*, 2008, pp. 183–184. See also the online library at http://www.centerforfinancialstability.org/amfm.php linking to Divisia monetary aggregates data and studies for over 40 countries throughout the world, including Australia, Canada, China, India, Japan, and the United Kingdom, among others.

In order to address climate change, environmental, and sustainability issues, monetary authorities could also include green bonds in the construction of Divisia monetary aggregates, as recently suggested by Binner et al. (2023). They identify groups of assets that form monetary aggregates composed of monetary assets and green bonds and include green bonds into the construction of Divisia as well as and simple sum monetary aggregates for the United States. They then test the direct effects of their green simple sum and Divisia monetary aggregates on aggregate demand in the United States and find that the green-augmented monetary aggregates provide additional information for aggregate demand.

Conclusion

In this work, we provide an investigation and explanation of the different behaviors among the CFS Divisia monetary aggregates and point out the years when those changes take place. The application of the recurrence plots and recurrence quantification analysis methodology provides useful results and explains changes in the dynamics of the Divisia monetary aggregates. We point out the changes in the dynamics locating the 2008 global financial crisis and the Covid-19 pandemic.

In addition, we also apply the visual boundary recurrence plot methodology, providing detailed information about the dynamics of the Divisia monetary aggregates. Indeed, the visual boundary recurrence rates point out the differences among the Divisia monetary aggregates around the outbreak of the Covid-19 crisis in 2020. The Covid-19 pandemic is especially relevant to the comparison of the different monetary aggregates, because of the relevance of credit and the credit crisis during that period and its aftermath. The credit card-augmented Divisia aggregates are especially informative during this period and can provide important insights.

The behavior of the credit card-augmented Divisia monetary aggregates and the credit card-augmented Divisia inside monetary aggregates in general is different from that of the conventional Divisa monetary aggregates at narrow levels of monetary aggregation, but not at broad levels of aggregation. The Divisia M1 aggregate presents a distinctly different behavior than the other Divisia aggregates; it captures a disruption in 2008–2009 while all the others Divisia aggregates capture a disruption in 2009–2011.

In the period from 2016 to 2019 there are less fluctuations in all of the Divisia monetary aggregates which shows more clearly in the visual boundary recurrence plots with large regions with strong and medium recurrent points. This study provides further support for the results in Liu et al. (2020) that the broad Divisia monetary aggregates, exhibiting less fluctuation than the narrow Divisia monetary aggregates, could be used for monetary policy and business cycle analysis and as part of financial monetary policies (see Ashraf et al. (2022) and Singh et al. (2018)). In particular, the new CFS Divisia monetary aggregates can and should play an important role, either as intermediate targets or indicator variables, for the conduct of monetary policy, in addition to that of the short-term nominal interest rate.

In this regard it should also be noted that recently Isakin and Serletis (2023), motivated by the significant financial innovation and growth of the shadow banking system and the development of digital currencies in recent years, augment each of the CFS original Divisia monetary aggregates with unobserved assets and construct a new set of Divisia monetary aggregates. They argue that the augmented (with unobserved assets) Divisia monetary aggregates are more appropriate measures of money in terms of capturing the relationship between velocity and the opportunity cost of holding money.

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Competing interests
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